PROCEEDINGS

of the

American Society

of

Civil Engineers

(INSTITUTED 1852)

PAPERS AND DISCUSSIONS

VOL. 63

JANUARY TO DECEMBER, 1937

LANCASTER, PA.
PUBLISHED BY THE SOCIETY

1937

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Water

PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 63 JANUARY, 1937 NO. 1

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except June and July, at 99–129 North Broadway, Albany, N. Y., by the American Society of Civil Engineers, Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, December 28, 1931, at the Post Office at Albany, N. Y., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy.

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NOTE.—The closing dates herein published, are final except when names of prospective discussers are registered for special extension of time.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

GRAPHICAL DISTRIBUTION OF VERTICAL PRESSURE BENEATH FOUNDATIONS

By Donald M. Burmister,[1] Assoc. M. Am. Soc. C. E.

Synopsis

The most important part of the analysis of foundations deals with the settlement of structures due to the consolidation of compressible strata of clay and fine saturated silt located at some depth beneath the structure. This involves the practical application of the theory of elasticity in the determination of the state of stress in the underground. In the present state of knowledge the Boussinesq equation for the pressure at a point within the soil mass, due to a point load concentrated at the surface, is the logical basis for this determination. The practical side of the solution of such problems requires a simplification of procedure, in order to make the valuable method of analysis more available. A method is suggested which makes use of the ideas of the influence line and of graphical integration.

A study of the discussions of foundation problems appearing in recent years makes it seem reasonable to divide them into two general classes. The first class deals with near-surface phenomena, involving the behavior of the soil in the "disturbed zone" close to the individual footings. Bearing capacity is developed by an almost immediate compacting and settling of the soil to a stable condition with some lateral displacement, provided that a certain quite definite loading is not exceeded. The soil may have any moisture content from the dry state to saturation. The second class deals especially with the settlement, over a long period of time, of a structure underlaid by highly compressible saturated layers. The settlement due to the surface phenomena may be negligible compared with the latter.

The essential requisites for such analyses are the determination of the physical properties of the different soil strata and of their behavior under load by means of physical tests, and the determination of a fairly complete picture of the distribution of pressure in the underground due to foundation

Note.—Discussion on this paper will be closed in April, 1937, *Proceedings*.

[1] Instr., Civ. Eng., Columbia Univ., New York, N. Y.

loads. The theoretical aspects of the stress conditions for the surface problems, the points of agreement and disagreement, and the results of investigations have been discussed very completely by Messrs. J. A. Moyer, M. L. Enger, A. T. Goldbeck, W. S. Housel, J. H. Griffith, and A. E. Cummings, Members, Am. Soc. C. E., Dr. F. Kögler, Dr. A. Scheidig, Dr. O. K. Froehlich, and others. Experience has shown that the settlement of full-scale footings is often considerably less and sometimes more than is indicated by extrapolation of small-scale test curves for granular soils. Hence, the important load-area-settlement relations have yet to be completely defined; likewise, the effects of lateral displacement of the soil and of differences of physical properties for full-scale footings are subject to further clarification.

It is well to consider certain questions in order to justify the use of the Boussinesq equation, which represents the only means available for the solution of the second class of foundation problems. The major settlement of structures in this class is due to consolidation of saturated compressible layers, which may be at considerable depth. At a comparatively shallow depth beneath a footing, two to three times its greatest width, the surface loading, due to individual footings, loses its isolated character and merges into a single pressure distribution for the structure as a whole. This occurs below the "disturbed zone" as defined by Kögler and Scheidig. Furthermore, the distribution of the contact pressure, which varies with both the elastic properties of the footing and the soil, becomes unimportant[2].

It is suggested by D. P. Krynine[3], M. Am. Soc. C. E., that the concentration factor, n, in the modified Boussinesq equation[2],

$$p_z = \frac{n\,p}{2\,\pi\,R^2}\,\frac{(z)^n}{(R)} \qquad (1)$$

is a function of depth, tending toward a value of 3 as depth increases, and that the differences between theory and the results of investigations are merely surface phenomena confined to the disturbed zone. Experience, thus far, seems to indicate that for finer grained soils possessing considerable cohesion and situated at depths at least greater than two to three times the greatest width of the footing, the soil may behave in accordance with the theory of elasticity; and the Boussinesq equation, for which $n = 3$, should give pressures in reasonably close agreement with the actual, if they could be determined in the natural soil.

It would follow from this fact that the principle of superposition of loads may be applied in order to obtain the effect of distributed loads. Glennon Gilboy, Assoc. M. Am. Soc. C. E., has investigated[4] the proper degrees for subdividing a footing into units of such size that the loads on each unit may be considered a point load as assumed in the Boussinesq equation, and has indicated the accuracy that may be attained by such subdivision and superposition of point loads.

[2] "Distribution of Stresses Under a Foundation", by A. E. Cummings, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1072.

[3] *Loc. cit.*, p. 1089.

[4] Progress Rept., Special Committee on Earths and Foundations, *Proceedings*, Am. Soc. C. E., May, 1933, p. 789.

The detailed analysis of the pressure distribution under a large structure becomes exceedingly long and tedious and involves the determination of a complete picture of the state of stress with respect to the compressible layer.

The table of Boussinesq coefficients, K, corresponding to values[3] of $\frac{r}{z}$, for determining the vertical intensity of pressure at a point in the underground due to a point load concentrated at the surface in the equation,

$$p_z = \frac{K\,p}{z^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

has placed a useful and valuable tool in the hands of the foundation engineer.

The following method is suggested as a practical simplification of the procedure for determining the stresses beneath a structure. The distribution of vertical pressure on a horizontal plane, mn, at some depth, z, due to a unit point load, p_0, at the surface at Point O, is shown by the typical curve in Fig. 1. The intensity of vertical pressure, p_z, at any point (B, Fig. 1) is given by the ordinate, ab, to the curve at that point. This distribution curve may be used conveniently as an influence line, as suggested by S. Timoshenko[5], for a unit load, p_0 of 1 ton. Then, for a load, p_1, at some point, O_1, the vertical pressure at Point A is given by the product of the ordinate, cd, due to a unit point load directly over A, by the load, p_1. It follows that the pressure, p_z, at Point A for a series of point loads, is given by the sum of the ordinates directly under the point loads, multiplied by p.

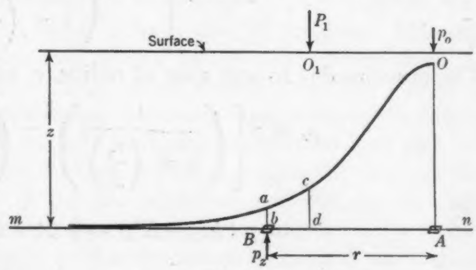

FIG. 1

This idea may be extended to a uniformly distributed line load of p tons per ft, covering all, or any part, of the foundation. The pressure, p_z, at Point A is then given by the area of the distribution curve directly under the loading, multiplied by the uniform loading of p tons per ft.

If this pressure-distribution curve is now revolved about its center at Point A, at which the pressure is to be computed, a bell-shaped surface is formed. Using this area as an influence surface, the pressure, p_z, at Point A is now given by the volume directly under the uniformly loaded area. As the distribution curve is revolved it sweeps out circular segments on the footing area, which are contours of equal pressure. By summing the products of the lengths of each arc by its average pressure intensity and multiplying by the footing load (p tons per sq ft), the total pressure at Point A is obtained. This is simply a graphical method of integration (see Fig. 2), and is the clue to the method proposed herein. The total pressure at Point A due to each

[5] "Theory of Elasticity", by S. Timoshenko, Engineering Societies Monographs, p. 88.

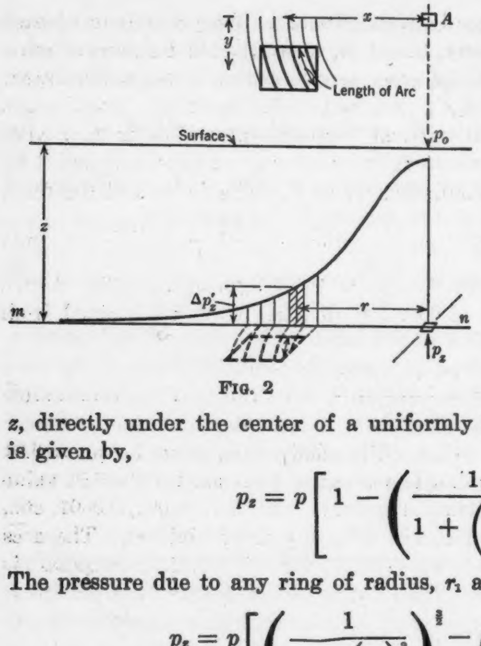

FIG. 2

ring of 1 ft width, is equal to the product of the pressure represented by the shaded area of the distribution curve, having an average ordinate, $\Delta p'_z$ multiplied by $2\pi r$, in which r is measured to the center of gravity of the shaded area. The pressure due to each of these rings may be determined in a very simple manner, because the Boussinesq equation can be readily integrated for circular areas. The pressure at a depth, z, directly under the center of a uniformly loaded circular area of radius, r, is given by,

$$p_z = p\left[1 - \left(\frac{1}{1 + \left(\frac{r}{z}\right)^2}\right)^{\frac{3}{2}}\right] \text{..................(3)}$$

The pressure due to any ring of radius, r_1 and r_2, is then,

$$p_z = p\left[\left(\frac{1}{1 + \left(\frac{r_2}{z_2}\right)^2}\right)^{\frac{3}{2}} - \left(\frac{1}{1 + \left(\frac{r_1}{z_1}\right)^2}\right)^{\frac{3}{2}}\right]$$

$$= p\,(K'_2 - K'_1) = p \times \Delta K' \text{........................(4)}$$

Because a graphical integration method greatly simplifies the computation of pressures due to distributed loads on a foundation, the writer has constructed charts for depths of $z = 10$, 20, 40, and 80 ft. A sample tabulation

TABLE 1.—SAMPLE TABULATION OF VALUES OF $\Delta K'$ AND THE DEGREES OF ARC FOR $p_z = 0.001$ FOR CONSTRUCTING PRESSURE CHARTS

Ring No.	DEPTH, z, IN FEET							
	10		20		40		80	
	$\Delta K'$	Degrees of arc	$\Delta K'$	Degrees of arc	$\Delta K'$	Degrees of arc	$\Delta K'$	Degrees of arc
1	0,01418	24.3	0.0037	96.0	0.0009		0,0002	.....
2	0.0423	8.51	0.0110	32.58	0.0023	156.7	0.0007	.....
3	0.0642	5.61	0.0180	30.00	0.0046	81.8	0.0012	300
4	0.0783	4.60	0.0243	14.81	0.0064	56.25	0.0016	225
5	0.0848	4.24	0.0298	13.08	0.0082	43.9	0.0021	171.5
6	0.0851	4.25	0.0344	10.46	0.0098	36.7	0.0025	.....
7	0.0807	4.46	0.0378	9.52	0,0114	31.6	0.0030	.....
8	0.0737	4.88	0.0405	8.89	0.0129	27.9	0.0035	.....
9	0.0654	5.51	0.0420	8.57	0.0142	25.3	0.0039	.....

of values of $\Delta K'$, which were computed for the purpose of constructing these charts, is given in Table 1. It is seen from the foregoing discussion that the

pressure, Δp_z, at Point *A* is directly proportional to the percentage of each ring swept out in Fig. 2. Each ring, therefore, is subdivided for interpolation into such unit lengths of arc that the pressure represented is some convenient value as 0.01, 0.02, 0.03 tons per sq ft, etc., and multiples of 10 thereof. The construction of a chart of equal vertical pressure intensities is now very simple. For example, $\Delta K'$ for $z = 10$ ft and a ring of radii of 6 ft and 7 ft, that is, Ring No. 7, is equal to 0.0807 ton per sq ft. The degrees of arc for a pressure intensity of 0.01 ton per sq ft would be, $\dfrac{360° \times 0.01}{0.0807} = 44.6.$ Interpolation is simply in direct proportion to the length of arc. The degree of arc for the pressure intensity of 0.01 for all other rings is obtained from the general equation.

$$\alpha = \frac{360° \times 0.01}{\Delta K'} \quad \text{........................(5)}$$

A smooth curve drawn through these points defines the curve for $p_z = 0.01$ ton per sq ft. The curve for $p_z = 0.01$ is one-tenth of the arc for the 0.01 value on each ring. Hence, having constructed the curves for 0.01, 0.001, 0.0001, etc., any degree of subdivision may be obtained by a pair of dividers. The arcs are laid out at the center of each ring which, for all practical purposes, is the center of gravity of the pressure on the ring. Such a chart[6] for $z = 10$ ft and a unit uniform load, *p*, of 1 ton per sq ft on each ring, is shown in Fig. 3.

The total pressure at Point *A* due to a uniformly distributed load of 1 ton per sq ft on a footing of any shape, located anywhere within the range of the chart, is now given by the sum of the arc intercepts enclosed on the chart by the footing. The graphical integration is actually accomplished by revolving the footing layout about Point *A*, so that the reference sides of the footing, *ab* and *cd* (Fig. 3), are brought successively for each ring to the base line, *AB*, where the arc intercepts can then be readily interpolated. The sum of these intercepts, multiplied by the actual loading in tons per square foot, gives the total pressure due to the footing. The interpolation between pressure curves on the chart is directly proportional to the length of arc intercepted, and is readily made by inspection.

If the point, *A*, falls within the loaded area, a certain number of complete rings may be contained within the area. The pressure due to these rings is tabulated below the ring numbers. The pressure due to the remaining area is found in this case by interpolating the intercepts as before. The total pressure at Point *A* due to all the footings within the limits of the chart is obtained by repeating the process for each and summing the effects. This is a simple graphical process and a simple adding machine computation.

The foundation plan is made on tracing paper to the same scale as the horizontal distances on the chart. Each successive point at which the pressure is to be determined in the soil, is pinned down at Point *A* on the proper chart for depth. If the foundation plan is regular, these points would be most conveniently placed on the center line beneath each footing. The procedure for

[6] A full set of charts, for $z = 10$, 20, 40, and 80 ft., may be included with the closing discussion if such procedure then appears warranted.

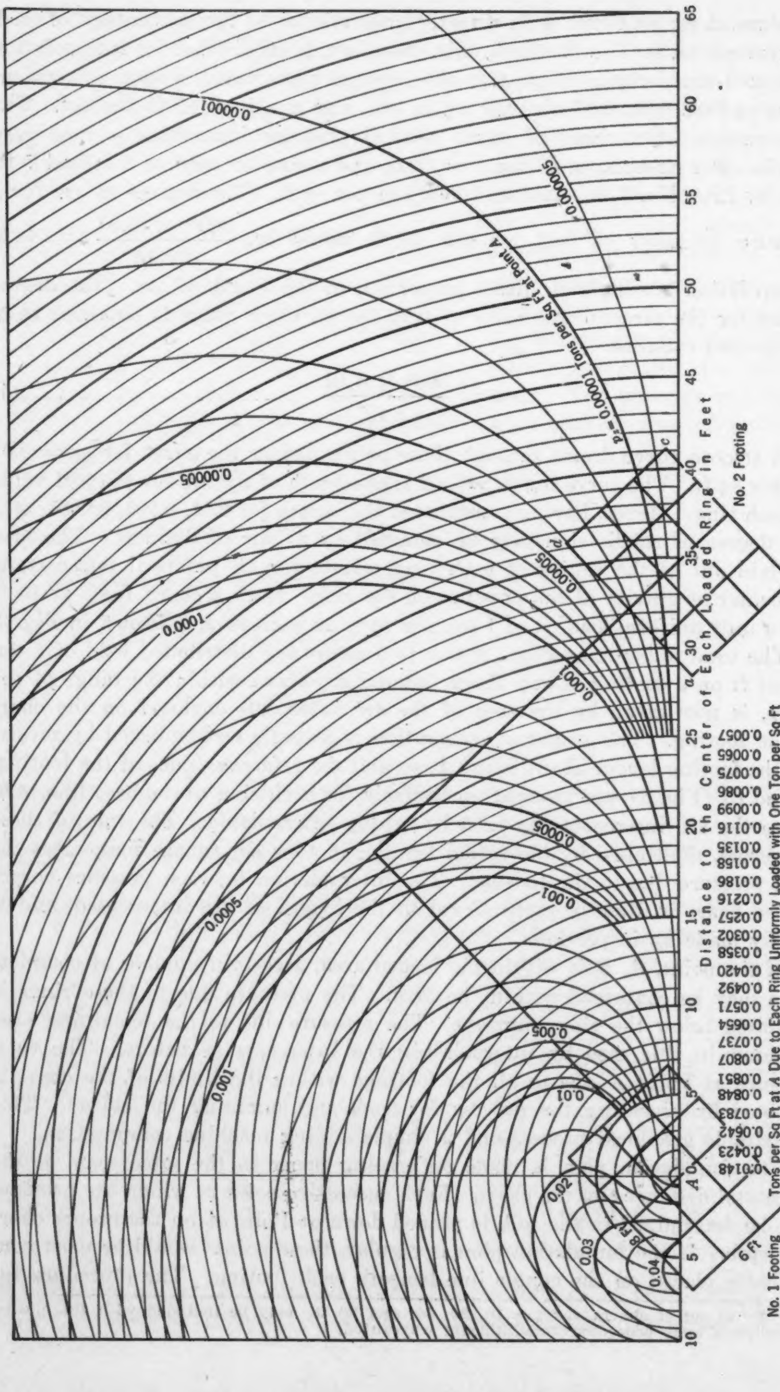

FIG. 3.—PRESSURE CHART FOR $z = 10$ FEET

the analysis of foundation pressures is indicated in Fig. 4. The pressure at Point *A* due to all footings having an appreciable influence may now be obtained by revolving the foundation plan about Point *A* and summing the effects for each footing, successively. An orderly and logical arrangement of the work will materially facilitate the computation. Most layouts have many

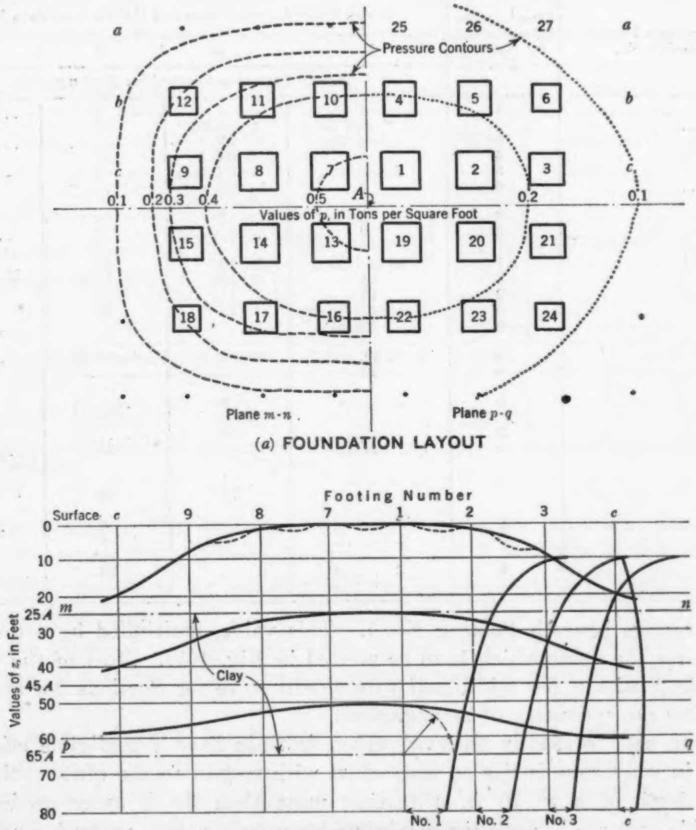

FIG. 4.—PROCEDURE FOR ANALYZING FOUNDATION PRESSURES

symmetrical arrangements of footings as to location, size, and loading. If these similarities are noted, duplication is avoided and the work is decreased. A sample arrangement of some of the duplicate values to illustrate this problem is given in Table 2. For example, the value obtained for the pressure beneath Footing No. 1 due to pressures exerted by Footing No. 2, when multiplied by 3, takes care of the influence of Footings No. 2, 7, and 19 in the third column. Thus, a set of values indicated by the first and second columns, when completed, serves to fill out the remaining four columns, thus

giving the data necessary for determining the distribution of pressure for Section *c-c* at any given elevation, *z*. The sum of all the values found for Footing No. 1 (that is, in the third column complete of Table 2) gives the

TABLE 2.—Tabulation of Duplicate Values for Any Depth Along Section *c-c* (Fig. 4).

The pressure beneath Footing No.	Due to pressures exerted by Footing No.	Is the Same as the Pressure Under Footings Nos.			
		1	2	3	*c*
		Due to Pressures Exerted by Footings Nos.			
1	1	1	2	3	....
1	2	2, 7, and 19	1 and 20	2	....
1	3	3 and 22	23	....	....
1	4	4	3 and 5	21	3
1	5	5 and 10	4 and 21	5	21
1	6	6	24	....	24
7	2	8	7	1	2
7	3	9	....	7	....
13	1	13 and 20	19	20	....
13	2	14	13	19	20 and 5
13	3	15	11	13	4
13	5	17	16	22	23
13	6	18	....	....	....
19	3	11, 16, 23, and 21	22 and 10	4 and 23	....
19	6	24	....	....	....
2	6	....	6	....	6
8	2	....	8	....	1
8	5	....	....	10	....
14	2	....	14	....	19
14	5	....	17	16	22
3	6	....	....	6	....
3	3	....	....	3	....
4	6	....	....	24	....

total pressure beneath Footing No. 1. This value, multiplied by 3 tons per sq ft gives the value which is to be plotted in Fig. 4(*b*). Most of the values in Table 2, with a few additional values, will serve for Sections *a-a* and *b-b*, by noting the symmetry of arrangement.

It can also be readily observed which footings have a negligible effect by noting in what zone it lies on each chart with respect to the center point, *A*, For a depth of $z = 10$ ft, a footing more than 50 ft away produces a negligible pressure because the maximum pressure contours covered equal only 0.00001 ton per sq ft, whereas a footing at Point *A* includes four and one-half full rings of 0.08 maximum pressure. For the example shown footings that are at a greater distance from the one considered (Point *A*), than the following values will not affect materially the third significant figure, which is about the limit of plotting:

Depth, in feet, below the ground surface	Distance, in feet, from any given point
10	50
20	60
40	70
80	80

The diagonal limit of Fig. 3 is at 70 ft.

Examples of the measurement of pressure at a depth, $z = 10$ ft, and $p = 3$ tons per sq ft, are given in Table 3.

TABLE 3.—EXAMPLE OF THE MEASUREMENT OF PRESSURE AT A DEPTH, $z = 10$ FEET, AND $p = 3$ TONS PER SQUARE FOOT (SEE FIG. 3).

MEASUREMENT OF PRESSURE BENEATH FOOTINGS NOS. 1, 2, AND 24 TO PRESSURE EXERTED BY:					
Footing No. 1		Footing No. 2		Footing No. 24	
Remarks	"Influence" pressures	Remarks	"Influence" pressures	Remarks	"Influence" pressures
Four full rings.............	0.0142 0.0423 0.0642 0.0783	Eight segments.	0.00160 0.00130 0.00105 0.00085 0.00070 0.00058 0.00048 0.00039	Eight segments.	0.000002 0.000005 0.000007 0.000007 0.000006 0.000004 0.000002 0.000001
Approximately one-half a ring......................	0.0424				
Four segments @ 0.0045 =	0.0180				
		Three-fourths of 0.00033.....	0.00025		
Total "influence" pressures.	0.2594	....	0.00710	....	0.000038
Total pressure, $p = 3$ tons per sq. ft. $\times$ the total "influence" pressure.....	0.7782	....	0.02130	....	0.000112*

* Negligible.

The pressure distribution on a vertical section through Section *c-c* along the center lines of Footings Nos. 1, 2, and 3, is shown in Fig. 4(*b*). It is a simple matter to lay out, by dividers, the distribution of pressure on the horizontal lines, *m-n* and *p-q*, which define the limits of the clay layers. In fact, the pressure at such depths only as are necessary to determine the distribution of pressure within the clay layer, defined by the planed *m-n* and *p-q*, need be determined, as, for example, for $z = 20$, 40, and 80 ft. Similar diagrams may be drawn for Sections *a-a* and *b-b*. The distribution of the pressure on any horizontal plane, such as *m-n* and *p-q*, may then be easily drawn by interpolating contours of equal pressure from curves, such as Fig. 4(*b*), for Sections *a-a*, *b-b*, and *c-c*. These contours have been sketched in in Fig, 4(*a*). The pressures are given in tons per square foot. When the relations between pressure and settlement are obtained from consolidation tests, the pressure contours will define curves of settlement.

Table 4 contains a comparison of values obtained by pressure charts, such as Fig. 3, by the subdivided area method, and by the single, concentrated point load method. The pressures are computed for each footing separately at depths of 10, 20, 40, and 80 ft, along the center line of Footing No. 1 (see Fig. 5).

TABLE 4.—COMPARISON OF PRESSURES UNDER FOOTING NO. 1, BY THE SUB-DIVIDED AREA METHOD, AND BY THE POINT-LOAD METHOD OBTAINED BY CHARTS SUCH AS FIG. 3 (SEE FIG. 5)

Depth, in feet	DUE TO FOOTING NO. 1			DUE TO FOOTING NO. 2		
	By pressure charts	Load applied at twelve points*	Load applied at a single point	By pressure charts	Load applied at twelve points*	Load applied at a single point
10	0.1853	0.1942	0.2290	0.00096	0.00097	0.00091
20	0.0544	0.0549	0.0572	0.00369	0.00373	0.00367
40	0.0133	0.0142	0.0143	0.00499	0.00518	0.00521
80	0.0034	0.0036	0.0036	0.00265	0.00264	0.00263

* Subdivided area method.

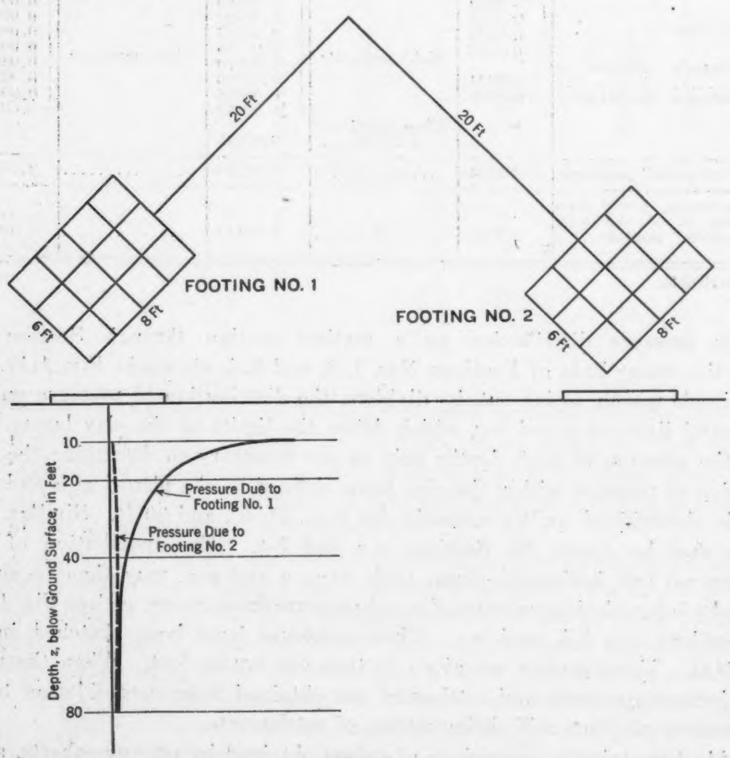

FIG. 5.

CONCLUSION

These results show that the point load method is considerably in error, especially close to the footing, whereas the differences between the chart method and the subdivided area method in general, are small. The saving in time of computation, however, is the important factor. The recent trend toward the use of such labor-saving computing charts in all branches of

engineering is indicated by the number of such charts which have appeared from time to time in engineering literature. It is hoped that these charts will prove to be useful and by saving time and labor will make possible a more complete stress analysis in the soil beneath foundations. This would mean that the field of uncertainties would be narrowed down to other considerations and the correlation between the behavior of structures and theoretical and experimental soil mechanics would indicate the necessary modifications and re-adjustments.

The influence-line, graphical integration method may be extended to other problems for which the stress distribution is essential wherever the equation may be integrated for circular areas or by using the method outlined in Fig. 2 and the discussion immediately following.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

STRUCTURAL ANALYSIS BASED UPON PRINCIPLES PERTAINING TO UNLOADED MODELS

By Otto Gottschalk,[1] Esq.

Synopsis

When the stresses in a structure are investigated by the use of unloaded models, known deformations are produced at a section at which the stresses are desired, and the corresponding displacements are measured at the point at which the load is applied in the actual structure. The ratios between the latter and the former deformations give the required stresses for a unit load, and when the original deformation is taken as unity, the model forms the influence line of stress. To obtain a moment, a known or unit rotation is produced at the section at which the stress is required; a normal displacement is produced to obtain a shear or a reaction; and an axial displacement is produced to obtain a direct tension or compression. The geometrical analysis of these operations without the actual use of models leads to fundamental simplifications, and it is the purpose of this paper to establish the few simple equations necessary for the use of this method of analyzing statically indeterminate structures.

Definitions and Notation

The fundamental beam with a constant moment of inertia used in this analysis is the member, AB, shown in the several illustrations. The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials,[2] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

a = a definite value of x; a' = a definite value of x'; a'' = distances measured along a cantilever from the support;

f = the intercept at End A of the tangent to the elastic curve, at End $B = \alpha_{AB}\, L$; similarly, $f' = \alpha_{AB}\, L$;

Note.—Discussion on this paper will be closed in April, 1937, *Proceedings*.

[1] Buenos Aires, Argentine Republic.

[2] A.S.A.—Z10*a* 1932.

I = moment of inertia;

K_A = the sum of the relative stiffness values of the ends of all members meeting at a joint; and, therefore, $K_A = S + S_A$; and, $K_B = S' + S_B$;

k = stiffness of a member $= \dfrac{I}{L}$;

L = span length;

M = bending moment; M_{AB} = moment at End A of Beam AB; M_{BA} = moments at End B of Beam AB; and are positive in the same directions as their corresponding rotations (α);

m = ratio of end rotations; for rotations at Ends A and B, respectively: $m = \dfrac{\alpha_{BA}}{\alpha_{AB}} = \dfrac{f}{f'}$; and $m' = \dfrac{\alpha_{AB}}{\alpha_{BA}} = \dfrac{f'}{f}$ for rotations originating at A and B, respectively;

N = a substitution factor (see Equation (21));

P = total concentrated load applied at any joint of the actual structure;

R = a ratio $= \dfrac{\Delta}{L}$;

S = relative stiffness value $= (1 - 0.5\ m)\,k$ = the resistance to rotation of the left end of a member; $S' = (1 - 0.5\ m')\,k$ = the resistance to rotation of the right end of a member; S_A = the sum of the relative stiffness values of all the members meeting the left end of a member at a joint, but not including the end of the member in question. Since the right ends of the members connect with the left end of a member under consideration, $S_A = S'_1 + S'_2 + S'_3 + \ldots +$, etc.; S_B is a quantity similar to S_A, for the right end of a member, and, correspondingly, $S_B = S_1 + S_2 + S_3 + \ldots +$, etc.;

V = total shear;

W = total uniform load on a beam;

x = variable abscissa of the influence line, measured from the left support; $x' = L - x$;

y = variable ordinate of the influence line; y_c = the ordinate at the center of the beam; y_m = the mean value of y for any influence line;

α = the angular rotation of the end of a member relative to the final direction of the axis. It is positive when the rotation is such that tension is produced in the "upper" fibers. As will be seen from the analysis all members have a "left" and a "right" end, both of which are definitely determined; therefore, the terms "upper" and "lower" are also definitely determined;

Δ = the deflection of the left end of a member relative to the right end and is positive when the left end is "raised" relative to the right end;

ϵ = energy; internal work exerted against the work involved in the diminishing reaction (see Equation (26));

θ = the angular rotation of the end of a member relative to the original direction of its axis, and is positive in the same direction as α.

Introduction

In hypothetical models, as in actual models, the deformations are assumed to be indefinitely small, so that the projection of any deformed member or part of it on its original axis is equal to its original length; that is, in Fig. 1, the distance, *A-B*, remains unchanged after rotation at Point *E*.

The present analysis will be limited to skeletons or frames with vertical columns and horizontal beams, each of constant moment of inertia and all of the same homogeneous material, with constant modulus of elasticity. As in other, similar methods of analysis, the structure is first restrained against side-sway. The structure is then released, the effect of side-sway computed, and the stresses are corrected. For example, let *AB* be a beam within the typical skeleton shown in Fig. 1 in which the bending moment, M_E, at Point *E* is required. If the skeleton were merely simply supported at Points *A* and *B*, a unit rotation at Point *E* would produce the deformation, $C'AE'BD'$, in the model as shown in broken lines, in Fig. 1. Then (with the angles between members fixed at all joints, but with the joints free to rotate) Points C' and D' are returned to their original position. The shape of the frame ($CAE''BD$), will then represent the final influence line for bending moment.

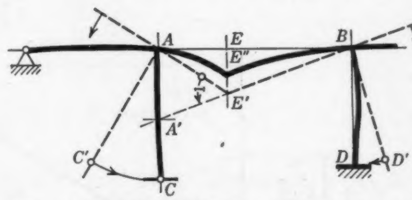

FIG. 1.—A RIGID FRAME; INFLUENCE LINE FOR MOMENT AT POINT *E*.

It will be seen from Fig. 1 that all branches of the influence line for M_E are simple geometrical curves, produced by the rotation of the broken ends at Point *E* and by the restraint at the far ends, except $AE''B$, which is a combination of the triangle, $AE'B$, corresponding to free supports at *A* and *B* and the eventual restraint at Points *A* and *B*. These rotations, restraints, and displacements produce typical bending moments in the several parts of the model, which admit of an easy analysis of the curves formed. It should be kept in mind, however, that influence lines may be considered as geometrical curves which can be obtained by geometrical methods. Equations (1) to (21) which follow, have been obtained experimentally by applying visible displacements and unit rotations to models composed of elastic splines. The resulting curves have been analyzed in Appendix II. The basic theory of unloaded models is derived in Appendix I.

Fundamental Equations of Simple Influence Lines

When an elastic spline (see Fig. 2(*a*)), pin-connected at Points *A* and *B*, is rotated about Point *A* through an angle equal to $\frac{f'}{L}$, the corresponding rotation at Point *B* is equal to $\frac{f}{L} = \frac{f'}{2\,L}$; or $m = \frac{f}{f'} = 0.5$. This experiment may be checked by abstract theory: Rotation at Point *A* produces a

bending moment in the spline which is a maximum at Point A and which decreases uniformly to zero at Point B. At Point A, the reaction of the beam, loaded with that triangular moment area, is twice that at Point B and, because the inclination of the tangents to the elastic line are proportional to the reactions of this moment area, $f' = 2f$.

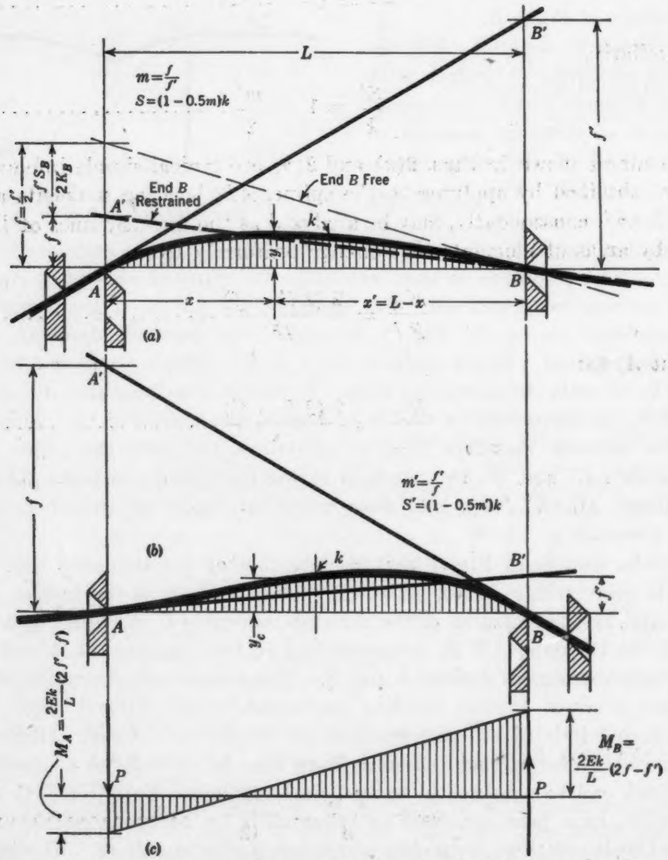

FIG. 2.—SIMPLE INFLUENCE LINES

However, when other members join at Point B to create a total relative stiffness, S_B, the rotation of the spline is restrained proportionately as shown by Line BA' (Fig. 2(a)) and, therefore:

$$m = \frac{f}{f'} = \frac{0.5\,k}{S_B + k} \qquad (1)$$

Similarly, rotating the spline about Point B and restraining it at Point A (see Fig. 2(b)):

$$m' = \frac{f'}{f} = \frac{0.5\,k}{S_A + k} \qquad (2)$$

Referring back to Fig. 2(*a*), when the spline is released at Point *B* so that the value of f increases, the strain on the spline at Point *A* is reduced by one-half the corresponding increase; and since, when Point *B* is fixed and cannot rotate, $S = k$ (for $f = 0$ and $m = 0$):

$$\frac{S}{k} = 1 - \frac{m}{2} \qquad \text{(3a)}$$

and, similarly,

$$\frac{S'}{k} = 1 - \frac{m'}{2} \qquad \text{(3b)}$$

The curves shown in Figs. 2(*a*) and 2(*b*) are typical simple influence lines; they are obtained by applying to the spline, *AB*, bending moments at Points *A* and *B* and, consequently, may be analyzed as the moment lines of Beam *AB* loaded by areas of moments that change uniformly from:

$$\frac{M_A}{E\,I} = \frac{2\,(2\,f' - f)}{L^2} \qquad \text{(4a)}$$

at Point *A*, to:

$$\frac{M_B}{E\,I} = \frac{2\,(2\,f - f')}{L^2} \qquad \text{(4b)}$$

at Point *B* (see Fig. 2(*c*)). Thus, one may derive the very simple equation for general influence lines in terms of end rotations:

$$y = \frac{x\;x'}{L^3}\,(x\,f + x'\,f') \qquad \text{(5)}$$

At the center of the beam:

$$y_c = \frac{1}{8}\,(f + f') \qquad \text{(6a)}$$

and the mean value is:

$$y_m = \frac{2\,y_c}{3} = \frac{f + f'}{12} \qquad \text{(6b)}$$

When the beam, *AB*, is simply supported or pin-connected at Point *B*: $m = 0.5$; $S = 0.75\,k$;

$$y_c = \frac{3\,f'}{16} \qquad \text{(7a)}$$

$$y_m = \frac{f'}{8} \qquad \text{(7b)}$$

and,

$$y = \frac{x\,x'\,f'}{L^3}\left(\frac{x}{2} + x'\right) \qquad \text{(7c)}$$

and when End B is fixed: $m = 0$; $S = k$;

$$y_c = \frac{f'}{8} \quad \text{(8a)}$$

$$y_m = \frac{f'}{12} \quad \text{(8b)}$$

and,

$$y = \frac{x\,(x')^2 f'}{L^3} \quad \text{(8c)}$$

Similarly, when End A is simply supported: $m' = 0.5$; $S' = 0.75\,k$;

$$y_c = \frac{3f}{16} \quad \text{(9a)}$$

$$y_m = \frac{f}{8} \quad \text{(9b)}$$

and,

$$y = \frac{x\,x'\,f}{L^3}\left(x + \frac{x'}{2}\right) \quad \text{(9c)}$$

When End A is fixed: $m' = 0$; $S' = k$;

$$y_c = \frac{f}{8} \quad \text{(10a)}$$

$$y_m = \frac{f}{12} \quad \text{(10b)}$$

and,

$$y = \frac{x^2\,x'\,f}{L^3} \quad \text{(10c)}$$

Equations (1) to (10) are the fundamentals required for analyzing frames. They apply to all classes of influence lines and lead to new and useful analytical and graphical interpretations of these curves. Of special interest is the fact that the relative stiffness of end panels can be easily calculated from Equations (8) and (10) and that, therefore, the statical indeterminancy due to restraint at the support need no longer be a serious factor in computations. Side-sway is not taken into account in these equations, but the necessary correction can be made easily.

Linear Displacement

Fig. 3(a) shows a beam or column in its normal, unstrained position. If End A, with all of the part of the structure to the left, is translated vertically upward a distance, Δ, Beam AB will deform as shown in Fig. 3(b). The members to the left of Point A are considered to be restrained so that they

cannot move parallel to the original position of Member AB, but are free to move perpendicular to that axis, except that all members move the same distance, Δ. All members are considered free to rotate as much as their relative stiffnesses, S, will permit.

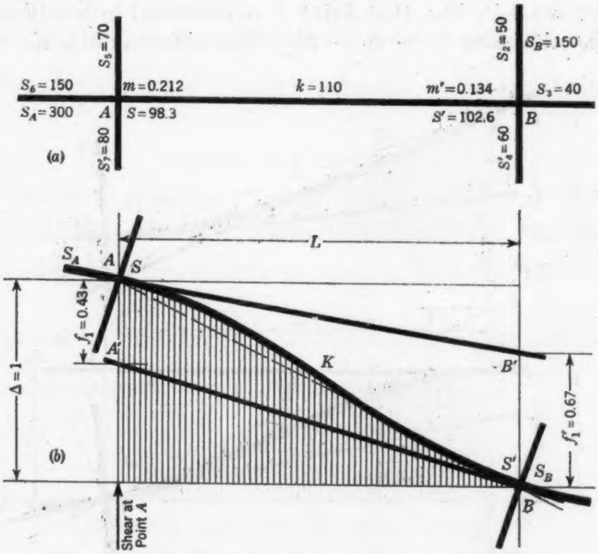

FIG. 3.—LINEAR DISPLACEMENT IN A RIGID FRAME

Joints A and B, then, tend to resist rotation and exert a counter rotation as shown in Fig. 3(*b*), so that the algebraic sums of the deformations are:

$$\frac{f}{\Delta} = -\frac{S_B}{K_B} + \frac{m\ S_A}{K_A} \quad \text{....................(11)}$$

and,

$$\frac{f'}{\Delta} = \frac{S_A}{K_A} - \frac{m'\ S_B}{K_B} \quad \text{....................(12)}$$

For example (referring to Fig. 3), let $k = 110$; $S_A = 70 + 150 + 80 = 300$; and $S_B = 50 + 40 + 60 = 150$. If Joint A is displaced vertically a distance, $\Delta = 1$, the rotations are: At Joint A, $\frac{f'}{L}$; and, at Joint B, $\frac{f}{L}$. The influence line for shear at End A of Beam AB, then will be determined as follows: $m = \frac{0.5 \times 110}{110 + 150} = 0.212$; $S = 110\left(1 - \frac{0.212}{2}\right) = 98.3$; $m' = \frac{0.5 \times 110}{110 + 300} = 0.134$; $S' = 110\left(1 - \frac{0.134}{2}\right) = 102.6$; $K_A = 98.3 + 300 = 398.3$; and $K_B = 102.6 + 150 = 252.6$. The net rotation, therefore,

will be: At Joint B, $-\dfrac{f}{\Delta} = \dfrac{150}{252.6} - \dfrac{0.212 \times 300}{398.3} = 0.434$; and, at Joint A, $\dfrac{f'}{\Delta} = \dfrac{300}{398.3} - \dfrac{0.134 \times 150}{252.6} = 0.674$.

Special Cases.—In Fig. 4(*a*) Joint B is restrained by a stiffness, S_B, composed of the stiffnesses, $S_2 + S_3 + S'_4$. The stiffness ratio, $K_B = S'_1 + S_B$.

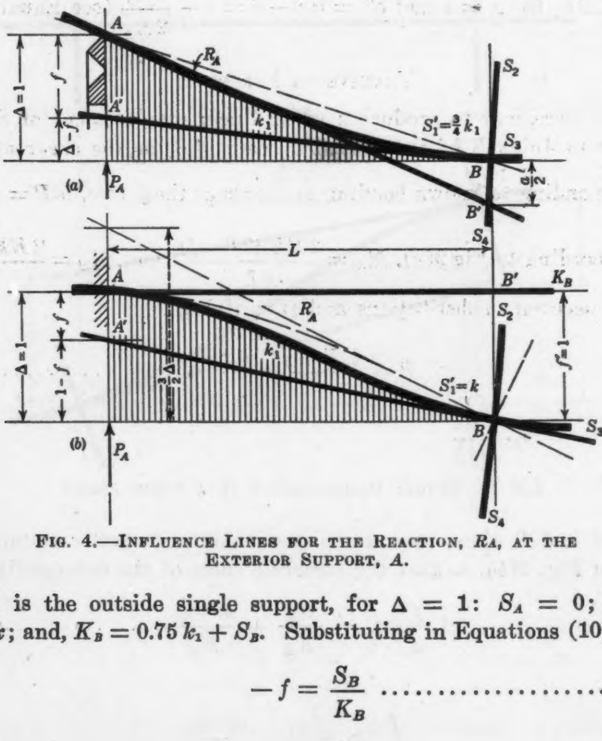

FIG. 4.—INFLUENCE LINES FOR THE REACTION, R_A, AT THE EXTERIOR SUPPORT, A.

If End A is the outside single support, for $\Delta = 1$: $S_A = 0$; $m' = 0.5$; $S' = 0.75\,k$; and, $K_B = 0.75\,k_1 + S_B$. Substituting in Equations (10) and (11):

$$-f = \frac{S_B}{K_B} \quad\text{.......................}(13a)$$

and,

$$f' = -\frac{S_B}{2\,K_B} \quad\text{.......................}(13b)$$

On the other hand, if Point A is fixed (see Fig. 4(*b*)), for $\Delta = 1$: $S_A = \infty$; $\dfrac{S_A}{K_A} = 1$; $m = \dfrac{k_1}{2\,K_B}$; and $m' = 0$, substituting these values in Equations (10) and (11):

$$-f = \frac{S_B}{K_B} - \frac{k_1}{2\,K_B} = 1 - \frac{3\,k_1}{2\,K_B} \quad\text{..............}(14a)$$

and,

$$f' = 1 \quad\text{.........................}(14b)$$

Equations (13) and (14) may also be obtained from Fig. 4 without reference to Equations (11) and (12); for example, in Fig. 4(*b*) if Support B were free, an upward displacement, Δ, at End A would rotate AB at Joint B through an angle, $\dfrac{3\,\Delta}{2\,L}$. If the resisting stiffness of the remaining three members meeting at Joint B is then applied, a counter rotation occurs, equal to $\dfrac{3\,\Delta\,S'_1}{2\,L\,K_B}$. Finally, for $\Delta = 1$ and $S'_1 = k_1$: $-f = 1 - \dfrac{3\,k_1}{2\,K_B}$ (see Equation (14)).

TRANSVERSE FORCES

The force necessary to produce a given linear displacement of Support A with respect to Joint B of the unloaded beam, AB, or the shear at the supports, corresponding to known bending moments at the joints, is $P = \dfrac{M_A - M_B}{L}$ in which, according to Fig 2(*c*), $M_A = \dfrac{2\,E\,k_1\,(2f' - f)}{L}$; and, $M_B = \dfrac{2\,E\,k_1\,(2f - f')}{L}$. Making the necessary substitutions and simplifying:

$$P = \frac{6\,E\,k_1\,(f' - f)}{L^2} \qquad \text{(15)}$$

INFLUENCE LINES FOR BENDING MOMENTS

The influence line for bending moment at any section, E, of a beam, AB (see Fig. 5), is obtained by applying a unit rotation at Point E. When Joints A and B are simply supported the unit rotation deforms the member, AB, as shown in Fig. 5(*a*), producing the broken beam, $AE'B$, so that $\Delta = a$ and $\Delta' = a'$.

The restraining stiffness of the connecting members of the two joints, however, exerts a counter rotation, producing the curve with end rotations, $a - f$ and $a' - f'$, shown in Fig. 5(*b*), so that, finally, in Fig 5(*c*):

$$f = a\,\frac{S'}{K_B} - m\,a'\,\frac{S_A}{K_A} \qquad \text{(16a)}$$

and,

$$f' = a'\,\frac{S}{K_A} - m'\,a\,\frac{S_B}{K_B} \qquad \text{(16b)}$$

General Ordinate and Mean Ordinate.—For $x < a$:

$$y = \frac{a'\,x}{L} - N \qquad \text{(17a)}$$

and, for $x > a$:

$$y = \frac{a\,x'}{L} - N \qquad \text{(17b)}$$

in which,

$$N = \frac{x\,x'}{L^3}\left[(a - f)\,x + (a' - f')\,x'\right] \qquad (18)$$

In general, the mean ordinate is expressed by,

$$y_m = \frac{a\,a'}{2\,L} - \frac{L - f - f'}{12} \qquad (19)$$

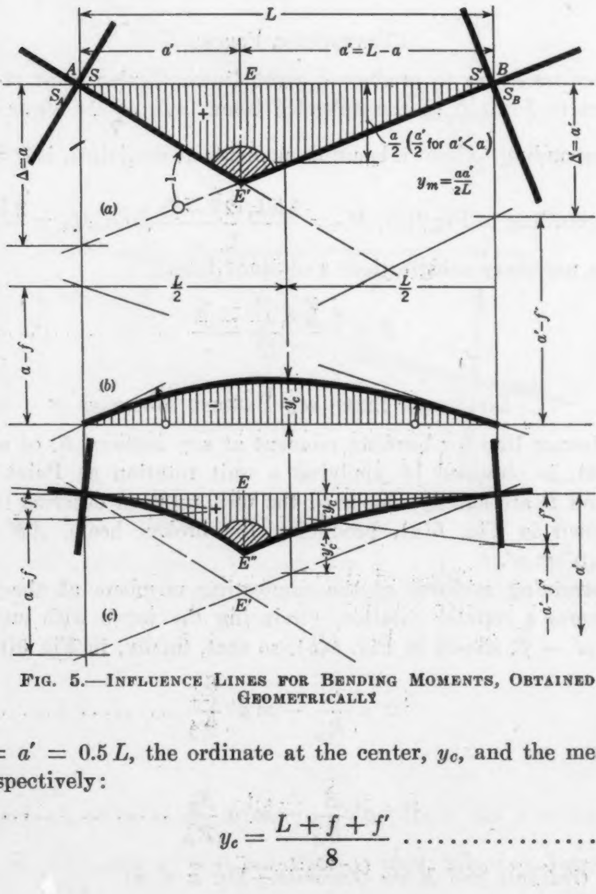

FIG. 5.—INFLUENCE LINES FOR BENDING MOMENTS, OBTAINED GEOMETRICALLY

When $a = a' = 0.5\,L$, the ordinate at the center, y_c, and the mean ordinate, y_m, are respectively:

$$y_c = \frac{L + f + f'}{8} \qquad (20a)$$

and,

$$y_m = \frac{2\,y_c}{3} - \frac{L}{24} \qquad (20b)$$

For $x = a$:

$$y = \frac{a\,a'}{L^3}\,(2\,a\,a' + a\,f + a'\,f') \qquad (21)$$

Here again it is demonstrated that influence lines, considered as purely geometric curves, can be derived by geometric methods alone. For example, the ordinate of Fig. 5(*b*) may be deducted from corresponding ordinates of Fig. 5(*a*) to secure the final curve of Fig. 5(*c*), without becoming involved in the theory of stresses or in stress functions. When $a = L$ and $a' = 0$, the

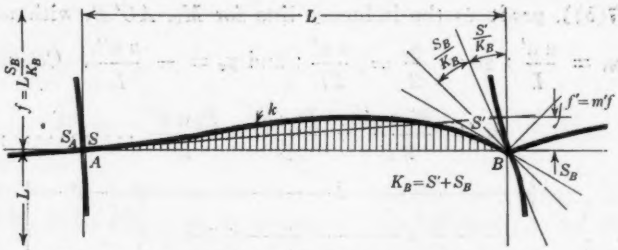

FIG. 6.—INFLUENCE LINE FOR BENDING MOMENT, M_{BA}.

result is an influence line for the end moment, M_B (see Fig. 6), for which the corresponding end rotations are:

$$f = \frac{L\,S_B}{K_B} \qquad (22a)$$

and,

$$f' = m'\,f \qquad (22b)$$

EXAMPLES

The value of the methods outlined in this paper may be illustrated by means of the following examples.

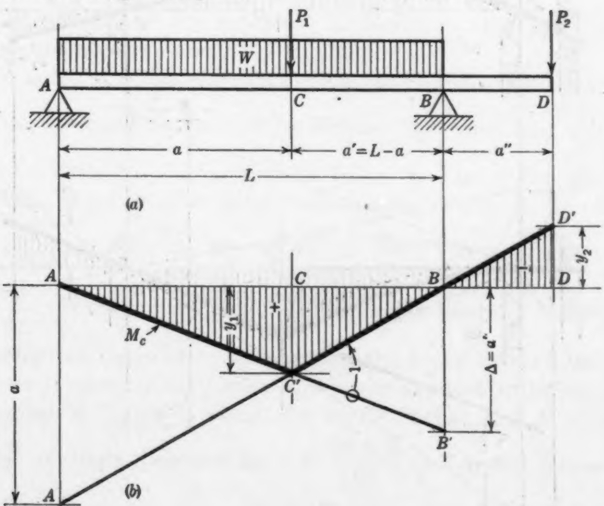

FIG. 7.—INFLUENCE LINE FOR BENDING MOMENT AT POINT C OF A SIMPLE BEAM

Example 1.—Fig. 7(*a*) represents a beam simply supported at Points *A* and *B*, with a cantilever extension beyond Point *B*. The bending moment M_C is required. Breaking the hypothetical beam at Point *C* and rotating one of the parts with respect to the other through an angle equal to $\frac{a'}{\Delta} = 1$ (see Fig. 7(*b*)), produces the influence line for M_C, $AC'B$, with major ordinates of: $y_1 = \frac{a\,a'}{L}$; $y_m = \frac{y_1}{2} = \frac{a\,a'}{2\,L}$; and $y_2 = -\frac{a\,a''}{L}$. Consequently,

$$M_C = \frac{W\,a\,a'}{2\,L} + \frac{P_1\,a\,a'}{L} - \frac{P_2\,a\,a''}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

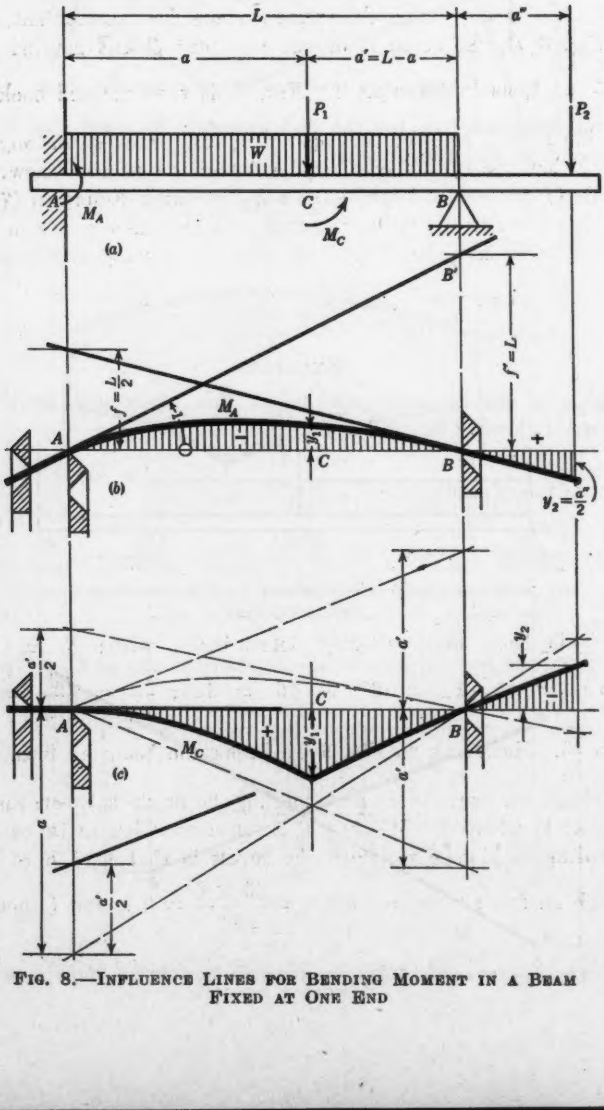

FIG. 8.—INFLUENCE LINES FOR BENDING MOMENT IN A BEAM FIXED AT ONE END

Example 2.—In Fig. 8(*a*), the beam in Example 1 has been fixed at End A. The bending moment M_A is required. Rotating the beam at Point A through an angle, $\frac{f'}{L} = 1$ (see Fig. 8(*b*)) and then translating Point B' back to its original position at Point B, $f = \frac{f'}{2} = \frac{L}{2}$ Then, from Equation (7); $y_m = \frac{L}{8}$; $y_1 = \frac{a\,a'\,(0.5a + a')}{L^2}$; $y_2 = \frac{f\,a''}{L} = \frac{a''}{2}$; and,

$$M_A = -\frac{W\,L}{8} - \frac{P_1\,a\,a'\,(0.5\,a + a')}{L^2} + \frac{P_2\,a''}{2} \quad \text{.........(24)}$$

Example 3.—To determine the influence line for the moment, M_C, at any Point C, Fig. 8(*a*), the beam is broken at Point C and rotated through an angle of $\frac{a'}{\Delta} = 1$, as in Example 1. End A is then rotated back until it is once more horizontal, and for this operation considered separately, the hypothetical beam has the form shown by the dotted curved line shown above AB in Fig. 8(*c*), ($f' = +\,a'$ and $f = +\,0.5\,a'$), for which Equation (7) gives the ordinates. For the first operation the ordinates are the same as in Example 1. The final ordinates are the differences between corresponding ordinates in each of the two steps, thus: $y_m = \frac{a\,a'}{2\,L} - \frac{a'}{8}$; $y_1 = \frac{a\,a'}{L} - \frac{a\,(a')^2\,(0.5\,a + a')}{L^3} = \frac{a^2\,a'\,(3\,L - a)}{2\,L^3}$; $y_2 = \frac{-\,a\,a''}{L} + \frac{a'\,a''}{2\,L} = \frac{-\,a''\,(3\,a - L)}{2\,L}$; and, finally,

$$M_C = W\,y_m + P_1\,y_1 + P_2\,y_2 \quad \text{..................(25)}$$

It is to be noted that the ordinates in Equation (25) can be obtained directly by means of Equations (16) to (22).

It is to be noted also that the effect of the load on the cantilever section is to produce positive bending moment in Member AB when $a < \frac{L}{3}$.

Example 4.—Consider the continuous beam in Fig. 9 for which M_B is required. This beam is elastically determinate, with: $S_1 = k_1 = 80$; $S_2 = 0.75\,k_2 = 45$; $k_B = 80 + 45 = 125$; $\frac{S'_1}{k_B} = \frac{80}{125} = \frac{16}{25}$; and, $\frac{S_2}{k_B} = \frac{45}{125} = \frac{9}{25}$. Referring to Fig. 9(*b*), break the beam at Point B; rotate one part through an angle of 1; and holding the angle between the two ends at this joint at 1, return End C back to a pin connection in its original position. According to Equation (22*a*), the result is that End B of Beam AB yields through an angle measured by $f_1 = \frac{20 \times 9}{25} = 7.2$, and (since the beam is fixed at Point A) $f' = 0$; $f'_2 = \frac{16 \times 16}{25} = 10.24$; and (since BC is simply

supported at Point C), $f_2 = \frac{10.24}{2} = 5.12$. From Equation (10): $y_{m_1} = \frac{f_1}{12} = 0.6$; and, $y_1 = \frac{14^2 \times 6 \times 7.2}{20^3} = 1.06$. Also, from Equations (7): $y_{m_2} = \frac{10.24}{8} = 1.28$. Finally, $y_2 = \frac{-5.1 \times 5.0}{16} = -1.6$; and, therefore, $M_B = 12 \times 0.6 + 15 \times 1.06 + 8 \times 1.28 - 5 \times 1.6 = 25.32$.

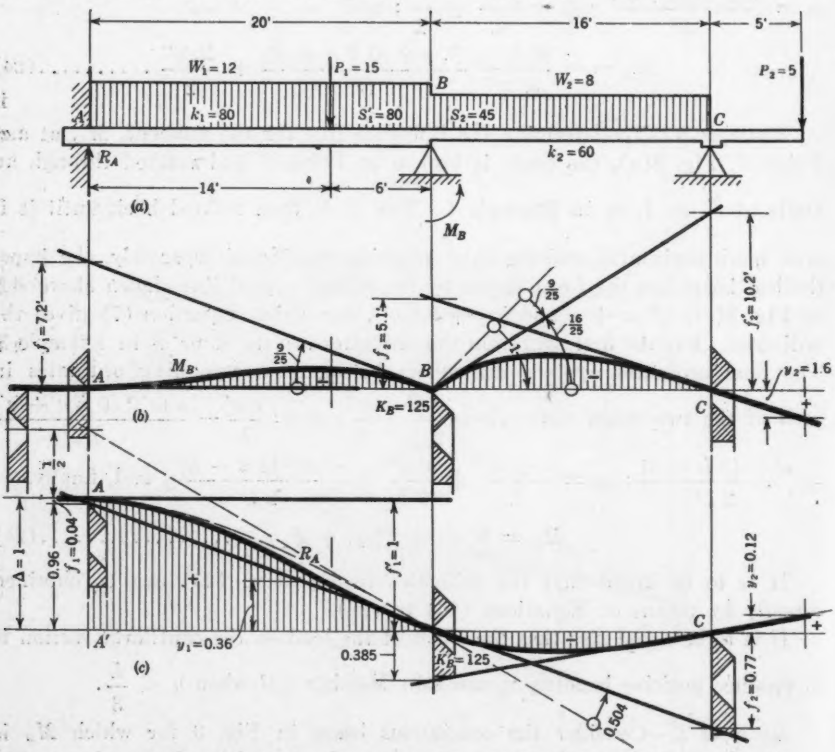

FIG. 9.—INFLUENCE LINES FOR A CONTINUOUS BEAM

Example 5.—The influence line for the reaction, R_A, in the beam shown in Fig. 9 may be determined by reference to Fig. 9(*c*). First, deflect End A upward a distance, $\Delta = 1$, without permitting it to rotate. Following the procedure demonstrated in connection with Fig. 4(*b*) and from Equations (14): $-f_1 = 1 - \frac{3 \times 80}{250} = 0.04$; $f'_1 = 1$; $-f'_2 = \frac{0.96 \times 16}{20} = 0.77$; $f_2 = \frac{f'_2}{2} = 0.385$; $y_{m_1} = \frac{1}{2} + \frac{1 - 0.04}{12} = 0.58$; $y_1 = \frac{6}{20} + 14 \times 6 \times \left(\frac{-14 \times 0.04 + 6 \times 1}{20^3}\right) = 0.63$; $y_{m_2} = -\frac{0.77}{8} = -0.096$; $y_2 = \frac{0.385 \times 5}{16} = 0.12$; and, $R_A = 12 \times 0.58 + 15 \times 0.36 - 8 \times 0.096 + 5 \times 0.12 = 12.2$.

Example 6.—Consider an unsymmetrical portal frame, loaded by a horizontal force as shown in Fig. 10(*a*). The values, H_A and H_D are required. The stiffness, S, of Columns AB and DC is equal to k, and, therefore, $S'_1 = 20$;

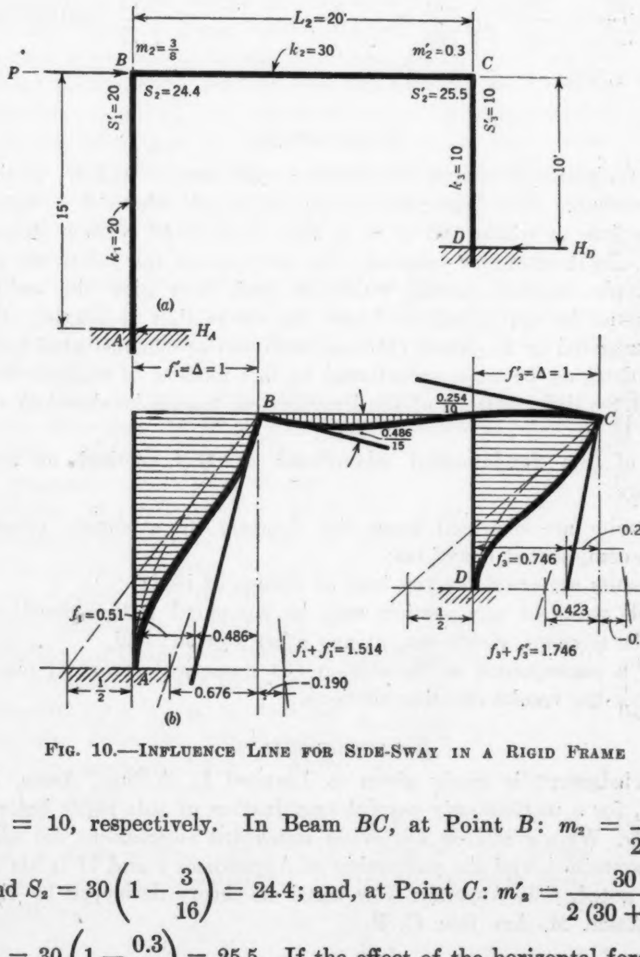

FIG. 10.—INFLUENCE LINE FOR SIDE-SWAY IN A RIGID FRAME

and $S'_3 = 10$, respectively. In Beam BC, at Point B: $m_2 = \dfrac{30}{2\,(30 + 10)} = \frac{3}{8}$; and $S_2 = 30\left(1 - \dfrac{3}{16}\right) = 24.4$; and, at Point C: $m'_2 = \dfrac{30}{2\,(30 + 20)} = 0.3$; and, $S'_2 = 30\left(1 - \dfrac{0.3}{2}\right) = 25.5$. If the effect of the horizontal force, P, is to deflect the frame sidewise any distance, Δ, with respect to Points A and D (assuming Joints B and C as pin-connected), the next step is similar to that shown in Fig. 3(*b*), with the result that, $f' = \Delta = 1$ and $f = -\dfrac{\Delta}{2} = -0.5$. Restoring the rigidity in the joints by applying counter-rotations: $1 - f_1 = \dfrac{3 \times 20}{2\,(20 + 24.4)} = 0.676$; and $1 - f_3 = \dfrac{3 \times 10}{2\,(10 \times 25.5)} = 0.423$, the adjustment being

in the ratio of $m'_2 = 0.3$, and $m_2 = 0.375$. Consequently, $f_1 = 1 - 0.676 + 0.3 \times 0.423 \times \frac{15}{10} = 0.514$; $f_3 = 1 - 0.423 + 0.375 \times 0.676 \times \frac{10}{15} = 0.746$; $f_1 + f'_1 = 0.514 + 1 = 1.514$; and, $f_3 + f'_3 = 0.746 + 1 = 1,746$. Therefore, $\frac{H_A}{H_D} = \frac{20}{10} \times \frac{1.514}{1.746} \times \frac{10^2}{15^2} = 0.77$; $\frac{H_A}{P} = \frac{0.77}{1.77} = 0.435$; and, $\frac{H_D}{P} = \frac{1.0}{1.77} = 0.565$.

Conclusions

When analyzing structures the relative stiffnesses, S and S', of the members are computed first, beginning at the end panels where $S = 0.75\,k$ when the end is free to rotate and $S = k$ when it is fixed. Then, imposing an imaginary displacement or rotation, the rotations at the joints are obtained in the simple manner shown, which at each load give the ordinate by which it must be multiplied to obtain the stress that is sought. Side-way must be computed by Equation (15) and analyzed as demonstrated by Fig. 10. Until the designer becomes accustomed to this method of analysis it is suggested that the deformations of the hypothetical models be sketched as shown in Figs. 7 to 10.

Some of the fundamental advantages of this method of structural analysis are:

(1) Results are obtained from the diagram by a simple process that eliminates complicated equations;

(2) Results are exact for any load or change of load;

(3) The stress at any section may be computed independently without knowing the moment, shear, etc., at any other section; and,

(4) As a consequence of Conclusion (3) numerical errors at one section do not affect the results of other sections.

Acknowledgments

Acknowledgment is freely given to Bernard L. Weiner, Assoc. M. Am. Soc. C. E., for a meticulously careful examination of this paper before publication. Mr. Weiner offered numerous important suggestions for clarifying the ideas contained, and the authorship of Appendices I and II is his entirely, except, as noted, that Appendix I is based on theory developed by Henry G. Babcock, Assoc. M. Am. Soc. C. E.

APPENDIX I

Theory of the Analysis of "Unloaded Models"

Models used for the analysis of structures may be divided roughly into two classes: "Loaded Models" and "Unloaded Models." The first type is simply a small-scale reproduction of the full-sized structure to which loads are

applied which are proportional to the loads that will presumably be carried by the actual structure. No loads are applied to the second type of model, but, instead, a known deformation is introduced at one point of the model and the deflection at some other point is measured. The stresses are obtained from the ratio of these two deformations as will be shown subsequently.

The two types of models differ in the manner in which the required information is obtained. In the first case, the action of the real structure is reproduced in the model and the data are obtained by direct measurement of the quantities involved. In the second case, the manipulation to which the model is subjected has no direct counterpart in the actual structure and the results are obtained by a mathematical relationship which exists between the deformations and the loads acting in the prototype A well known example of this latter type is that developed by George E. Beggs, M. Am. Soc. C. E.[3]

The derivation[4] which follows was developed by Henry G. Babcock, Assoc. M. Am. Soc. C. E., as a proof of the validity of the Beggs method, but it applies equally well to any other type of model, or to an imaginary model as is the case in this paper.

Let P (Fig. 11) be a load acting at any point, C, of a structure, in any direction whatever; let R be the component, in any assumed direction, of the reaction of Point A due to P; let R be gradually diminished to zero; let the point of application, A, of R be constrained to move in the line of action of R; let Δ_1 be the deflection of Point A when $R = 0$; and, let Δ_2 equal the component of that part of the deflection of Point C in the direction of P which is due to the removal of the reaction, R (thus, Δ_2 does not include the deflection of Point C due to the application of P while Point A is fixed).

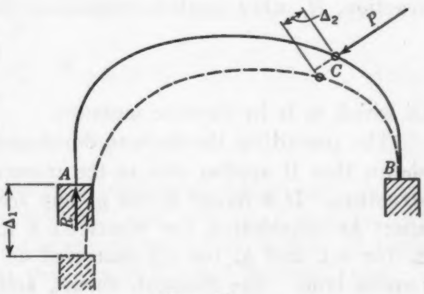

FIG. 11.

As Point C moves through the distance, Δ_2, in the direction of P, the total work done is $P\,\Delta_2$. Part of this work is done against the diminishing reaction, R, and the remainder, ϵ, is the internal work done on the structure. Evidently,

$$P\,\Delta_2 = \frac{R\,\Delta_1}{2} + \epsilon \qquad (26)$$

To evaluate ϵ, remove P and permit Point A to return to its original position. Now, apply a force at Point A, gradually increasing it from zero to $-R$; that is, in a direction opposite to that of $+R$. Since every force produces its own characteristic deflections at every point of a structure, independently of any other forces acting, it follows that the load, $-R$ acting alone will pro-

[3] *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1208.

[4] Not previously published.

duce the same deflections, Δ_1, at Point A as was produced by the removal of $+R$ when P was acting. It will also produce the same deflections at every other point of the structure and, hence, the same internal work, ϵ; but, without P acting, $\frac{R\,\Delta_1}{2}$ is the total external work and ϵ the total internal. Therefore,

$$\epsilon = \frac{R\,\Delta_1}{2} \qquad (27)$$

Substituting in Equation (26), $P\,\Delta_2 = \frac{R\,\Delta_1}{2} + \frac{R\,\Delta_1}{2}$; and,

$$R = P\,\frac{\Delta_2}{\Delta_1} \qquad (28)$$

Since the ratio, $\frac{\Delta_2}{\Delta_1}$, is independent of the value of P, it may be determined by moving Point A through any arbitrary known distance in the direction of R and measuring the corresponding component of the deflection at Point C in the direction of P.

The preceding derivation can be extended to include moments as well as forces. If a rotation, α_1, is substituted for Δ_1, and a moment, M, for the reaction, R, other symbols remaining the same:

$$M = \frac{P\,\Delta_2}{\alpha_1} \qquad (29)$$

in which α_1 is in circular measure.

The preceding theory was developed for a reaction, but it can be easily shown that it applies also to the moment, thrust, or shear at any point of a structure. If a model is cut at any section and the two cut ends are moved apart by translation (or rotation) a known amount relative to each other, Δ_1 (or α_1), and Δ_2 (or α_2) measured as before, Equations (28) and (29) still remain true. The moment, thrust, and shear can thus be found directly at any point of a structure. Finding reactions is, therefore, only a special case of finding the stresses at any point in general.

Application

The theory presented in this paper can be derived without reference to models. The usual theories deal directly with the loads and their reactions and stresses. Here, an indirect method based on the deflections of a model is used, and simplifications result. If instead of discussing more or less abstract deflections, a model is imagined to be manipulated in the same manner as a physical model is deformed by gages, and the relationship between the deflections established mathematically instead of by measurement as in a actual model, the understanding of the theory is simplified.

In the derivations which follow, the customary theories are used only in so far as they are necessary to develop the equations for the deflections. Once these relations are established, stresses are obtained by "model analysis."

APPENDIX II

Analytical Derivation of Equations

Fig. 12 shows a simply supported beam subjected to a typical deformation, which forms a simple influence line. It can be easily shown, by the usual methods,

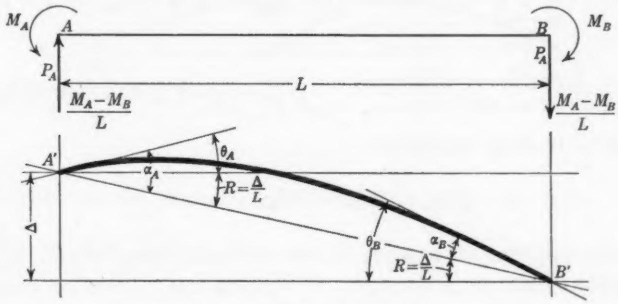

Fig. 12.

that: $\alpha_A = \frac{M_A}{3\ E\ k} + \frac{M_B}{6\ E\ k}$; and, $\alpha_B = \frac{M_A}{6\ E\ k} + \frac{M_B}{3\ E\ k}$; whence, $\theta_A = \frac{M_A}{3\ E\ k} + \frac{M_B}{6\ E\ k} - R$; and, $\theta_B = \frac{M_A}{6\ E\ k} + \frac{M_B}{3\ E\ k} + R$. Solving for M_A and M_B:

$$M_A = 2\ E\ k\ (2\ \theta_A - \theta_B + 3\ R) \qquad \text{(30a)}$$

and,

$$M_B = 2\ E\ k\ (-\ \theta_A + 2\ \theta_B - 3\ R) \qquad \text{(30b)}$$

Equations (30) are for the fundamental deformed beam used in the development of the "slope deflection method" and will be found in any standard text on statically indeterminate structures. The system of signs used herein, however, is different from that generally used in these texts.

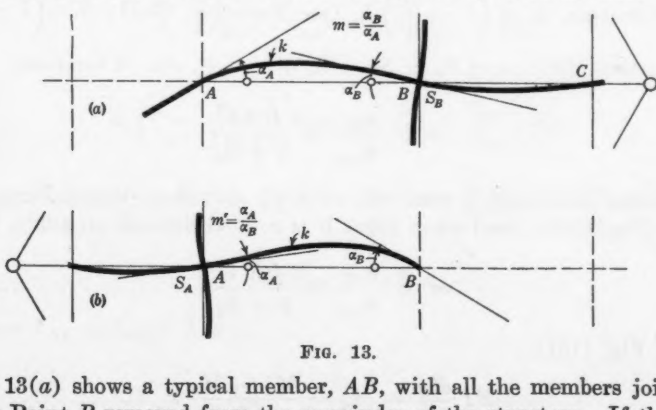

Fig. 13.

Fig. 13(*a*) shows a typical member, *AB*, with all the members joining the beam at Point *B* removed from the remainder of the structure. If the End *A*

is rotated through an angle, θ_{AB}, this part of the structure will deform as shown. The relation between θ_{AB} and θ_{BA} will now be found (side-sway being prevented so that $\theta = \alpha$) from Equations (30) and Fig. 14 as follows:

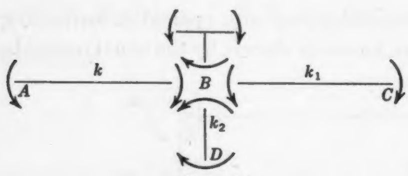

FIG. 14.

$$M_{BA} = E\,(4\,k\,\theta_{BA} - 2\,k\,\theta_{AB}) \;..(31a)$$

$$M_{BC} = E\,(4\,k_1\,\theta_{BC} - 2\,k_1\,\theta_{CB}) \;..(31b)$$

and,

$$M_{BD} = E\,(4\,k_2\,\theta_{BD} - 2\,k_2\,\theta_{DB}) \;..(31c)$$

Since $M = 0$ about any joint:

$$M_{BA} = M_{BC} + M_{BD} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(31d)$$

Since the rotation at a joint is the same at the ends of all members meeting at the joint:

$$\theta_{BC} = \theta_{BD} = -\,\theta_{BA} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(32)$$

and,

$$(k + k_1 + k_2)\,\theta_{BA} + 0.5\,k_1\,\theta_{CB} + 0.5\,k_2\,\theta_{DB} = 0.5\,k\,\theta_{AB} \;\ldots\ldots(33)$$

By definition, since $\theta = \alpha$ for this case, $\dfrac{\theta_{BA}}{\theta_{AB}} = m$; $\dfrac{\theta_{CB}}{\theta_{BC}} = \dfrac{\theta_{CB}}{\theta_{BA}} = m_1$; and, $\dfrac{\theta_{DB}}{\theta_{BD}} = \dfrac{\theta_{DB}}{\theta_{BC}} = m_2$. . Consequently, it follows that:

$$m = \frac{\theta_{BA}}{\theta_{AB}} = \frac{0.5\,k}{k + \left(1 - \dfrac{m_1}{2}\right)k_1 + \left(1 - \dfrac{m_2}{2}\right)k_2} \;\ldots\ldots(34a)$$

By definition, $S_1 = \left(1 - \dfrac{m_1}{2}\right)k_1$ (see Equation (3*a*)); $S_1 = \left(1 - \dfrac{m_2}{2}\right)k_2$ (see Equation (3*b*)); and $S_B = S_1 + S_2 + \ldots +$, etc. Therefore:

$$m = \frac{\theta_{BA}}{\theta_{AB}} = \frac{0.5\;k}{k + S_B} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(34b)$$

Similarly, when AB is removed, with all members joining Beam AB at End A, (Fig. 13(*b*)), and when Joint B is rotated through an angle, θ_{BA}:

$$m' = \frac{\theta_{AB}}{\theta_{BA}} = \frac{0.5\;k}{k + S_A} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(34c)$$

From Fig. (15):

$$E\,I\,\frac{d^2 y}{dx^2} = M_A - \frac{M_A - M_B}{L}\,x \;\ldots\ldots\ldots\ldots\ldots(35)$$

Integrating twice and remembering that $y = 0$ when $x = 0$ and $x = L$, and substituting the values of M_A and M_B from Equations (30):

$$y = \frac{x\,x'}{L^3}\,(x\,f + x'\,f') \quad \text{.....................} (36)$$

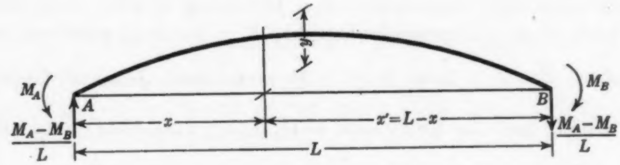

FIG. 15

Equation (36) is the formula of the elastic curve in terms of the end rotations.

Fig. 16 shows two beams, AB and BC, joined at Point B and rotated with respect to each other through an angle of 1, to produce the influence

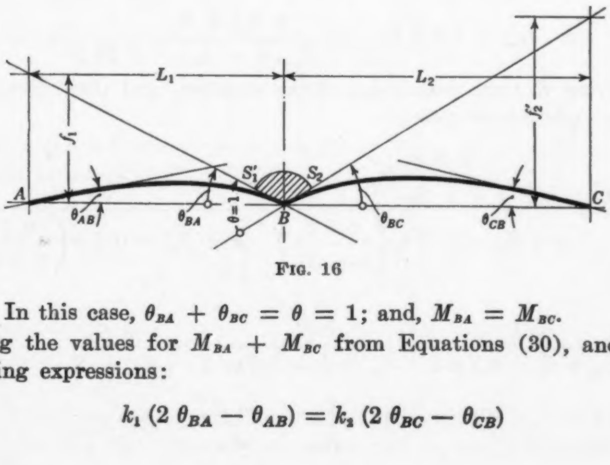

FIG. 16

line, M_B. In this case, $\theta_{BA} + \theta_{BC} = \theta = 1$; and, $M_{BA} = M_{BC}$.

Writing the values for $M_{BA} + M_{BC}$ from Equations (30), and equating the resulting expressions:

$$k_1\,(2\,\theta_{BA} - \theta_{AB}) = k_2\,(2\,\theta_{BC} - \theta_{CB})$$

or

$$k_1\left(1 - \frac{\theta_{AB}}{2\,\theta_{BA}}\right)\theta_{BA} = k_2\left(1 - \frac{\theta_{CB}}{\theta_{BC}}\right)\theta_{BC}$$

Since $\frac{\theta_{AB}}{\theta_{BA}} = m'_1$; $\frac{\theta_{CB}}{\theta_{BC}} = m_2$; $1 - \frac{m'_1}{2} = S'_1$, and $1 - \frac{m_2}{2} = S_2$:

$$S'_1\,\theta_{BA} = S_2\,\theta_{BC}$$

Since $\theta_{BA} + \theta_{BC} = \theta = 1$:

$$\theta_{BA} = \frac{S_2}{S_1 + S_2} = \frac{S_2}{K_B} \quad \text{....................} (37a)$$

and,

$$\theta_{BC2} = \frac{S'_1}{K_B} \quad \text{.......................... (37b)}$$

The following equations for side-sway are derived in a manner similar to that used for obtaining Equations (34). Referring to Fig. 3(*b*), using Equations (30) once more and remembering that $R = 0$ for all members except AB, for which $R = \frac{\Delta}{L}$, it follows from $\Sigma M = 0$ about Joint B (repeating the operations which lead to Equations (32), (33), and (34*a*) and (34*b*), that the last equations take the following forms when $R = \frac{\Delta}{L}$ instead of zero:

$$(k + S_B)\,\theta_{BA} - 0.5\,k\,\theta_{AB} = 1.5\,k\,R \quad \text{.............. (38a)}$$

and,

$$0.5\,k\,\theta_{BA} - (k + S_A)\,\theta_{AB} = 1.5\,k\,R \quad \text{.............. (38b)}$$

Solving these two equations simultaneously for θ_{BA}:

$$\theta_{BA} = 1.5\,k\,R\,\frac{0.5\,k + S_A}{(k + S_A)\,(k + S_B) - 0.25\,k^2} \quad \text{........... (39)}$$

Subtracting R from both sides of the equation and then dividing by R, simplifying and re-arranging:

$$\frac{(\theta_{BA} - R)\,L}{R\,L} = \frac{\dfrac{0.5\,k}{k + S_B}\,S_A}{k + S_A - 0.5\,k\left(\dfrac{0.5\,k}{k + S_B}\right)} - \frac{S_B}{k + S_B - 0.5\,k\left(\dfrac{0.5\,k}{k + S_A}\right)} \quad \text{. (40)}$$

and,

$$\frac{f}{\Delta} = \frac{m\,S_A}{S_A + k - 0.5\,m\,k} - \frac{S_B}{S_B + k - 0.5\,m'\,k} = \frac{m\,S_A}{S_A + S} - \frac{S_B}{S_B + S'}$$

$$= -\frac{S_B}{K_B} + \frac{m\,S_A}{K_A} \quad \text{................... (41a)}$$

Similarly,

$$\frac{m'}{\Delta} = \frac{S_A}{K_A} - \frac{m'\,S_B}{K_B} \quad \text{..................... (41b)}$$

The force, P_A, necessary to displace Point A in Figs. 3 and 4 is easily derived. Referring to Fig. 12, $P_A = \frac{(M_A - M_B)}{L}$. Substituting the values of M_A and M_B from Equations (30):

$$P_A = \frac{6\,E\,k}{L}\,(\theta_A - \theta_B + 2\,R) = \frac{6\,E\,k}{L}\,[(\theta_A + R) - (\theta_B - R)]$$

$$= \frac{6\,E\,k}{L}\,(\alpha_A - \alpha_B) \quad \text{...................... (42)}$$

and,

$$\frac{P}{E} = \frac{6\,k}{L^2}\,(-f + f') \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

Equation (43) can also be used to obtain the correction for side-sway.

Influence Lines for Bending Moments

Fig. 17 shows a typical member, AB, broken at Point G and rotated through an angle, θ, by means of the gage. The forces acting at Point G must be the moment, M, and a shear, V, shown in Fig. 18.

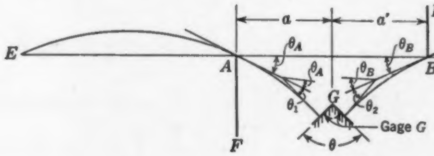

Fig. 17

From Fig. 17, the angle of the gage is:

$$\theta_A + \theta_1 + \theta_B + \theta_2 = \theta \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

Furthermore, the deflection normal to AB at Point G, must be the same for both broken members, AG and BG.

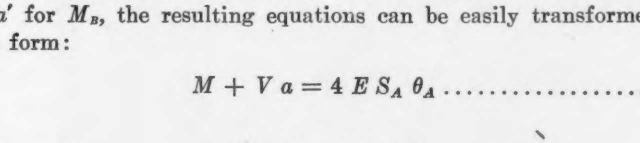

Fig. 18

Using Equations (30), once more and writing $M + V\,a$ for M_A, and $M - V\,a'$ for M_B, the resulting equations can be easily transformed to the following form:

$$M + V\,a = 4\,E\,S_A\,\theta_A \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45a)$$

and,

$$M - V\,a' = 4\,E\,S_B\,\theta_B \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45b)$$

or,

$$\frac{M\,L}{E} = 4\,(a'\,S_A\,\theta_A + a\,S_B\,\theta_B) \;\ldots\ldots\ldots\ldots\ldots\ldots (46a)$$

and,

$$\frac{V\,L}{E} = 4\,(S_A\,\theta_A - S_B\,\theta_B) \;\ldots\ldots\ldots\ldots\ldots\ldots (46b)$$

The equations for the angular and linear deflections at Point G, are:

$$\theta_A + \frac{M\,a}{E\,I} + \frac{V\,a^2}{2\,E\,I} + \theta_B + \frac{M\,a'}{E\,I} - \frac{V\,(a')^2}{2\,E\,I} = \theta \quad \text{........(47)}$$

and,

$$a\,\theta_A + \frac{M\,(a')^2}{2\,E\,I} + \frac{V\,a^3}{3\,E\,I} = a'\,\theta_B + \frac{M\,(a')^2}{2\,E\,I} - \frac{V\,(a')^3}{3\,E\,I} \quad \text{.......(48)}$$

Substituting Equation (46) in Equations (47) and (48):

$$(k + 2\,S_A)\,\theta_A + (k + 2\,S_B)\,\theta_B = k\,\theta \quad \text{...............(49)}$$

and,

$$3\,k\,a'\,\theta_A + 2\,(2\,a - a')\,S_A\,\theta_A = 3\,k\,a'\,\theta_B - 2\,(a - 2\,a')\,S_B\,\theta_B \quad \text{....(50)}$$

Solving Equations (49) and (50) for θ_A and θ_B, and remembering that $a' + a = L$; $L\,\theta_B = f$; $L\,\theta_A = f'$; $S = \dfrac{0.75\,k + S_A}{k + S_A}$; $K_A = S + S_A$; etc., the resulting equations can be put into the form given in Equations (16*a*) and (16*b*).

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

STANDARD PRACTICE IN SEPARATE SLUDGE DIGESTION

PROGRESS REPORT OF THE COMMITTEE OF THE SANITARY ENGINEERING DIVISION ON SLUDGE DIGESTION

Introduction

Satisfactory disposal of sludge is an important part of the treatment of sewage, from the standpoint of cost as well as of sanitation. It has long been a difficult problem, and many processes and combinations of processes have been experimented with and developed.

At present, primary sludge is usually digested in separate compartments or in the lower part of an Imhoff tank, dried on sand beds, and disposed of either as waste or as fertilizer. Mechanical dewatering has been suggested as an alternate to the sand-bed for large projects. Activated sludge is frequently dewatered, dried, and sold as fertilizer, but it, also, may be digested with primary sludge in separate tanks.

Although there is doubt as to the first use of the method known as separate digestion, several of the earliest sludge-treatment plants employed it. The late Rudolph Hering, M. Am. Soc. C. E.,[1] states that, in 1884 and 1885, separate digestion tanks were used in tests at the Pimlico Pumping Station, in London, England, under the direction of Mr. W. J. Dibdin. The method was again used about 1899, at the Lawrence (Mass.) Sewage Testing Station, and the results are discussed in the Annual Reports for that Station for 1899 and 1900.

In the first decade of the present century, Imhoff and Travis developed the 2-story tank. Most of the sludge-digestion plants installed during the second decade was of that type, but since about 1920 the trend has been decidedly toward the adoption of the separate system; at present, several hundred such plants are in operation in the United States.

Structures and equipment for separate digestion of sludge may account for from 20 to 35% of the total cost of the sewage treatment works. They include digestion tanks, storage tanks, pumping stations, drying beds, meters, boilers, buildings, etc. Ample capacity and good design as to detail of

NOTE.—Discussion on this report will be closed in April, 1937, *Proceedings*.

[1] *Engineering News*, August 14, 1913.

arrangement and of mechanical equipment are required, as a deficiency in these points not only hinders digestion, but may result in reducing the efficiency of sedimentation and other processes thus lowering the entire standard of plant operation.

Good design alone is not enough, however; even in the best plant, satisfactory operation depends upon the operator's knowledge of the basic principles involved, and upon his constant attention to the proper methods of testing and routine procedure.

Adoption of the separate digestion method and progress in sludge-treatment research have been so rapid that on the whole neither the principles of design nor the methods of operation have been thoroughly correlated. The purpose of this report, therefore, is to state the fundamental aspects of sludge digestion and to assemble in useful form worth-while information relating to the design and operation of separate sludge-treatment plants.

The report begins with a description of sewage and of the end products of digestion—gas and digested sludge. Important environmental factors in the digestion process are then discussed. Next, the various types of sludge-digestion and gas-utilization structures and equipment are described, and methods for computing the required capacity of tanks and heating units are presented. The following section outlines the various proposals for disposing of sludge after digestion, and the report closes with a discussion of operating routine. Two Appendices are included, giving data on the operation of typical plants.

I.—CHARACTERISTICS OF SEWAGE AND OF THE END PRODUCTS OF DIGESTION

SEWAGE

The water-borne wastes of a modern city are made up of such a great variety of compounds that only a general discussion of sewage characteristics is practicable. Data on the composition of sewage have been secured from thirty cities representing a total contributing population of 4 760 000, and are summarized in Tables 1 and 2. It will be noted that the average daily flow per capita varies from 308 gal to 48 gal, and that a marked trade-waste contribution was reported by ten cities, while this factor was negligible in the other nineteen. Fifteen of the cities reported combined sewerage systems, and fourteen, separate systems.

Waste products are present in sewage as solids, colloids, and solutions. The solids, in turn, may be classified as volatile or organic, and fixed or mineral (sometimes called "inert"); or, with respect to their settleability, they may be graded from the readily settleable to the practically non-settleable. Colloids ordinarily undergo a marked reduction when sewage treatment is carried to a high degree of purification. The reduction, however, can not be accomplished by sedimentation unless that process is preceded by some special treatment adapted to that end. The materials in solution, also, are obviously not susceptible to removal by plain sedimentation, although certain of the soluble ingredients of the incoming sewage may stimulate sedimenta-

tion. (They may also retard or accelerate the digestion of the sludge.) Any changes that the dissolved wastes undergo in their passage through the settling tanks is due to bio-chemical agencies.

One factor having an important bearing on the accuracy and comparability of analytical data is the state of comminution of the solids in the samples on which the data are based. A sample from sewage in which very little comminution has taken place may be too high or too low in solids, depending on whether or not the coarse particles happen to have been included. More truly representative samples may be obtained in cases where the sewage has been subjected to pumping, or to flow at high velocity, or even simply

TABLE 1.—Characteristics of Sewage of Typical Cities of the United States

(From Data Secured in 1930. Total Contributing Population, 4 760 000)

Item No.	City	Tributary population, in thousands (as of 1930)	Average sewage flow, in gallons per capita per day	Type of sewerage system‡	Trade waste content	Total Solids, in Parts per Million			Number of samples on which value is based§	Suspended Solids				
										Total Suspended Solids			Volatile Suspended Solids	
						Total	Volatile	Fixed		Actual, in parts per million	Parts per million adjusted to sewage flow of 100 gal per capita per day	Pounds per capita per day	Actual, in parts per million	Parts per million adjusted to sewage flow of 100 gal per capita per day
	(1)	2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1	Akron, Ohio.......	220	156	C	Yes	2 069	870	1 199	W	377	588	0.491	285	445
2	Alliance, Ohio......	24	97	S	No	880	406	474	52	174	169	0.140	124	120
3	Aurora, Ill..........	47	122	C	..	909	...	...	...	167	...	.....	125	...
4	Baltimore, Md.....	735	82	S	No	.....	...	...	...	266	...	0.182	...	...
5	Brockton, Mass.....	54	51	S	Yes	671	383	288	W	245	125	0.104	198	101
6	Canton, Ohio.......	100	97	S	No	.....	...	...	...	288	279	0.233	...	...
7	Cleveland, Ohio....	...	...	C	Yes	678	...	...	...	184	...	.....	...	...
8	Columbus, Ohio....	300	97	C	No	986	396	590	172	222	215	0.178	175	170
9	Dallas, Tex........	273	73	S	..	1 334	605	729	150	342	250	0.208	258	188
10	Decatur, Ill........	45	306	C	Yes	.....	...	...	365	200	612	0.510	100	306
11	Delaware, Ohio.....	10	48	S	No	.....	...	...	W	235	113	0.094	180	86
12	Elyria, Ohio........	21	71	C	..	666	297	369	250	224	159	0.133	171	121
13	Fitchburg, Mass....	40	58	C	No	398	200	198	...	140	81	0.068	88	51
14	Flint, Mich	130	75	S	..	860	484	376	365	225	169	0.141	168	126
15	Grand Rapids, Mich.	168	145	C	No	1 246	728	518	16	221	274	0.229	135	196
16	High Point, N. C...	25	97	S	Yes	738	...	...	365	224	217	0.181	...	...
17	Houston, Tex......	...	...	..	..	.....	...	...	...	246	...	.....	...	...
18	Indianapolis, Ind...	328	140	C	Yes	1 040	...	...	365	319	447	0.373	...	...
19	Lincoln, Nebr......	80	50	S	No	265	174	81	...	265	132	0.110	...	...
20	Madison, Wis......	32	130	S	No	785	...	...	19	156	203	0.169	121	157
21	Marion, Ohio......	23	70	S	No	1 261	...	...	W	238	167	0.139	180	126
22	Milwaukee, Wis....	700	121	C	Yes	1 021	471	550	615	328	397	0.331	218	264
23	Pasadena, Calif.....	115	74	S	No	...	...	...	W	290	215	0.179	247	183
24	Philadelphia, Pa....	175	188	C	No	.....	...	...	416	188	353	0.295	124	233
25	Pontiac, Mich......	56	91	S	Yes	809	436	373	184	359	327	0.273	181	165
26	Rochester, N. Y.*...	298	169	C	..	615	310	305	365	141	238	0.199	99	167
27	Rochester, N. Y.†...	15	117	S	No	689	355	334	W	166	194	0.163	121	142
28	Springfield, Mass...	67	112	C	No	.....	...	...	300	186	208	0.174	116	130
29	Syracuse, N. Y.....	200	142	C	No	.....	...	...	365	218	310	0.258	...	...
30	Worcester, Mass....	190	92	C	Yes	785	321	464	M	251	231	0.193	154	142
31	Dayton, Ohio......	...	92	..	..	....	...	...	...	202	186	0.155	167	154

* Irondequoit Works. † Brighton Works. ‡ C=combined, and S=separate systems. § W=Weekly composite and M=Monthly composite. In general, analyses were made on composite samples.

TABLE 1.—(*Continued*)

Item No.	Suspended Solids (*Continued*)		Settleable Solids						Five-Day Bio-Chemical Oxygen Demand at 20°C		Ether soluble matter, in parts per million	pH	Chlorine as chlorides, in parts per million	Immediate chlorine demand	N as NH_3 in parts per million
			In Cubic Centimeters per Liter		In Parts per Million										
	Volatile suspended solids, in pounds per capita per day	Fixed suspended solids, actual, in parts per million	Actual	Adjusted to sewage flow of 100 gal per capita per day	Actual	Adjusted to sewage flow of 100 gal. per capita per day	In pounds per capita per day	In percentage of total suspended solids	In parts per million	In pounds per capita per day					
	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)
1	0.371	1 199	....	.....	305	476	0.397	81	146	0.190	....	7.2	717	25.5	8.5
2	0.100	474	5.2	5.1	...	.....	.....	..	193	0.156	....	...	72	....	17.8
3	....	....	3.5	.....	...	.....	.....	..	120	.....	....	7.6	156	....	.....
4	....	....	....	.....	164	135	0.112	62	235	0.209	....	...	.....	....	.....
5	0.084	288	....	.....	...	.....	.....	..	...	.....	....	6.9	85	....	37.9
6	....	....	5.1	4.9	224	217	0.181	78	202	0.163	....	7.3	79	....	....
7	....	....	2.9	.....	...	.....	.....	..	163	.....	....	7.1	67	....	17.5
8	0.141	590	3.9	3.8	120	116	0.097	54	226	0.182	....	7.1	92	6.7	.....
9	0.157	729	9.1	6.7	...	.....	.....	..	445	0.271	....	7.5	.....	....	13.2
10	0.255	....	2.8	8.5	...	.....	.....	..	154	0.392	....	7.0	.....	9.5	8.0
11	0.074	....	5.3	2.5	...	.....	.....	..	344	0.138	....	8.0	.....	....	.....
12	0.101	369	3.5	2.5	...	.....	.....	..	171	0.101	....	...	53	....	16.3
13	0.043	198	....	.....	93	54	0.045	66	...	.....	....	...	43	....	10.5
14	0.105	376	5.0	3.7	...	.....	.....	..	280	0.175	48.0	7.6	99	12.0	7.9
15	0.163	518	4.4	.....	...	.....	.....	..	169	0.274	....	7.1	.....	....	.....
16	....	....	....	.....	...	.....	.....	..	358	0.289	....	...	.....	....	16.9
17	....	....	....	.....	...	.....	.....	..	193	.....	....	7.6	.....	....	18.2
18	....	....	....	.....	...	.....	.....	..	338	0.395	39.3	7.7	.....	7.9	.....
19	0.072	81	3.3	1.7	...	.....	.....	..	...	.....	....	...	.....	....	.....
20	0.131	....	6.5	8.5	...	.....	.....	..	216	0.234	....	7.4	.....	....	14.1
21	0.105	....	3.7	2.6	...	.....	.....	..	270	0.158	....	7.4	89	....	21.1
22	0.220	550	....	.....	...	.....	.....	..	269	0.272	52.4	7.3	.....	....	.....
23	0.152	....	9.1	6.7	...	.....	.....	..	257	0.159	....	7.2	.....	....	19.0
24	0.194	....	4.6	8.6	...	.....	.....	..	204	0.320	52.0	...	78	....	10.0
25	0.137	373	6.6	6.0	...	.....	.....	..	213	0.162	....	...	.....	....	.....
26	0.139	305	3.1	8.3	...	.....	.....	..	233	0.329	....	7.1	69	....	8.9
27	0.118	334	7.1	5.2	...	.....	.....	..	244	0.239	....	7.1	49	....	8.3
28	0.108	....	4.1	4.6	...	.....	.....	..	166	0.155	....	7.1	51	....	12.1
29	....	....	....	.....	148	210	0.175	68	...	.....	....	7.0	.....	....	.....
30	0.118	464	10.1	9.3	...	.....	.....	..	212	0.163	....	6.8	96	....	14.6
31	0.129	....	5.6	5.1	...	.....	.....	..	...	.....	....	...	.....	....	.....

to flow through a great length of sewer, for in such cases the suspended-matter content is likely to be more evenly distributed.

Other conditions influencing the reliability of data are: (1) Correctness of the measurement of flow and of the count of the contributing population; (2) the method of sampling and the number of samples taken; and (3) the accuracy of the analytical methods.

It will be noted that nineteen cities reported settleable solids in cubic centimeters per liter, and six in parts per million. The latter method has the advantage of expressing the results as a definite quantity, but the data are not as valuable as they would be if this method were in general use, because the results cannot be compared with those expressed in cubic centimeters per liter.

Digested Sludge

The characteristics of digested sludge depend largely upon whether the contributing sewerage system is separate or combined, and upon the pro-

TABLE 1.—(*Continued*)

Item No.	Free ammonia, in pounds per capita per day	Organic Nitrogen: Total: In parts per million	Organic Nitrogen: Total: In pounds per capita per day	Organic Nitrogen: Suspended: In parts per million	Organic Nitrogen: Suspended: In pounds per capita per day	Organic Nitrogen: Dissolved, in parts per million	Iron as Fe, in Parts per Million: Total	Iron as Fe, in Parts per Million: Suspended	Iron as Fe, in Parts per Million: Fixed	Composition of Sludge: Specific gravity	Composition of Sludge: Percentage, H_2O	Composition of Sludge: pH	Composition of Sludge: Percentage of volatile matter	Composition of Sludge: Percentage of ether-soluble matter
	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)	(38)	(39)	(40)	(41)	(42)	(43)
1	0.011	21.0	0.027	8.8	0.011	12.2	....	....	....	1.08	89.9	6.5	51.1	10.6
2	0.014	18.8	0.010	6.4	0.005	6.4	....	....	....	1.02	95.0	7.4	57.0	.....
3	......	......	......	......	......	......	....	....	....	....	....	....	....	.....
4	......	......	......	......	......	......	....	....	....	....	93.9	6.7	61.2	.....
5	0.009	......	......	......	......	......	....	....	....	1.02	94.0	5.3	....	.....
6	......	......	......	......	......	......	....	....	....	1.03	94.0	6.7	65.0	.....
7	......	10.7	......	8.4	......	2.3	10.4	....	....	1.05	86.8	7.1	40.3	8.3
8	......	......	......	......	......	......	....	....	....	....	....	....	....	.....
9	0.008	29.0	0.018	......	......	......	....	....	....	....	95.0	6.9	....	.....
10	0.020	13.0	0.033	......	......	......	....	....	....	....	93.0	7.0	38.0	3.5
11	......	......	......	......	......	......	....	....	....	1.03	94.1	....	57.8	.....
12	0.010	7.4	0.004	3.7	0.002	3.7	....	....	....	1.04	92.7	7.2	42.0	.....
13	0.005	......	......	......	......	......	....	....	....	1.05	85.6	7.1	34.4	.....
14	0.005	28.0	0.017	......	......	......	....	....	....	....	92.8	6.6	48.3	10.1
15	......	......	......	......	......	......	....	....	....	....	95.1	7.4	67.7	.....
16	0.014	10.2	0.008	......	......	......	....	....	....	....	88.4	7.8	40.5	6.3
17	......	......	......	......	......	......	....	....	....	1.05	....	....	65.0	.....
18	......	28.3	0.039	......	......	......	6.4	....	....	1.07	80.0	....	45.0	3.1
19	......	......	......	......	......	......	....	....	....	....	89.2	....	....	.....
20	0.015	18.5	0.020	......	......	......	....	....	....	1.02	95.0	7.4	50.0	10.0
21	0.012	13.3	0.008	7.9	0.005	5.4	....	....	....	1.02	92.8	8.1	50.0	8.0
22	......	31.8	0.032	......	......	......	35.0	....	....	....	....	....	....	.....
23	0.012	20.8	0.013	......	......	......	....	....	....	....	....	....	....	.....
24	0.016	26.0	0.041	......	......	......	....	....	....	1.03	90.4	....	48.0	17.7
25	......	......	......	......	......	......	....	....	....	....	91.6	7.0	53.9	.....
26	0.012	......	......	......	......	......	....	....	....	....	92.8	6.6	49.6	11.6
27	0.008	......	......	......	......	......	....	....	....	....	88.7	6.6	46.9	8.3
28	0.011	19.6	0.013	......	......	......	....	....	....	....	90.4	7.0	40.7	.....
29	......	......	......	......	......	......	....	....	....	....	98.0	7.0	45.0	.....
30	0.011	13.3	0.010	6.7	0.005	6.6	47.2	24.0	23.2	1.03	93.3	6.8	48.8	.....
31	......	......	......	......	......	......	....	....	....	....	....	....	....	.....

TABLE 2.—Résumé of Sewage Characteristics, Based on a Sewage Flow of 100 Gallons per Capita per Day

(Computed from Data in Table 1 for Typical Cities of the United States)

Item	Number of sewage works reporting	Contributing population, in thousands	Parts per Million: Maximum	Parts per Million: Minimum	Parts per Million: Average	Pounds per Capita per Day: Maximum	Pounds per Capita per Day: Minimum	Pounds per Capita per Day: Average
Total suspended solids	27	3 711	612	81	254	0.51	0.07	0.21
Volatile suspended solids	22	2 978	445	51	171	0.37	0.04	0.14
Settleable solids	19	2 144	9.3*	1.7*	5.4*	.....	.....	.....
Settleable solids	6	1 595	476	54	20	0.40	0.04	0.17
Bio-chemical oxygen demand, 5 days, at 68° F	22	3 337	473	121	262	0.39	0.10	0.22
pH-value	23	3 437	8.0†	6.8†	7.3†	.....	.....	.....
Immediate chlorine demand	6	1 213	39.8	6.5	.....	.....	.....	.....
Nitrogen as free ammonia	·18	1 937	34.5	6.1	14.3	0.020	0.005	0.012
Organic nitrogen	16	2 606	48.8	5.2	22.1	0.041	0.004	0.018

* Values are in cubic centimeters per liter. † Values are not parts per million.

portion of industrial wastes and the degree of treatment of the sewage. Sludge from combined sewers usually contains more mineral matter and less moisture than sludge from separate sewers. Abnormal quantities of oil, iron, or other chemicals may be due to industrial wastes. Fine screening, skimming, or removal of detritus in preliminary tanks will reduce the quantity of coarse suspended solids in the sludge.

Color is somewhat of an indicator of the degree of digestion, well-digested sludge being usually quite black or very dark gray. Color, however, depends somewhat upon local conditions—particularly upon the type and quantity of industrial wastes. The moisture content of digested sludge should be less than that of fresh sludge, although the reduction in some cases is not great. In general, well-digested sludge has the following characteristics:

Physical.—

(*a*) Texture: Uniform, black or dark colored, with no visual evidence of fresh sewage solids, such as feces, paper, or grease.

(*b*) Density: Thick; the moisture content of sludge on withdrawal from the tanks is generally not more than 90 per cent. (Sludge with as much as 95% moisture, however, may be thoroughly digested.)

(*c*) Odor: Faintly tarry; practically unobjectionable. After draining or drying, well-digested sludge should have an earthy or musty odor.

(*d*) Ability to drain on sand-beds: Good.

Chemical.—

(*a*) Mineral matter content: Substantially greater (expressed as a percentage) than that of the fresh sewage solids.

(*b*) Volatile matter content: 50%, or less than 50%, of the volatile matter content of fresh sludge.

(*c*) Organic content (of sludge removed from the drying beds): Not generally more than 50 per cent. The bio-chemical oxygen demand of this content should be substantially lower than that of the fresh solids.

(*d*) Hydrogen-ion concentration: pH usually between 7.0 and 7.6; that is on the alkaline side of neutrality.

It should be noted that the process of digestion reduces the value of the sludge for use as fertilizer by as much as two-thirds, because of the gasification of organic matter containing nitrogen.

Gas

Gas produced during the digestion of sludge consists principally of methane (CH_4) carbon dioxide, and nitrogen, but includes also small quantities of hydrogen, hydrogen sulfide, and other gases. Although the composition depends upon the character of the sewage and the stage to which digestion has progressed, the particular sources of the various constituents are not definitely known.

Table 3 shows the analyses of gases collected at various sewage works. The average composition, based on the data there given, is approximately 69% methane, 22% carbon dioxide, 6% nitrogen, and 3% hydrogen. In general, gas obtained at German installations contains more hydrogen and hydrogen sulfide than that produced in the United States.

TABLE 3.—AVERAGE ANALYSES AND HEAT VALUES OF GASES COLLECTED AT SEWAGE TREATMENT WORKS*

(Gas from Separate Digestion Tanks Unless Otherwise Noted)

Location	Date	Percentage by Volume							Heat value, in British thermal units per cubic foot
		CH_4	CO_2	N_2	O_2	H_2	H_2S	Other gases	
Antigo, Wis.	1930	62.0	31.4	3.4	0.6	2.6	......	.....	596 [l]
Aurora, Ill.[a]	1930	51.8	32.3	13.4	0.4	0.0	Slight	2.1 [i]	534 [l]
Aurora, Ill.[b]	1930	48.2	28.9	19.8	0.8	0.0	Slight	2.2 [i]	495 [l]
Baltimore, Md.	1930	.....	22.6–31	.....	.....	.....	......	.....	675–779 [l,m]
Berlin, Germany (Wassmannsdorf)	1929–30	69.59	17.77	8.36	0.0	4.15	0.13	.....	719
Birmingham, England (Saltley)	1930	69.29	28.02	1.89	0.26	0.05	0.0	0.49	685 [l]
Bochum, Germany	1929–30	64.2	22.1	7.1	.....	6.6	......	.....	687
Chicago, Ill. (Calumet)	1926–27	76.6	14.7	8.2	0.5	.....	......	.....	780
Dayton, Ohio	1930	76.6	14.0 [g]	9.0	0.4	0.0	Some [g]	.....	777
Decatur, Ill	.....	70–75	15–20	4–6	0	1–3	0–2	.....	700 [l]
Elyria, Ohio[c]	1930	85.0	12.8	0.3	0.7	1.2	Not run	.....	900 [l]
Elyria, Ohio[d]	1930	69.0	29.8	0.1	0.5	0.6	Not run	.....	740 [l]
Essen-Frohrhausen, Germany[e]	1929–30	59.8	29.6	6.0	.....	4.6	......	.....	619
Essen-Frohrhausen, Germany[c]	1929–30	63.3	27.1	5.1	.....	4.5	......	.....	631
Essen-Nord, Germany	1929–30	69.0	20.8	7.0	.....	3.2	......	.....	715
Essen-Nordwest, Germany	1929–30	68.3	19.3	6.8	.....	5.6	......	.....	704
Essen-Rellinghausen, Germany	1930	68.3	28.3	3.4	0.1	.....	0.03–0.2	.....	732
Gelsenkirchen-Nord, Germany	1929–30	71.0	20.0	8.0	.....	1.0	......	.....	730
Grand Rapids, Mich	1931	65.30	31.80	1.34	0.10	1.12	......	0.34 [j]	707 [l]
Halle, Germany (Tafelwerder)[f]	1929–30	69.8	29.0	1.1	0.0	0.0	0.1	.....	787
Halle, Germany (Tafelwerder)[c]	1929–30	72.9	24.6	1.6	0.6	0.0	0.3	.....	.....
Hattingen, Germany	1930	76.0	16.1	2.3	.....	5.6	......	.....	818
High Point, N. C. (East Side)	1929–30	.....	28.0	.....	.....	.....	......	.....	683
Iserlohn, Germany	1930	72.6	23.1	4.2	0.1	.....	......	.....	.....
Kettwig, Germany	1929	75.8	14.4	9.6	0.0	.....	......	.....	815
Langendreer, Germany	1930	81.5	18.0	0.5	.....	.....	......	.....	876
Leipzig-Wahren, Germany	1930	70.5	27.0	1.9	.....	0.6	0.01	.....	742
Los Angeles, Calif. (Experimental Station)	1930	68.5	12.0	17.5 [h]	2.0	0.0	0.0	.....	689
Munich, Germany (Grosslappen)	1930	77.0	16.0	7.0	.....	Trace	Trace	.....	753
Nürnberg, Germany (Nürnberg-Sud)	1930	.....	23.1	.....	0.7	.....	......	.....	722
Oberhausen, Germany	1929–30	66.4	23.0	6.4	.....	4.2	......	.....	697
Plainfield, N. J. (Joint)	.....	65.8	30.6	3.6	0.0	0.0	Slight	.....	.....
Stuttgart, Germany (Mühlhausen)	1930	70.6	23.2	1.45	0.2	4.475	0.075	.....	772
Toronto, Ont., Canada (North Toronto)	1930	59.26	29.35	0.50	0.19	7.70	0.00	3.00 [k]	622

* Values for German localities were converted from calories per cubic meter.
[a] Sample taken October 17. [b] Samples taken October 6. [c] Gas from Imhoff tanks. [d] Gas from separate digestion tanks. [e] Gas from Prüss digestion tank. [f] Gas from supplementary tanks. [g] H_2S included with CO_2 percentage. [h] Air included with N_2 percentage. [i] CO. [j] Illuminants. [k] 0.77% CO; 2.23% illuminants. [l] Calculated at sewage treatment works. [m] Calculation based on laboratory work relating to sludge digestion.

Heat value varies almost directly with the percentage of methane, although the hydrogen content, while small, has some effect. The average heat value of gas from the installations reporting in Table 3, is 730 Btu per cu ft.

In their early studies, Buswell and his associates assumed that the weight of gas produced was equal to the summation of the weights of protein, grease,

TABLE 4.—Data Relating to Gas Collection at Sewage Treatment Works*

Location	Period of Observation		Contributing population	Sewage flow, in million gallons daily	Suspended solids removed, in parts per million	Characteristics of Raw Sludge		Type of Digestion Tanks from Which Gas Was Collected		General Data on Digestion			Quantity of Gas Collected, in Cubic Feet			
	Year	Length of period				Volatile solids, percentage by weight of total solids	Moisture content (percentage)	Type	Heated	Temperature, in degrees Fahrenheit	pH	Time, in days	Average daily	Per million gallons daily Column (16) / Column (5)	Per pound of volatile matter added (Col. (16) × 100) / (Col. (5) × Col. (6) × Col. (7) × 8.33)	Per capita daily, Column (16) / Column (4)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
Antigo, Wis.	1930	1 yr.	5 600	0.594	258 [a]	....	95.5	Dorr	Yes	80.0	7.2	.....	7 240	12 000	....	1.29
Aurora, Ill.	1930	11 months	43 000	4.89	......	....	.....	Dorr	Yes	97.6	.....	180 [a]	33 190	6 800	....	0.77
Baltimore, Md.	1930	1 yr.	12 000 [a]	......	206	77.3	94.6	Separate (covered)	Yes	73.4	6.9	.....	5 130	....	....	0.43 [a]
Berlin, Germany (Wassmannsdorf)	1929–30	1 yr.	435 762	17.0	278	75.7	95.7	Imhoff	No	62.5	7.3	60–90	186 740	11 000	6.3	0.43
Birmingham, England (Saltley)	1930	1 yr.	......	36.0	380	56.8	91.75	Separate (uncovered) [e]	No [f]	46.6	7.5	90 [a]	128 000	3 600	2.0	.....
Bochum, Germany	1929–30	1 yr	183 000	13.39	......	64.0	94.0	Imhoff	No	......	6.8	80	34 800	2 600	....	0.19
Charlotte, N. C. (Sugar Creek)	.....	....	45 000	4.0	200 [d]	....	95.0	Separate (uncovered)	No	......	7.0	120	......	....	....	0.5 [h]
Chicago, Ill. (Des Plaines)	1927–28	1 yr.	1 000 [a]	......	......	75.0	.....	Dorr	Yes	73.3	7.3	180	400	....	....	0.40 [a]
Chicago, Ill. (Calumet)	1926–27	1 yr.	4 400	1.4	63	55.0	.....	Imhoff	No	58.0	6.3–7.0	150	1 930	1 400	4.8	0.44
Chico, Calif.	.....	....	10 000	1.0	......	....	.....	Dorr	Yes	70–85	6.8–7.2	.....	11 000 [a]	11 000 [a]	....	1.10 [a]
Dayton, Ohio	1930	1 month	200 982	17.40	119 [i]	85.0	.....	Imhoff	No	63.0	6.9	90	67 200	3 900	4.6 [a]	0.33
Decatur, Ill.	1929	1 yr.	112 000 [b]	11.35	88	60.0	95–97 [a]	Imhoff	No	70–85 [g]	6.7	.....	49 000	4 300	9.8	0.44
De Kalb, Ill.	1930	3½ months	8 545	0.75	350	45.0	98.5	Dorr	Yes	90.0	8.0–8.2	.....	11 140	14 900	11.3	1.30
Elyria, Ohio	1930	7¾ months	13 000 [a]	0.9 [a]	140 [a, d]	85.0	...	Imhoff	....	......	....	.....	......	....	....	.....
								Downes	Yes	78.8	7.1	45 [a]	10 000	11 100	11.2 [a]	0.77 [a]
Essen-Frohnhausen, Germany	1929–30	1 yr.	26 000	1.19	......	70.0	95.0	Pruss (separate)	Yes	61.5	7.3	90	12 700	10 700	....	0.49
								Imhoff (separate)	No	55.9	7.3	170	5 400	4 500	....	0.21
Essen-Nord, Germany	1929–30	1 yr.	120 000	15.32 [e]	......	59.0	92.0	Imhoff (separate). Pruss (separate)	No	60.8	7.2	100	48 700	3 200	....	0.41
Essen-Nordwest, Germany	1929–30	1 yr.	50 000	......	......	58.0	95.0	Imhoff	No	59.0	7.0	110	9 740	....	....	0.19

* Quantities and temperatures for German localities were converted from cubic meters or degrees Centigrade.
[a] Approximate. [b] About 45 000 human population plus 67 000 population (equivalent of industrial wastes). [c] High iron content. [d] Mixture of primary and activated sludge.
[e] Gas collected from portion only of contents. [f] Sludge which has been warmed by steam is added in water. [g] Winter and summer, respectively, due to warm industrial wastes.
[h] Assumed value. Gas quantities not available due to leakage. [i] In addition, packing-house wastes were partly treated. [j] Cooling water from gas engines was used to heat two digestion chambers. [k] Caused by warm wastes from wool washery. [l] Not all the gas was metered. [m] Three Imhoff, two separate, one Neustadt, one Stuttgart, and one supplementary tank. [n] Sludge heated before being added to tanks.

TABLE 4.—(*Continued*)*

Location	Period of Observation: Year	Period of Observation: Length of period	Contributing population	Sewage flow, in million gallons daily	Suspended solids removed, in parts per million	Characteristics of Raw Sludge: Volatile solids, percentage by weight of total solids	Characteristics of Raw Sludge: Moisture content (percentage)	Type of Digestion Tanks from Which Gas Was Collected: Type	Type of Digestion Tanks: Heated	General Data on Digestion: Temperature, in degrees Fahrenheit	General Data on Digestion: pH	General Data on Digestion: Time, in days	Quantity of Gas Collected, in Cubic Feet: Average daily	Per million gallons daily Column (16) / Column (5)	Per pound of volatile matter added Col. (16) × 100 / Col. (5) × Col. (6) × Col. (7) × 8.33	Per capita daily, Column (16) / Column (4)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
Essen-Rellinghausen,																
Germany...........	1930	4 months	50 000	6.9	..... d	....	93.6	Imhoff (supple-								
								mentary sepa-								
								rate)..........	No	57.2–59.0	7.2–7.8	55	28 200	4 100	....	0.56
Fond du Lac, Wis.......	1929–30	....	26 449	1.75	......	....	.....	Dorr............	Yes	80.0	7.0	180	18 800	10 700	....	0.71
Fort Worth, Tex........	1930	7 months	148 500 i	8.17	286 d	76.0	97.25	Dorr............	Yes	88.0	7.15	.....	85 250	10 400	5.8	0.57
Gelsenkirchen-Nord,																
Germany............	1929–30	1 yr.	100 800	6.92	h......	....	96.0	Imhoff (separate).	Partly	68.0	7.0	100	26 000	3 800	....	0.26
Grand Rapids, Mich....	May,															
	1931	1 month	168 592	20.7	85	65.7	94.0	Dorr............	Yes	84.0	7.75	20	159 980	7 700	16.6	0.95
Hagen, Germany.......	.....	....	90 000	7.9	......	....	.....	Imhoff (supple-								
								mentary).......	....	......	.....	.....	26 500	3 400	....	0.29
Halle, Germany (Tafel-																
werder).............	1929–30	1 yr.	190 000	6.9	340	67.0	96.0	Imhoff (supple- {	....	......	6.8–7.0	60–90				
								mentary).... {	Yes	68.0–77.0	7.3	15	75 400	10 900	5.8	0.40
Hattingen, Germany....	1930	7 months	17 000	0.7	246 d	70.0	92.0	Imhoff with aerat-					9 400	13 400	9.4	0.55
High Point, N. C. (East								ors...........	Yes	60.8	7.5–8.0	30	7 620	3 000	3.5	0.21
Side)...............	1929–30	13 months	36 745	2.51	160	64.8	93.8	Separate........	Yes	77.0	7.0	90 a	......	....	....	......
Iserlohn, Germany......	1930	1 yr.	19 000	0.9	437 d	....	92.0	Imhoff..........	No j	57.2	7.2	.....	7 940	8 800	....	0.42
Kettwig, Germany......	1929	1 yr.	6 300	0.7	......	....	.....	Imhoff..........	No	62.6 h	7.2–7.3	80	2 510	3 600	....	0.40
Langendreer, Germany..	1930	1 yr.	30 000	2.1 c	......	....	90–95	Imhoff..........	No	......	7.5	.....	2 840	1 400	....	0.10
Leipzig-Wahren,																
Germany...........	1930	1 yr.	19 000	0.66	..... d	67.0	94.0	Imhoff (with aer-								
								ators).........	No	......	7.2	60	6 500	9 900	....	0.34
Los Angeles, Calif......																
(Experimental Station)	1930	7 months	1 750	0.13	370 d	70.0	95.0	Dorr............	Yes	85.0	7.1	25	1 410	10 800	5.0	0.81
Middletown, N. Y.....	.....	....	21 276	......	......	70.0 a	.....	Downes.........	Yes	78–80	.....	.....	17 000 a	....	....	0.80 e
Munich, Germany (Gross-																
lappen).............	1930	1 yr.	570 000	70.74	......	....	92–93	Separate (uncov-								
								ered).........	No	52.5–54.3	7.2	99–100	266 000 a	3 800	....	0.47 e
Neuheim, Germany.....	1930	1 yr.	6 000	0.4	......	....	.....	Imhoff..........	No	50.5	7.4	.....	1 450	3 600	....	0.24
New Castle, Pa........	1930	1 yr.	48 674	4.31	......	....	.....	Imhoff..........	No	59.5	6.9	.....	25 400	5 900	....	0.52 l

TABLE 4.—(*Continued*)*

Location	Period of Observation: Year	Period of Observation: Length of period	Contributing population	Sewage flow, in million gallons daily	Suspended solids removed, in parts per million	Characteristics of Raw Sludge: Volatile solids, percentage by weight of total solids	Characteristics of Raw Sludge: Moisture content (percentage)	Type of Digestion Tanks from Which Gas Was Collected: Type	Type of Digestion Tanks from Which Gas Was Collected: Heated	General Data on Digestion: Temperature, in degrees Fahrenheit	General Data on Digestion: pH	General Data on Digestion: Time, in days	Quantity of Gas Collected, in Cubic Feet: Average daily	Quantity of Gas Collected, in Cubic Feet: Per million gallons daily Column (16) / Column (5)	Quantity of Gas Collected, in Cubic Feet: Per pound of volatile matter added (Col. (16) × 100) / Col. (5) × Col. (6) × Col. (7) × 8.33	Quantity of Gas Collected, in Cubic Feet: Per capita daily, Column (16) / Column (4)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
Nürnberg, Germany (Nurnberg-Süd)	1930	1 yr.	190 000	11.3	127	64.1	93.5	Imhoff	No	56.3	7.2	80[a]	46 400	4 100	6.1	0.24
Oberhausen, Germany	1929–30	1 yr.	30 000	1.24	......	65.0	93.0	Separate	No	59.2	7.4	90	11 760	9 500	....	0.39
Plainfield, N. J. (Joint)	.....	....	50 000	3.3	128	76.0	95.0	Downes	Yes	70.0	7.2–7.4	40	16 000[a]	4 900[a]	6.0[a]	0.32[a]
Sioux Falls, S. Dak.	.....	....	......	......	......	....	.....	....	....	......	.....	.....	......	....	....	.....
Springfield, Ill.	1930	....	71 864	5.0	......	....	.....	Downes	Yes	82.0	6.9	.....	50 000	10 000	....	0.70
Sturgis, Mich.	1930	7 months	6 950	0.5	......	....	.....	Dorr	Yes	76.0	7.2	.....	3 740	7 500	....	0.54
Stuttgart, Germany (Mühlhausen)	1930	1 yr.	360 000	17.1	442	59.6	92.0	Miscellaneous[m]	No	59.0	7.74	120	132 000	7 700	3.5	0.37
Toronto, Ont., Canada (North Toronto)	1930	6½ months	56 000	5.08	164[d]	77.9	96.5	Dorr	No[n]	82.0	7.2	40[a]	28 080	5 500	5.2	0.50
Velbert, Germany	1930	1 yr.	18 000	0.6	340	60.0	.....	Imhoff	No	52.0	7.5	180	4 890	8 100	4.8	0.27
Waco, Tex.	.....	....	52 848	3.5	235[d]	65.0	96.5	Dorr	Yes	108.0	7.5	40[a]	48 000	13 700	10.8	0.91
Waukesha, Wis.	1929–30	1 yr.	17 176	1.75	......	....	96.0	Dorr	Yes	74.0	7.4	180	12 000	6 900	....	0.70
Wetter, Germany	1929	1 yr.	6 000	0.2	......	....	.....	Imhoff	No	54.3	.....	.....	1 790	9 000	....	0.30

cellulose, and crude fiber digested. Buswell later found that the higher fatty acids of grease produce more than their equivalent weight of gas, because more or less water enters into the reaction. The quantity depends upon the number of carbon atoms in the acid. Buswell and Pearson obtained 1.25 lb of gas per lb of volatile matter digested, at Champaign-Urbana, Ill.

Information relating to quantities of gas collected at various sewage works has been assembled in Table 4. Gas from separate sludge digestion tanks in Germany averages 0.41 cu ft per capita per day for unheated tanks and 0.53 cu ft per capita per day for heated tanks. In the United States, excluding Antigo, Wis., Chico, Calif., and DeKalb, Ill., the quantity from heated digestion tanks averages 0.61 cu ft per capita per day. Where an activated sludge mixture is digested, the average quantity is 0.69 cu ft. At Antigo, Chico, and DeKalb, the gas production is much higher, possibly because of the nature of the industrial wastes in the sewage.

II.—ENVIRONMENTAL FACTORS

Among the environmental factors that have an extremely important influence on the bio-chemical processes of sludge digestion may be mentioned the temperature at which digestion is carried on, the hydrogen-ion concentration and the percentage of return sludge (reaction and seeding), and the trade-waste content. Other environmental factors of less importance include pressure and density. All these factors are considered in the following paragraphs. Attention should first be called, however, to the fact that the properties of the influent to the sludge digestion equipment may themselves have a decided effect on digestion. These properties depend on the nature of the processes to which the influent has already been subjected. Thus, if the influent comes directly from primary settling tanks through which crude sewage flows, it will consist almost entirely of fresh, or primary, sludge. On the other hand, if primary settling tanks are operated as septic tanks, a partly digested sludge will result, that may be termed septic sludge; it will ordinarily be a mixture of relatively fresh and quite well-digested sludge, the proportion depending largely upon operating conditions. Still other general types of digestion-equipment influent are secondary sludge (from final or secondary settling tanks following filters) and activated sludge (from settling tanks following aeration tanks).

Temperature

Temperature has been found to have a considerable influence on the time required for sludge digestion, as it determines to a large extent the type of organism that predominates in the sludge. The importance of this factor has led to considerable investigation.

Until a few years ago, prior to the time of temperature control, sludge digestion was carried on at or near the temperature of the sewage, or from about 50°F to 70°F, and in the early investigations of temperature control, the optimum temperature was considered to be about 82°F. Recent investigations have indicated, however, that a temperature of 110°F to 130°F makes

possible a still shorter digestion period. Digestion at these higher temperatures has been designated as thermophilic digestion. Less information is available on results at intermediate temperatures. There are not enough operating data from large-sized plants to justify a final conclusion as to the best and most economical temperature. Present information indicates that temperatures between 80°F and 95°F are desirable. Savings in cost and improvement in operating conditions with temperatures in excess of 100°F have not been established.

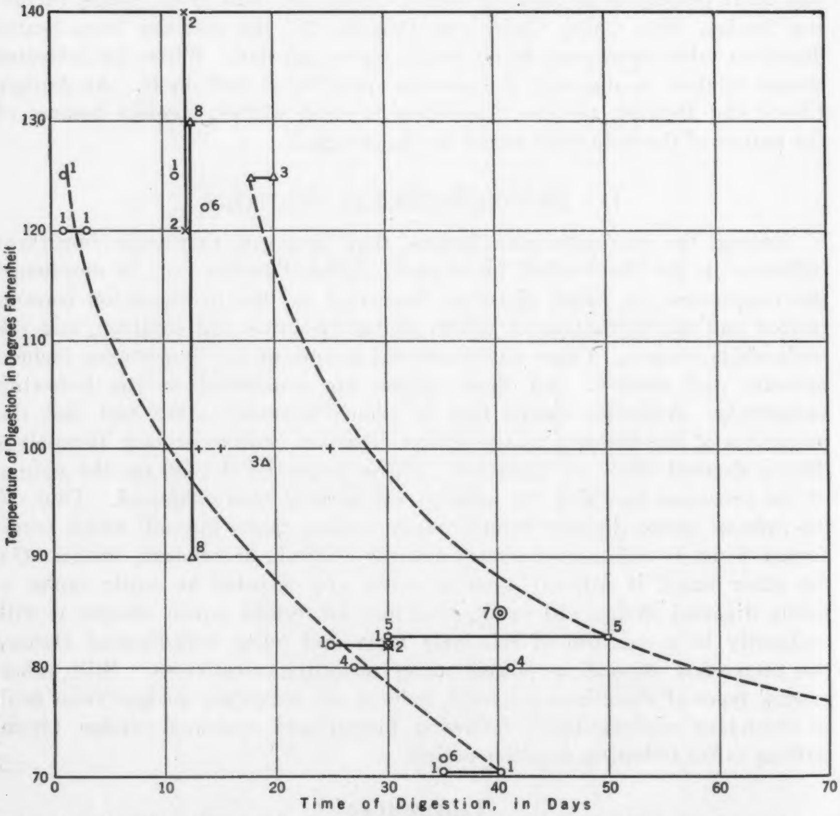

FIG. 1.—EFFECT OF TEMPERATURE ON SLUDGE DIGESTION: 1, HEUKELEKIAN (*Sewage Works Journal,* January, 1931); 2, NEW JERSEY SEWAGE EXPERIMENT STATION (AM. SOC. C. E., COMMITTEE, 1930); 3, RUDOLF AND HEUKELEKIAN (*Industrial and Engineering Chemistry,* January, 1936); 4, HEUKELEKIAN (*Sewage Works Journal,* July, 1930); 5, KEEFER AND KRATZ (*Sewage Works Journal,* July, 1930); 6, HEUKELEKIAN (*Sewage Works Journal,* April, 1930); 7, INDICATED AT GRAND RAPIDS, MICH.; 8, AURORA, ILL. (*Water-Works and Sewerage,* June, 1931)

Some data on the relation between temperature and the time required for digestion are illustrated by Fig. 1. In addition to the effect of temperature on the time, and hence on the capacity, required to bring about a satisfactory reduction in volatile matter, the effect on quantity and rate of production of gas is also important. The rate of gas production appears

to be greater at the higher temperatures, and Willem Rudolfs, M. Am. Soc. C. E., states that the total quantity per unit of volatile mater is also greater, although the evidence supporting such a conclusion is as yet limited.

The percentage of reduction in 108 days in the volatile content of sludge, unseeded but with pH-adjustment, at different temperatures, was stated by Rudolfs as follows:

Temperature: In degrees Centigrade	In degrees Fahrenheit	Percentage reduction in volatile matter
10	50.0	16
18	64.4	26
24	75.2	58
29.5	85.1	64
35	95.0	54

Heukelekian[2] states that with temperature control at 70° F, the digestion period is about 35 to 40 days, but he declares confidently that it is possible to draw ripe sludge 24 hr after the addition of the raw material, provided there is ripe active thermophilic sludge and the temperature is maintained at approximately 120° to 125°F. According to Heukelekian:

"The sludge will be just as ripe as that drawn at the end of 35 to 40 days at 70°F. It will have an ash content of 45 percent, or more, a pH of 7.8 or more, and a B.O.D. [bio-chemical oxygen demand] per percent of organic matter of 1 500 ppm, or less."

In discussing the foregoing, Rudolfs states:

"If under ordinary 82°F temperature 50 to 60 percent of the volatile matter is decomposed in 25 to 30 days, about 70 to 75 percent is destroyed at 120°F in 1 to 3 days * * * if at 82°F the quantity of gas amounts to 6 cubic feet per pound of dry volatile matter, it will be about 7 cubic feet under thermophilic conditions."

At Aurora, Ill., digestion is reported to have been accomplished in 12.5 days at temperatures ranging from 90°F to 130°F.

Keefer and Kratz[3] found that, for Baltimore, Md., sludge, seeded with 50-day digested sludge in the ratio of 2 parts of raw to 1 of seeding sludge, digestion was more rapid at 37°C than at 27°C, but the difference in rate was not great.

In 1930 a committee of the Sanitary Engineering Division of the Society summarized the results obtained at the New Jersey Sewage Experiment Station, as follows:

Temperature in degrees Fahrenheit	Number of days required for digestion
55	120
68	42
82	30
120–140	12
170	11

[2] "Digestion of Solids between Thermophilic and Non-Thermophilic Range," *Sewage Works Journal*, September, 1933. (Heukelekian found little difference between digestion at 20°C (82°F) and 42°C (108°F).)

[3] "Digesting Sludge at 37° C," *Sewage Works Journal*, March, 1933. (Keefer and Kratz found some advantage in digestion at 37° C (98.6° F) as compared to digesting at 28° C (82° F).

The actual operating temperatures in several large sludge digestion tanks are shown in Table 5.

TABLE 5.—DIGESTION TANK TEMPERATURES, IN DEGREES FAHRENHEIT

Month, in 1933	Aurora, Ill.	Springfield, Ill.	Rockford, Ill.	Grand Rapids, Mich.	Peoria, Ill.	Erie, Pa. (1932)
January	89	73	80	83	89	67
February	83	75	81	87	89	70
March	82	81	82	84	89	71
April	75	80	86	83	90	67
May	74	80	92	88	90	....
June	78	85	92	91	87	78
July	85	94	88	94	89	81
August	89	93	90	90	86	79
September	90	94	91	89	86	81
October	88	95	92	89	91	81
November	87	....	87	85	89	75
December	90	88	93	85	90	71

Among the considerations that should be taken into account in selecting an operating temperature, and in designing digestion tanks, are:

(*a*) The balance between the cost of providing extra capacity for digestion at a lower temperature and that of extra equipment for heating and insulating at a higher temperature, due consideration being given to the value of the additional gas.

(*b*) The capacity of the selected type of tank for being heated and for retaining heat.

(*c*) The method of heating and rate of transmission of heat.

(*d*) The effect of higher water temperatures on the rate at which sludge cakes on heating coils.

(*e*) The effect on the heat balance of the concentration of solids in the tank sludge and in the fresh sludge.

(*f*) The heat lost with the sludge liquor and the possibility of some recovery by heat exchange.

(*g*) The effect of more active gas release on the area and depth of the tank and on the formation of scum.

(*h*) The need for and the relative cost of winter storage.

(*i*) The fact that the total size of sludge digestion and storage tanks is not directly proportional to the time required for digestion.

(*j*) The effect of the duration of low winter temperatures on the relative costs of operation at different temperatures.

REACTION AND SEEDING

Freshly settled sewage solids are normally alkaline or neutral. The pH-value may vary from 6.8 to 7.2 for solids from domestic sewage. Within the first 24 hr this reaction begins to change toward the acid side, and the longer the solids are held in a settling tank the more acid their reaction becomes. If no seeding sludge is present the solids undergo a period of acid fermentation

before true digestion occurs. If suitable seeding sludge is present this acid phase is shortened or eliminated. There are, therefore, two courses of sludge digestion. Acid sludge is developed and digestion delayed when a digestion process, either Imhoff or separate, is started. After ripe sludge is developed this acid condition is restricted and digestion is greatly accelerated. If solids are held for many hours or days, either in sewers or in plain sedimentation tanks, acid sludge will be produced and discharged into the digestion tanks.

The formation of acid in unseeded sludge in process of digestion was studied in Germany 15 or 20 yr ago. Thumm and Reichle, in 1914, noted an acid reaction accompanying poor digestion, and recommended the addition of lime to the sewage.

The first study of the pH-value of fresh solids undergoing digestion was reported by Rudolfs in 1924. Solids collected from the Plainfield, N. J., sewage, which is from separate sewers, were digested at 20°C. Initially 6.4, the pH dropped to a minimum of 4.9 in 9 days, increased to 6.0 in 48 days, to 6.8 in 66 days, and to 7.0 in 120 days. In studies by Fair[4], of the digestion of fresh sewage solids from separate sewers at Brockton, Mass., the initial pH of 6.4 dropped to the very low value of 4.7 in 20 days, recovered to 6.4 in 35 days, to 6.8 in 60 days, and reached a maximum of 6.9 in 70 days. Keefer and Rudolfs report very prolonged acidity with unseeded Baltimore sludge from separate sewers. Solids from the primary settling tanks were pumped into a large open tank and allowed to digest without any seeding material. The pH initially was 6.1 and the volatile matter 65 per cent. The pH fluctuated between 5.5 and 7.3 for the first 200 days, then dropped to 5.6 in 300 days, and 5.2 in 400 days, and slowly rose to a maximum of 6.8 in 830 days. The prolonged acidity was probably due to lack of seeding material and to the low temperature in winter.

On the other hand, the sludges used in the experiments reported by Zack and Edwards, from combined sewers contributing to the Des Plaines Treatment Works of the Sanitary District of Chicago, did not produce high acidities even when unseeded. The fresh solids had a pH of 6.3 when incubated at 25°C, and this value rose to 7.0 in 13 days. The fresh activated sludge had an initial pH of 7.3, which decreased to 6.6 in 10 days and then quickly rose to 7.0 in 20 days. The volatile content of the fresh solids was 67%, and that of the activated sludge was 60 per cent.

The acidity produced by unseeded solids apparently increases greatly if the initial pH is low and the volatile matter high. From a practical standpoint this means that solids high in organic matter, from separate sewers tend to produce greater acidity than solids from combined sewers normally lower in organic matter. Proper seeding, therefore, is more necessary with highly organic solids.

Storage of solids in settling tanks is undesirable, since the acidity increases, and lengthens the period of digestion if seeding material is insuffi-

[4] "Time and Rate of Sludge Digestion and Their Variation with Temperature," *Sewage Works Journal*, January, 1934. (Fair and Moore find four zones of temperature with optimum for non-thermophilic at 33°C (91°F).)

cient. Fair and Klein show that digestion of unseeded solids is greatly retarded if the fresh solids are held until high acidity develops. They state that "it seems desirable that the settling solids reach the digestion tanks as soon as possible."

Undoubtedly, therefore, it is desirable, in separate digestion, to remove the solids from the settling tank as promptly as possible. The production of acid sludge should be prevented throughout the entire digestion cycle; therefore, the initial solids should not be allowed to become acid if it is practicable to prevent it.

Sewage solids do become acid, however, when there is not enough ripe sludge available to prevent acid decomposition. This occurs (1) when digestion tanks are first started: (2) when ripe sludge is lost by improper operation; or (3) when acid industrial wastes are present. From the results of analyses of gas from various digestion tanks, Buswell concludes that a high CO_2 content of the gas may serve as an index of improper seeding conditions, as correlated with acidity shown by pH.

When ripe sludge is not available, lime is commonly used for neutralization of acidity. The experience of a number of plant operators as reported by Bachmann indicates that liming is a useful remedy under abnormal conditions, but hardly to be recommended, from a practical standpoint, for normal operation.

When well-digested sludge is mixed in proper amount with fresh sludge, the reaction of the mixture and the course of digestion are different from those of unseeded sewage solids. Dr. Imhoff states:

> "Basically different from acid fermentation is the second kind, that establishes itself in well-conditioned digestion tanks. It results in the formation of carbon dioxide, nitrogen and, above all, methane (marsh gas), the latter constituting about 75 percent of the gas by volume. The pH-value consistently remains slightly above the neutral point. The process is odorless and is called alkaline or methane fermentation."

The low pH of unseeded solids is caused primarily by the formation of fatty acids that decompose very slowly, but that must be decomposed before digestion is complete. In the presence of well-digested sludge, these acids decompose more quickly, and, in addition, the reserve alkalinity or buffer action of the digested sludge neutralizes them to a varying degree, depending upon the composition of the inorganic solids in the digested sludge.

The proportion and the character of digesting sludge in the tank are important. Rudolfs found in his earlier work that, at 70°F, the daily additions of fresh organic solids should not exceed 2% of the weight (dry basis) of the organic matter in the digesting sludge. In a later work, he has shown that this ratio might be increased greatly (the ratio in general falls between 2 and 6%) by increasing the temperature and, if necessary, adding lime. The use of lime is not recommended as a routine procedure, however.

Ripe sludge used for seeding must obviously be neutral or slightly alkaline in order to provide the proper buffer action or to neutralize the acid reaction of raw sludge. Its moisture content, within normal limits, is immaterial, but it should be actively digesting sludge.

Trade Waste Content

Trade or industrial wastes originate from a great variety of manufacturing processes and have widely differing characteristics. When such wastes are

TABLE 6.—Effects of Various Industrial Wastes on Sludge Digestion

Source of wastes	Quantity of sludge produced	Characteristics of sludge	Remarks
Bleacheries	Small	Usually alkaline; may contain disinfectants.	The waste contains soaps and decoloring agents.
Canneries	Usually large (depends on material canned).	Contains both carbonaceous and nitrogenous organic matter; develops organic acids in digesting.	It is usually necessary to provide preliminary treatment of cannery wastes before mixing with domestic sewage. These wastes are likely to cause foaming and scum.
Creameries and dairies	Depends on treatment process.	Fats and sugars break down forming organic acid, delaying digestion and causing obnoxious odors.	Organic acids that are generated coagulate the casein, which rises to the top as scum that does not digest readily.
Corn-starch plants	Variable, depending on process at corn - products plant.	Contains residual SO_2 which breaks down during digestion to form H_2S.	There is a tendency for sludge to bulk in aeration tanks.
Coke, coal, gas, and creosoting plants	Small	Contains phenols and other compounds having germicidal characteristics.	This sludge will not digest. Normal domestic sewage sludge will digest if this type of industrial waste does not exceed 2% by volume.
Dye-houses	Small	Usually acid, and likely to produce offensive odors.	Will not interrupt digestion if dilution is large.
Meat packing plants.	Large	Organic matter of high nitrogen content. Also, fats, hair, and paunch manure.	Much of the packing-house sludge digests with difficulty, if at all. Treatment at the source is desirable before mixing with domestic sewage.
Oil (crude oil, fuel oil, and spent crank-case oil)	Variable	Relatively inert chemically.	Interferes with digestion by surrounding the sludge particles, preventing proper drainage, and forming undesirable scum.
Paper and pulp mills (strawboard waste)	Very large	Contains fiber and chemical residues. Sulfite wastes are germicidal.	Sludge digests slowly if at all. Most of the solids in waste from paper mills have commercial value and may be recovered by use of "save-alls", or similar equipment.
Pickle. Liquor; Iron	Variable.	Strongly acid. If neutralized by alkalinity of the sewage or by lime, iron compounds precipitate and form sludge.	The precipitated iron compounds do not interfere with digestion of the domestic sewage solids. The iron floc may be quite bulky. Treatment at the source is desirable.
Tanneries	Large	Contains considerable organic matter. Reaction may be acid or alkaline depending on the part of the process contributing the waste.	If properly neutralized, tannery wastes do not interfere materially with sludge digestion. Hair and lime residues do not digest.
Wool scouring	Medium	Has a very foul odor and contains dirt, grease, organic matter, soaps, and fiber.	When mixed with domestic sewage these wastes increase the bulk of the sludge to be digested and create masses of floating scum that do not digest readily.

mixed with domestic sewage the quantity of sludge is increased, the extent of increase depending on the characteristics of the particular wastes. The effect of industrial wastes on the process of digestion may vary over a wide range from somewhat beneficial to very detrimental, depending on the characteristics of the industrial sewage, the pre-treatment it has received, and the relative quantities of industrial waste and domestic sewage.

The effects of various industrial wastes on sludge digestion are summarized in Table 6.

Pressure

The pressure on sewage solids undergoing digestion results mainly in common practice from the depth of the digestion tank. In one of the shallowest 2-story tanks (at Schenectady, N. Y.) the liquid depth is 13.8 ft, and in one of the deepest (at Rochester, N. Y.), it is 33.9 ft. Most separate digestion tanks have depths of 20 to 30 ft. The pressure may be changed by applying a negative pressure (vacuum) or a positive pressure to the surface of the digesting sludge.

The major possible effects of pressure on sludge digestion are: (*a*) The effect on the biological life and its activities; (*b*) the effects on the gas produced by digestion, as, for example, the rate of gas production, the solubility of the gas, the size of the bubbles, and their motion in the liquid; and (*c*) the effect on the density of the sludge.

Other possible effects, believed to be of minor importance, are the pressure on the viscosity and on the surface tension.

Pressures likely to obtain in practical sludge digestion appear to have no effect on bacteria. There is some difference of opinion on the effect of very high pressures. Fischer states:

"High pressure seems to be absolutely without influence upon bacteria. The processes of putrefaction and alcoholic fermentation go on without interruption under pressures of from 300 to 500 atmospheres, * * *."

Dr. F. Sierp[5] made experiments on the digestion of sludge at different depths. Samples containing one part of fresh sludge to two parts of water were placed in vessels at depths of 3.0, 1.0 and 0.15 m (9.8, 3.3 and 0.5 ft), respectively, and observed for 40 days. At the end of this period the sludge characteristics of the three samples were very similar.

Tests on the vacuum degasification of Imhoff tanks have been made by Steel and Zeller[6] at the Texas Agricultural and Mechanical College, College Station, Tex., on two Imhoff tanks treating the college sewage. The gas vents of one tank were placed under a vacuum of from 1.75 to 3.7 ft of water, generated by an automatically operated blower. Observations were made daily for five weeks of the change in H_2S, suspended matter, and bio-chemical oxygen demand of the sewage, and weekly for sixty days of the solid and ash content and the bio-chemical oxygen demand of the sludge. The authors concluded that degasification has no appreciable effect on the tank effluent nor on the digestion of the sludge.

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[5] *Technische Gemeindeblatt*, Vol. XXIX, p. 267 (1927).
[6] *Sewage Works Journal*, Vol. 2, p. 29.

The effect of pressure on the quantity of gas held in solution has also been considered. The quantity of gas that will be dissolved in a liquid at a given temperature is directly proportional to the absolute pressure, and where a number of gases are in contact with a liquid, the quantity of each that will dissolve is proportional to its respective partial pressure. Regardless of the pressure on the tank, however, the gas lost in the sludge supernatant will be less, in general, than 1.0% of the gas produced, provided only that the sludge supernatant is withdrawn from a point where the pressure is 18 in., or less, of water.

The effect of pressure on sludge digestion may be summarized as follows:

(1) Pressures likely to obtain in practice appear to have no effect on bacterial action causing sludge digestion.

(2) Pressures other than normal do not shorten the digestion time.

(3) Degasification under vacuum does not produce more gas nor does it have an appreciable effect on the tank effluent or on the digestion of the sludge.

(4) Pressures in the tank with reference to its effect on the quantity of gas passing into solution and drawn off in the sludge supernatant are of negligible influence.

(5) In general, depth tends to produce dense sludge, but density is not directly proportional to depth because of the influence of other factors.

Moisture Content

There are ample data from operating sludge digestion tanks to show that the density of the sludge is greatest near the bottom of the tank as indicated on Figs. 2 and 3. The density of the sludge does not increase uniformly from top to bottom of the tank, and probably varies at different times at the same depth. The density of sludge as withdrawn from tanks of different depth appears to be very irregular and, within limits, a deep tank does not necessarily yield a sludge with a higher percentage of solids than does a shallow tank.

Experience indicates that the moisture content of sludge has less effect upon digestion than other factors. Fisher compared rates of digestion of raw sludges containing 6, 12, and 18% solids, and concluded that within these limits there was no retardation of rate of digestion in the more concentrated sludges. There appears to be some direct correlation between moisture and content of volatile matter, with indications that activated sludge may hold greater moisture than sedimentation tank sludge, for a given volatile content.

III.—TYPES OF DIGESTION TANKS

Tanks Built of Earth.—The earliest large-scale development of separate sludge digestion tanks was at Birmingham, England, in 1912. This installation followed approximately fifty years of efforts to eliminate the odor arising from crude sewage and, later, from septic tank effluent. It included a total of fifty tanks, of which thirty were constructed of embankments of engine ash, in some cases faced with concrete paving. The remaining twenty tanks were of brick masonry.

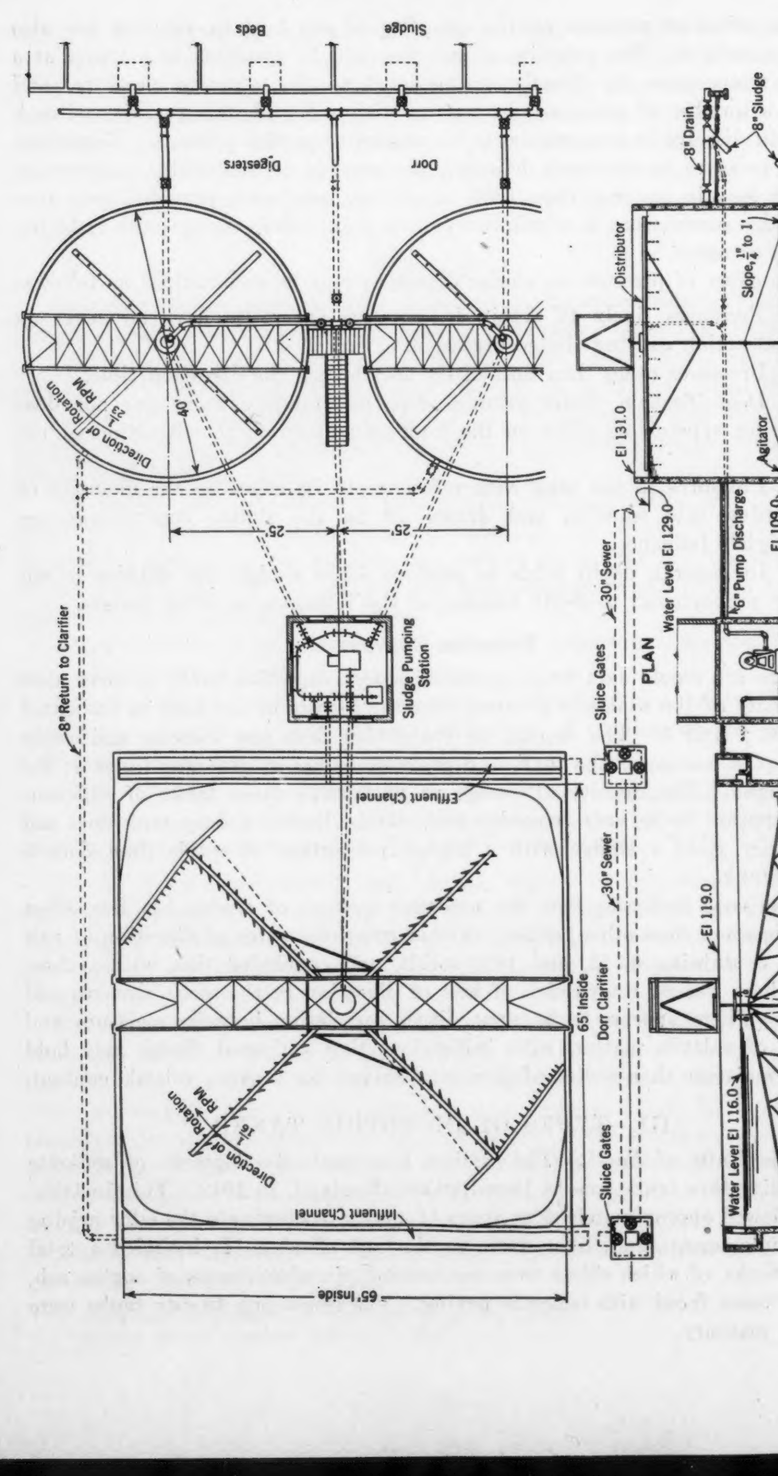


FIG. 2.—OPEN DIGESTION TANKS WITH ROTARY STIRRING EQUIPMENT. SEDIMENTATION TANK AT LEFT

Birmingham has a population of about 1 000 000. The dry-weather sewage flow in 1929 was 38 400 000 gal per day, of which 34 800 000 gal was treated. Digestion is carried out in two stages, sludge when partly digested being

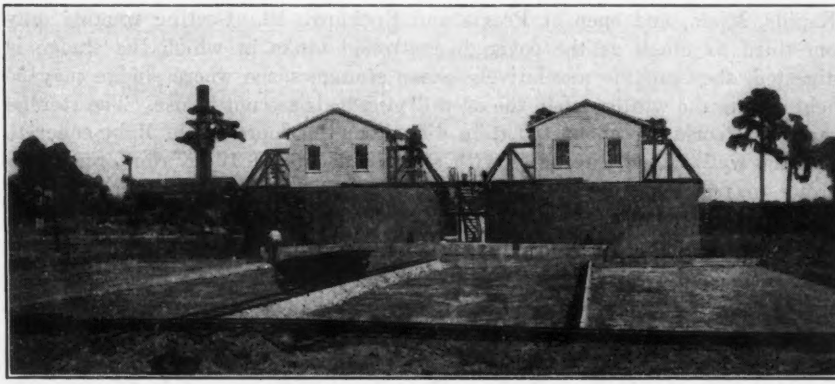


FIG. 3.—TYPICAL INSTALLATION OF OPEN SLUDGE DIGESTION TANKS WITH ROTARY STIRRING EQUIPMENT, LAKELAND, FLA.

pumped from primary tanks to secondary tanks, where the process is completed. The aggregate capacity of the 50 tanks is 340 000 cu yd, individual tanks varying from 1 200 to 17 000 cu yd. The older tanks are 12 to 14 ft deep; some of the newer ones are 20 ft deep.

At several places in the United States, earth tanks or lagoons have been used for handling undigested sludge, but their operation is not, in general, analogous to that of the tanks at Birmingham. However, at Indianapolis, Ind., "deep pit digestion" has been used for both settled and activated sludge. There are seven digestion pits or lagoons about 300 ft square, diked to give a depth of 7 to 10 ft. The pit capacity may amount to 14 cu ft per capita, or more.

Open Masonry Tanks Without Mechanical Equipment.—By far the greatest number of separate digestion tanks in this country are of masonry, and it is in connection with concrete tanks that the major improvements in design and operation have been made in recent years. Concrete tanks have many advantages over the less expensive earth tanks, the most important of which are permanence, space economy, and adaptability to the requirements of operation and structural design.

The earliest concrete tanks were open at the top and had no mechanical equipment. Usually, there was no attempt to control temperature, either by insulating the tanks with earth embankments or by heating the sludge. As a result, relatively high per capita capacities were required.

The sewage works at Baltimore, Md., is the first outstanding example of separate sludge digestion in the United States. Its first unit was built in 1909–1910, and consisted of three open, rectangular tanks, each 140 ft long, 103 ft wide, and varying from $13\frac{1}{2}$ to $15\frac{1}{2}$ ft in depth. Additional tanks have since been installed. In 1912 there was available a digestion-tank capacity of

11.60 cu ft per capita, and in 1930, with a tributary population of 735 000, the capacity was 3.7 cu ft per capita. In that year the sewage, domestic in character, amounted to 60 650 000 gal per day.

Concrete tanks are used for storing digested sludge with covers at Grand Rapids, Mich., and open at Peoria and Rockford, Ill. Costing roughly only one-third as much as the covered, controlled tanks in which the sludge is digested, they provide a relatively cheap storage space where sludge may be held during the winter, when the open drying beds are not in use. The storage tanks at Peoria are about 130 ft in diameter; the floors are of light concrete and the walls, lined internally with steel plate, are of 12-in. reinforced concrete, to resist earth pressure. The tanks are not heated and require no particular attention.

Open Tanks with Rotary Stirring Equipment—Experiments with mechanically cleaned digestion tanks were begun at Rochester, N. Y., in 1921. At that plant the open digestion tank was equipped with a stirring device consisting of two sets of horizontal arms attached to a central vertical shaft supported by a bridge across the tank. The lower set of arms concentrated the sludge in the bottom of the tank and gradually moved it to an outlet pipe at the center, while the upper arms prevented the formation of scum and distributed the incoming sludge over the surface.

In 1923, the first full-scale municipal plant incorporating this type of equipment was built at Brownsville, Tex. The digestion tank is circular, 35 ft in diameter, and 16 ft deep at the center. The agitator is supported from a steel superstructure spanning the tank, and consists of a gear-driven vertical shaft with two sets of arms. The top arms, constructed of steel channels, are placed just above the normal level of the liquid in the tank. Raw sludge is introduced through an inlet well mounted at the top of the shaft and flows outward along the channels, dropping into the tank through a series of holes. Short chains, hung from the upper arms, extend into the liquid a foot or more and serve to prevent the accumulation of scum. The power unit for driving the agitator is mounted on the truss.

The success of the Brownsville unit led to the building of a number of similarly equipped open tanks in all parts of the United States (see Table 7, and Figs. 2 and 3). Since about 1925, however, this type has yielded to the mechanically equipped, covered tank.

Tanks with Fixed Covers but Without Stirring Equipment.—The desirability of controlling odors and of collecting gas has doubtless been the chief cause of the rapid adoption of covered tanks in preference to open ones, although the closed units have other advantages as well. They are markedly superior in retaining heat, and, therefore, promote bacterial activity. Furthermore, by operating with the under side of the cover under a slight pressure, scum can be kept submerged; the digestion of floating solids is thus accelerated and the formation of a sometimes troublesome thick scum layer prevented.

Among the tanks with fixed covers should be mentioned the Kremer-Kusch design, (Fig. 4) developed in Germany and in use in a number of German

cities. The largest plant making use of it was built in 1920 at Hildesheim, a city of about 60 000 population. The single example in the United States was built at Palmyra, N. J., in 1922.

TABLE 7.—OPEN SLUDGE DIGESTION TANKS WITH ROTARY STIRRING EQUIPMENT*

Ref. No.	Location	Date installed	Design population	Tank Dimensions		Number of tanks	Volume of one tank, in cubic feet	Total tank volume, in cubic feet	Tank volume, in cubic feet per capita
				Diameter, in feet	Water depth, feet and inches				
1	Brownsville, Tex.	1923	15 000	35	15–0	1	14 400	14 400	1.0
2	Bartow, Fla.	1924	4 000	26	15–0	1	7 950	7 950	2.0
3	Vero Beach, Fla.	1925	4 000	30	16–8½	1	11 800	11 800	3.0
4	Lakeland, Fla.	1926	25 000	40	20–0	2	25 100	50 200	2.0
5	Fort Lauderdale, Fla.	1926	30 000	50	17–0	2	33 400	66 800	2.2
6	Fort Myers, Fla.	1927	8 000	55	15–0	1	35 600	35 600	4.4
7	Miami, Fla.	1927	Experimental	20	13–0	1	....	....	....
8	Haines City, Fla.	1927	3 000	26	13–9	1	7 300	7 300	2.4
9	Conway, Ark.	1927	7 500	35	20–9	1	20 000	20 000	2.7
10	Lake City, Fla.	1929	3 000	24	14–0	1	6 330	6 330	2.1
11	Carlsbad, Calif.	1929	2 000	20	15–0	1	4 700	4 700	2.3
12	Witten (Ruhrverband) Germany	1929	......	65	....	1	....	....	....
13	Atwater, Calif.	1930	3 600	22	15–0	1	5 700	5 700	1.6
14	Hartford, Wis.	1924	5 500	38½	14	1	16 400	16 400	3.0
15	Sheboygan, Wis.	1924	5 000	36	15	1	15 300	15 300	3.0
16	Kiel, Wis.	1925	2 000	25	15	1	7 370	7 370	3.7
17	Winterset, Iowa	1926	4 000	30	14–9	1	10 400	10 400	2.6
18	Moberly, Mo.	1927	10 000	30	23–0	1	16 200	16 200	1.6
19	Monroe, N. C.	1927	7 000	35	16–0	1	15 400	15 400	2.2
20	Blackwell, Okla.	1927	12 000	45	24–0	1	38 200	38 200	3.2
21	Salina, Kans.	1927	17 000	45	21–6¾	1	34 400	34 400	2.0
22	Pratt, Kans.	1927	5 200	30	21–0	1	14 900	14 900	2.9
23	Dunsmuir, Calif.	1927	2 500	38	15–0	1	17 000	17 000	6.8[a]
24	Easton, Pa.	1927	20 000	75	15–0	1	66 200	66 200	3.3
25	Roseville, Calif.	1927	10 000	40	16–0	1	20 100	20 100	2.0
26	Gilroy, Calif.	1927	4 000	50	24–6	1	48 100	48 100	12.0[a]
27	Moberly, Mo.	1928	4 000	35	20–6	1	19 700	19 700	4.9
28	Aurora, Colo.	1929	2 600	26	15–0	1	7 950	7 950	3.1
29	El Cajon, Calif.	1929	1 500	18	12–0	1	3 050	3 050	2.0

* Installations Nos. 1 to 13 are without cover of any kind; Installations Nos. 14 to 29 are open-type tanks, but have covers of boards laid on I-beams spanning space between superstructure and tank walls.

[a] Large capacity to handle trade wastes discharged into sewerage system.

These tanks are a development of the old Kremer sedimentation tank by the addition of a Kusch sludge cylinder below the apex of the steep hopper bottom, and a Kusch sludge digestion tank adjacent to the sedimentation tank. Sludge is transferred from the bottom of the cylinder to the digestion tanks by hydrostatic head. Two chambers are provided, rapid digestion and gasification taking place in the first, which is comparatively large and shallow. When partly digested, the sludge is removed to the second chamber, smaller in area, but deeper than the first, where the digestion process is completed.

Tark states that for a sedimentation tank 15 to 20 ft square, the usual diameter of the sludge cylinder is about 4½ ft, and that because the area of contact between the sludge in the cylinder and the sewage is small, the sludge may be retained in the cylinder for three or four days. The effect of this detention period and the depth of the cylinder is to secure a concentrated sludge with about 88% moisture.

Covered Tanks with Stirring Equipment.—Although a few covered tanks with stirring equipment are square in plan, by far the greater number are circular. The floor is cone-shaped, usually with a slope of about 1 to 48

toward a sump in the center. A pipe under the floor, terminating in the sump, permits the withdrawal of digested sludge. The roof is usually of concrete, and either slopes slightly upward toward the center or is dome-shaped.

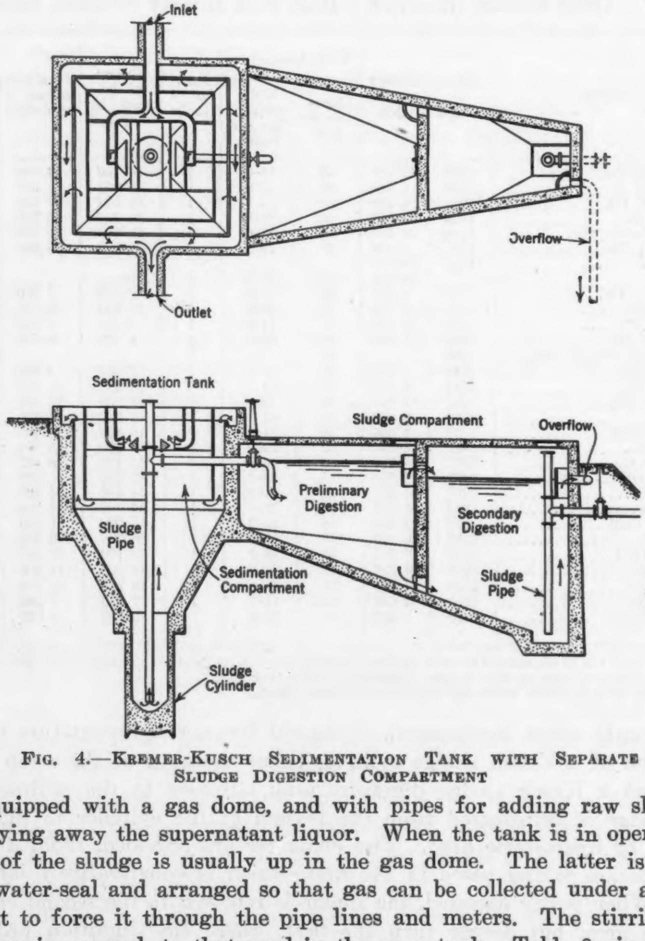
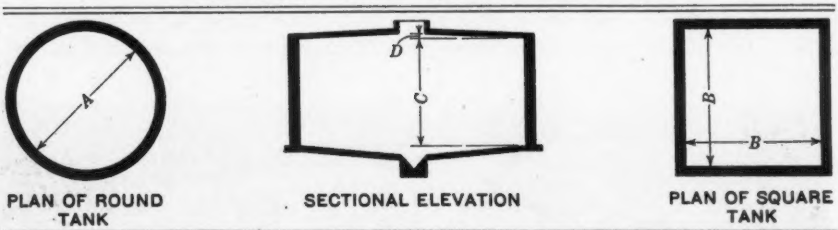


FIG. 4.—KREMER-KUSCH SEDIMENTATION TANK WITH SEPARATE SLUDGE DIGESTION COMPARTMENT

It is equipped with a gas dome, and with pipes for adding raw sludge and for carrying away the supernatant liquor. When the tank is in operation the surface of the sludge is usually up in the gas dome. The latter is equipped with a water-seal and arranged so that gas can be collected under a pressure sufficient to force it through the pipe lines and meters. The stirring device is similar, in general, to that used in the open tank. Table 8 gives data on plants using this type of equipment.

Tanks with Floating Covers.—Floating covers were first developed to permit the collection of gas at plants originally built with open tanks, but there is now at least one type of floating cover intended for use in new construction. Covers have been added to existing tanks, for example, at Plainfield, N. J., and Birmingham, England. The Plainfield covers are of wood; those at Birmingham are of concrete. The concrete covers (Fig. 5) are each 10 ft wide by 20 ft long, and are placed side by side, completely covering the tanks. At the center of each cover is a pyramidal gas dome, connected to the mains by flexible piping.

TABLE 8.—COVERED SLUDGE DIGESTION TANKS WITH ROTARY TYPE STIRRING EQUIPMENT, DESIGNED FOR GAS COLLECTION



Location	Date installed	Design population	TANK DIMENSIONS				Number of tanks	Volume of one tank, in cubic feet	Total tank volume, in cubic feet	Tank volume, in cubic feet per capita
			A	*B*	*C*	*D*				
Antigo, Wis	1926	11 000	50	..	17.33	0.50	1	34 710	34 710	3.2
Sioux Falls, S. Dak	1926	53 000	85	..	25.00	0.83	3	145 200	435 600	8.2[a]
Storm Lake, Iowa	1927	5 000	30	..	23.00	0.33	1	16 380	16 380	3.3[b]
Sedalia, Mo	1927	21 000	35	..	21.00	0.58	2	20 460	40 920	2.0
Fond du Lac, Wis	1927	35 000	.....	50	16.00	0.67	2	40 000	80 000	2.3
Springfield, Mo	1928	47 500	65	..	27.17	0.67	1	91 650	91 650	1.9
Northampton, Pa	1928	10 000	50	..	15.00	0.50	1	30 130	30 130	3.0
Aurora, Ill	1928	65 000	.....	50	16.50	0.50	3	41 250	123 750	1.9
Waukesha, Wis	1928	27 000	50	..	18.50	0.50	1	37 000	37 000	1.4
Salem, Ohio	1928	15 000	50	..	20.00	0.50	1	39 950	39 950	2.7
Fort Worth, Tex	1928	225 000	75	..	25.00	0.75	3	112 660	337 990	1.0[c]
Topeka, Kans	1928	40 000	65	..	23.00	0.67	2	77 830	155 650	3.9
Red Bank, N. J	1928	16 000	45	..	20.00	0.42	1	32 330	32 330	2.0
Petoskey, Mich	1928	9 000	40	..	16.00	0.42	1	20 430	20 430	2.3
Liberal, Kans	1929	3 500	30	..	16.00	0.33	1	11 430	11 430	3.3
San Antonio, Tex	1929	300 000	75	..	25.00	0.75	4	112 660	450 650	1.5
Waco, Tex	1929	75 000	75	..	25.00	0.75	1	112 660	112 660	1.5
De Kalb, Ill	1929	14 000	40	..	18.00	0.42	1	22 940	22 940	1.6
Lombard, Ill	1929	10 000	40	..	20.00	0.42	1	25 450	25 450	2.5
Chico, Colo	1929	20 000	40	..	18.00	0.42	1	22 940	22 940	1.1
Sturgis, Mich	1929	14 000	40	..	18.00	0.42	1	22 940	22 940	1.6
Grand Rapids, Mich	1929	250 000	70	..	24.50	0.75	4	96 200	384 800	1.5
Klamath Falls, Ore	1929	25 000	50	..	25.00	0.50	1	49 800	49 800	2.0
Cheektowaga, N. Y	1929	5 000	30	..	14.33	0.33	1	10 250	10 250	2.0
Ottawa, Kans	1929	12 000	35	..	20.25	0.33	1	21 900	21 900	1.8
Peoria, Ill	1929	140 000	85	..	29.00	0.83	4	167 900	671 600	4.8[a]
Toledo, Ohio	1930	600 000	85	..	29.00	0.83	8	167 900	1,343 200	2.2
Los Angeles, Calif	1930	3 000	19.50	..	15.00	0.21	1	5 000	5 000	1.7[d]
Greensboro, N. C	1930	35 000	60	..	23.00	0.58	1	66 190	66 190	1.8
Sparks, Nev	1930	7 000	30	..	19.00	0.33	1	13 550	13 550	1.9
Horicon, Wis	1930	3 000	26	..	16.00	0.25	1	8 560	8 560	2.8
Erie, Pa	1930	200 000	85	..	22.00	0.83	3	128 180	384 530	1.9
Ithaca, N. Y	1930	32 000	55	..	18.67	0.58	1	45 260	45 260	1.4
Galesburg, Ill	1930	35 000	35	..	17.00	0.33	3	16 610	49 830	1.5
Freehold, N. J	1930	8 000	30	..	17.00	0.33	1	12 140	12 140	1.5
Reno, Nev	1930	25 000	60	..	23.00	0.58	1	66 190	66 190	2.5
Abilene, Kans	1930	6 000	33	..	22.00	0.33	1	18 800	18 800	3.0
Carroll, Iowa	1930	7 000	35	..	18.00	0.33	1	17 570	17 570	2.5
Spring Lake, N. J	1930	6 925	.....	35	12.00	0.33	1	14 700	14 700	2.1
Spartansburg, S. C	1930	30 000	45	..	19.00	0.42	2	30 740	61 480	2.0
Wilmar, Minn	1930	9 000	40	..	18.00	0.42	1	22 940	22 940	2.5
Lima, Ohio	1931	70 000	85	..	22.00	0.53	2	128 180	256 350	3.6
Tulsa, Okla	1931	......	45	..	24.00	0.42	1	38 690	38 690	....
Tulsa, Okla	1931	......	45	..	19.00	0.42	1	30 740	30 740	....
Boulder City, Nev	1931	3 000	.....	24	13.00	0.25	1	7 490	7 490	2.5
Huron, S. Dak	1931	......	50	..	21.00	0.50	2	41 910	83 820	....
Muskegon Heights, Mich	1931	......	40	..	18.00	0.42	1	22 940	22 940	....
Toronto, Ont., Canada	1928	50 000	40	..	20.50	....	4	26 050	104 200	2.1
Johannesburg, South Africa	1929	......	40	..	22.00	....	4	27 970	111 880	....
Johannesburg, South Africa	1929	......	28	..	15.00	....	1	9 360	9 360	....
Essen, Germany	1929	......	40	..	27.88	....	1	....	....	....
Mildura, Australia	1930	8 000	40	..	15.00	....	1	19 170	19 170	2.4
Kitchener, Ont., Canada	1930	40 000	65	..	22.00	....	2	74 500	149 000	3.7

[a] Large capacity to handle industrial wastes. [b] No heating system provided. [c] Including equivalent industrial load. [d] Experimental unit.

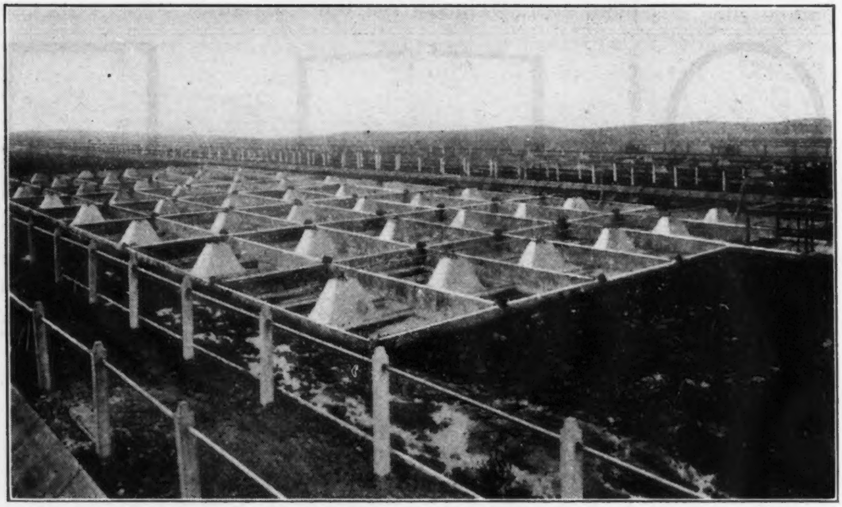


FIG. 5.—FLOATING CONCRETE COVERS ON SLUDGE TANKS, AT BIRMINGHAM, ENGLAND

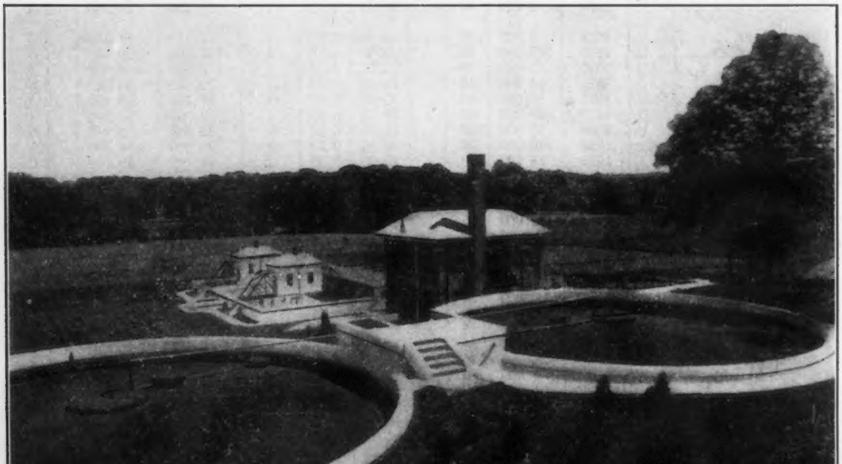


FIG. 6.—TYPICAL INSTALLATION OF SLUDGE DIGESTION TANKS WITH FLOATING COVERS, AT ELYRIA, OHIO

The floating cover designed for new construction is shown in Figs. 6 and 7. The framework consists of a system of radial trusses extending from a gas dome at the center to a point about 2 in. from the tank wall. Attached to the lower chords is a gas-tight and water-tight diaphragm of steel plate,

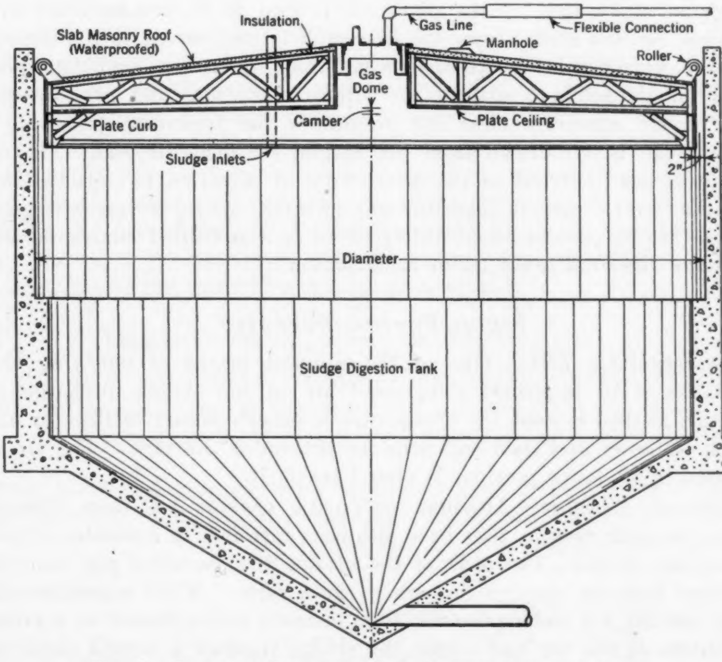


FIG. 7.—SEPARATE DIGESTION TANKS WITH FLOATING COVER

which floats on the surface of the liquid. On the upper chords rests an insulating roof of wood and water-proofed concrete At the outer ends of the trusses is a vertical curb of steel plate, extending from the top chords to below the lower chords and making a gas-tight and water-tight joint with the diaphragm. To insure free vertical movement, the cover is equipped with rollers that bear on the concrete walls of the tank.

Multi-Stage Digestion.—Multi-stage digestion consists of a system of two or more sludge digestion tanks, each connected with the other for the progressive and automatic transfer of the sludge and gas during digestion.

The primary tank may be equipped with a fixed steel dome cover and with agitators for keeping the sludge in a homogeneous condition which is considered by some to produce digestion and gas generation at the optimum rate. The primary tank may be equipped with a hot-water heat exchanger or other equipment for heating the digesting sludge to the desired temperature. The secondary or tertiary tanks may also be equipped with steel covers designed to serve as gas-holders of varying capacities.

Settled solids from the sedimentation tank enter the first stage of digestion. This primary tank is at all times kept filled to a constant level with sludge at a proper temperature and in a homogeneous condition by the mixers. The gas produced during digestion is collected at one point on the dome roof and taken to the secondary tank for storage.

Approximately 90% of the digestion is said to be accomplished in the first stage and the sludge from the first tank is transferred to the secondary tank by hydrostatic head. In the second stage, quiescent conditions prevail which allow the solids to settle to the bottom of the tank and leave the supernatant liquor relatively clear for return to the treatment process. The settled sludge is withdrawn from the bottom for final disposal. The small quantity of gas produced in the final stages of digestion is collected in the gas-holder cover where it joins the gas produced in the primary stage.

There are at present about thirty cities in the United States that have multi-stage digestion tanks under construction.

Sludge Pumping Equipment

Compressed-Air Lifts.—One of the simplest means of removing sludge from tanks is to introduce compressed air in the sludge discharge pipe. When this method is used, the sludge pipe is usually placed vertical, or nearly so, with its lower and open end near the bottom of the tank. The air pipe, perforated in its lower portion, is placed inside it.

Pneumatic Ejectors.—Although pneumatic ejectors are more commonly used for pumping sewage, they have also been installed in a number of places for pumping sludge. The body of the ejector is a cast-iron pot into which the sludge flows by gravity through a check-valve. When a predetermined level is reached, air under pressure, from either a rotary blower or a pressure tank, enters at the top and forces the sludge through a second check-valve into the discharge pipe. Ejectors can be made to operate either automatically or manually.

Diaphragm Pumps.—These pumps are made in two different designs. The first, not generally used for pumping sludge, has a free discharge; the second, more common, is used where the sludge must be pumped against a head. The pump consists of a cast-iron pressure chamber with inlet and outlet ports and a flexible diaphragm, the periphery of which is securely fastened to the pump body. An up-and-down motion is imparted to the diaphragm, through shafting, by an electric motor or gasoline engine. The ports are usually equipped with rubber ball-valves and renewable rubber valve seats. An air-pressure chamber to smooth out the pulsations of the discharge is provided just outside the pump.

Reciprocating Pumps.—A reciprocating pump consists essentially of a cylinder equipped with a piston or plunger and suction and discharge valves. Both ball-valves and metal flap-valves are used. In order to facilitate valve cleaning, the pump-casing is usually provided with removable hand-hole plates held in place by wing-bolts. Valve passages should be amply large, with as few bends as possible and free from any projections that might col-

lect fibrous or suspended materials. Where the pump is operating against moderate or high heads, an ample air chamber should be provided in the discharge main as close to the pump as possible.

Centrifugal Pumps.—Centrifugal pumps have been widely used for handling all kinds of sludge. Single and double-suction impellers, both open and enclosed, have been tried. The best adapted to sludge pumping is probably the single-suction, enclosed impeller, secured to the end of the pump shaft. With this arrangement the sludge cannot come in contact with the shaft, whereas in double-suction pumps the shaft is in direct contact with the sludge, and stringy materials frequently wrap around it and form obstructions. The pumps should operate at speeds not greater than 800 rpm, and preferably at 720 rpm, or less, in order to reduce the wear at the inlet of the impeller. One or more easily removable hand-hole plates in the pump-casing, through which obstructions can be removed, will minimize operating difficulties. Pumps with horizontal split cases have the advantage of being easily disassembled for inspection, cleaning, or repairs.

Equipment for Collecting and Utilizing Gas

General.—Although gas from digesting sludge has been utilized extensively only during the past four or five years, the practice is by no means new. Perhaps the first use made of it was at Exeter, England, where, in 1895,

TABLE 9.—Utilization and Sale of Gas at Sewage Treatment Works

Location	Period Reported		Quantity, in Thousand Cubic Feet*		
	Date	Length	Heating buildings	Heating sludge tanks	Operating gas engines
(1)	(2)	(3)	(4)	(5)	(6)
(*a*) Plants Reporting Quantities					
Berlin, Germany (Wassmannsdorf)	1929–30	1 yr.	12 400 [a]	...	31 800 [c]
Birmingham, England (Saltley)	1930	1 yr.	...	...	46 750
Bochum, Germany	1929–30	1 yr.	106	...	...
Chicago, Ill. (Des Plaines)	1927–28	1 yr.	...	145 [b]	...
Chico, Calif.	...	1 day	...	2 to 3	...
Dayton, Ohio	Dec., 1930	1 month	434	...	...
Decatur, Ill.	...	1 day	5 to 25	...	...
Essen-Frohnhausen, Germany	1929–30	1 yr.	106	...	1 766 [d]
Essen-Nord, Germany	1929	1 yr.	...	...	...
Essen-Nordwest, Germany	1929–30	1 yr.	...	...	...
Essen-Rellinghausen, Germany	1930	4 months	...	...	...
Fort Worth, Tex.	1930	7 months	Some	6 820	...
Gelsenkirchen-Nord, Germany	1929–30	1 yr.	...	1 059	...
Hagen, Germany	1930	1 yr.	460	...	...
Halle, Germany (Tafelwerder)	1929–30	1 yr.	...	2 100	2 800
Hattingen, Germany	1930	7 months	...	...	1 314
Iserlohn, Germany	1930	1 yr.	71	...	2 380
Kettwig, Germany	1929	1 yr.	...	...	...
Landendreer, Germany	1929	1 yr.	18	...	...
Leipzig-Wahren, Germany	1930	1 yr.	26	...	883
Munich, Germany	1930	1 yr.	...	...	...
Nürnberg, Germany (Nürnberg-Süd)	1930	1 yr.	530	...	...
Oberhausen, Germany	1929–30	1 yr.	106	...	...
Stuttgart, Germany (Mühlausen)	1930	1 yr.	...	...	...
Velbert, Germany	1930	1 yr.	...	...	...
Waukesha, Wis.	1930	1 yr.	1 460	548	...

TABLE 9.—(*Continued*)

Location	Quantity, in Thousand Cubic Feet (*Continued*)*			Amount Received†	
	Incinerating screenings	Miscellaneous	Sold to local gas works	Total, in dollars	Dollars per thousand cubic feet ‡
(1)	(7)	(8)	(8)	(10)	(11)
(a) Plants Reporting Quantities (*Continued*)					
Berlin, Germany, (Wassmannsdorf	1 910	1 180[e]	None	....	....
Birmingham, England (Saltley)	....	....	None	....	....
Bochum, Germany	....	....	None	....	....
Chicago, Ill. (Des Plaines)	....	....	None	....	....
Chico, Calif	....	....	None	....	....
Dayton, Ohio	465	....	None	....	....
Decatur, Ill	....	....	None	....	....
Essen-Frohnhausen, Germany	....	....	Electricity sold	191	....
Essen-Nord, Germany	....	138[f]	17 110	4 416	0.26
Essen-Nordwest, Germany	....	....	3 231	743	0.23
Essen-Rellinghausen, Germany	....	49[g]	3 390	780	0.23
Fort Worth, Tex	2 693	....	None	....	....
Gelsenkirchen-Nord, Germany	....	....	None	....	....
Hagen, Germany	....	....	None	....	....
Halle, Germany (Tafelwerder)	....	....	22 600	5 730	0.25
Hattingen, Germany	....	....	None	....	....
Iserlohn, Germany	....	71	None	....	....
Kettwig, Germany	....	....	920	286	0.31
Langendreer, Germany	....	....	932	193	0.21
Leipzig-Wahren, Germany	....	....	None	....	....
Munich, Germany	....	....	97 110	29 838[h]	0.31[i]
Nürnberg, Germany (Nürnberg-Süd)	....	....	16 406	2 000	0.12
Oberhausen, Germany	....	....	None	....	....
Stuttgart, Germany (Mühlausen)	....	....	48 200	26 047	0.54
Velbert, Germany	....	....	1 784	550	0.31
Waukesha, Wis	....	....	None	....	....

Location	Gas Was Used For:					Remarks
	Heating buildings	Heating sludge tanks	Operating gas engines	Incinerating screenings	Miscellaneous	
(12)	(13)	(14)	(15)	(16)	(17)	(18)
(b) Miscellaneous Plants§						
Antigo, Wis	Yes	Yes	No	....	....	....
Aurora, Ill	Yes	Yes	No	Proposed[k]	Laboratory	Excess wasted
Baltimore, Md	No	Yes	No	....	....	....
Charlotte, N. C. (Sugar Creek)	Yes	No	Yes	....	....	....
DeKalb, Ill	Yes	Yes	No	....	....	....
Elyria, Ohio	No	Yes	No	....	....	....
Fond du Lac, Wis	Yes	Yes	No	....	Water-still	Received $20 per month from sale of distilled water.
Grand Rapids, Mich	Yes	Yes	No	....	....	....
Greensboro, N. C	Yes	Yes	No	....	....	....
High Point, N. C	No	Yes	No	....	....	....
Middletown, N. Y	....	Yes	....	....	....	....
Neheim, Germany	Yes[j]	No	No	....	....	....
New Castle, Pa	Yes	No	No	....	....	....
Plainfield, N. J. (Joint)	No	Yes	Yes	....	....	....
Springfield, Ill	Some	Some	No	....	....	Mostly wasted
Sturgis, Mich	No	Yes	No	....	....	Excess blown off to furnace chimney
Toledo, Ohio (Bay View Park)	....	Proposed	....	....	....	....
Toronto, Ont., Canada (North Toronto)	No	Sludge	No	....	Laboratory	....
Waco, Tex	No	Yes	No	....	....	....
Wetter, Germany	No	No	No	....	....	....

* Quantities for German localities were converted from cubic meters. † Based on average value of Reich Mark in 1930 (23.87 cents). ‡ Column (10 ÷ Column (9). § Works where quantities were not available. None of the gas was sold. [a] Also for heating gas-holder. [b] In addition, 113 300 cu. ft. of city gas was used. [c] 65-, 335-, and 750-hp engines for driving sewage pumps. [d] 50-hp gas engine-generator. [e] 830,000 cu. ft. for street lighting; remainder for cooking. [f] Bottled and sold for explosive in coal mine. Received $279. [g] Laboratory. [h] Amount for December was estimated. [i] Cost of laying main and erecting gas-holder was borne by gas-works. [j] Attendant's house. [k] Also incinerating scum.

Cameron, the Borough Engineer, collected it and burned it in a street lamp. In 1902, James, at the leper colony at Matunga, India, constructed a gas-tight cover over two compartments of the septic tank, provided a gas-holder, and used the gas for cooking, lighting the compound, and running a ½-hp engine. The first American installation utilizing sewage gas was made in 1915 at the Peachtree Sewage Works, at Atlanta, Ga. Collectors were set in the gas vents of the Imhoff tanks, and sufficient gas was obtained to supply the plant laboratory and the dwelling of the operator.

Sludge gas is now used extensively for heating sludge tanks and buildings, and operating gas engines. Miscellaneous uses include lighting and incineration of screenings. In Germany, where it has perhaps been applied to a greater variety of purposes than elsewhere, quantities of it are frequently sold to the local gas plants. Table 9 summarizes the data on utilization obtained from a number of plants in both Germany and the United States.

Equipment for Collection of Gas

Gas hoods for separate digestion tanks are located above openings at high points in the roof slab. Generally, they are provided with separate water-seals, to insure against loss of gas around the hood in case the level of the liquid in the tank is lowered. Scum grids are used infrequently. In a few installations the hood consists simply of a casting tightly fastened to the roof slab without any seal.

A typical circular gas hood suitable for digesters as large as 55 ft in diameter is illustrated in Fig. 8, and Fig. 9 shows a typical rectangular hood for use on larger tanks. Data on hoods at twenty-five plants are given in Table 10.

Explosions in covered tanks containing digesting sludge are a very real hazard, but one that may be avoided if air is prevented from mixing with the gas in explosive proportions. Floating covers furnish an adequate and entirely automatic means of excluding air, since they rise and fall with the change of liquid level, and maintain a constant pressure in the dome. For tanks with fixed covers, the dome may be con-

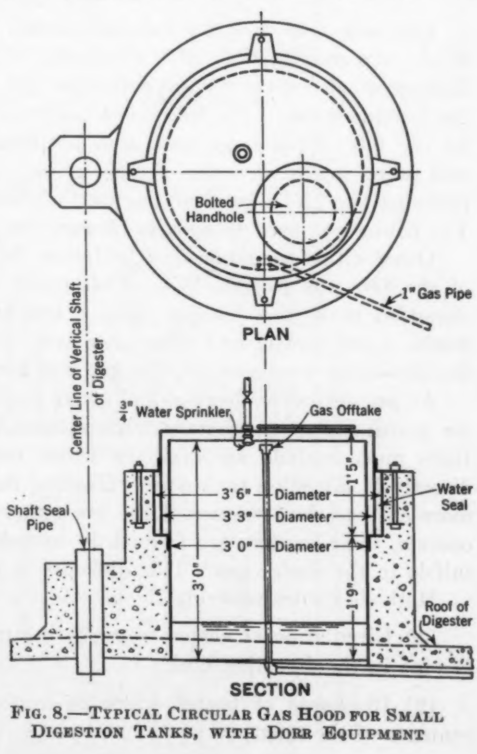
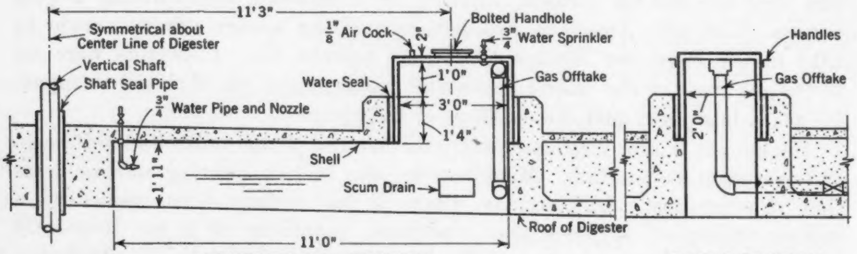


Fig. 8.—Typical Circular Gas Hood for Small Digestion Tanks, with Dorr Equipment

nected to a gas-holder, so that whenever sludge or liquor is drawn from the tank, the void will be filled by gas flowing back from the holder instead of by air entering through the water-seal. It should be noted that this



LONGITUDINAL SECTION CROSS SECTION

FIG. 9.—TYPICAL RECTANGULAR GAS HOOD FOR LARGE DIGESTION TANKS, WITH DORR EQUIPMENT; FIXED COVERS

method of protection fails if the gas-holder is allowed to become empty. Either the gas meters installed between dome and holder must be of a type that permits free flow of gas in both directions, or they must be equipped with a by-pass containing a check-valve.

SLUDGE-HEATING EQUIPMENT

Gas was first used for heating sludge, in February, 1926, at Plainfield, N. J. An earthen tank with a capacity of 25 000 cu ft was provided with a floating wooden cover for gas collection and a coil of iron pipe for circulating the heating water. The boiler was equipped for use of both gas and soft coal. In the fall of the same year, two additional sludge tanks were constructed and were heated by the sludge gases. The Plainfield installation also included two 15-hp gasoline engines, modified to use the sludge gas for fuel. The power was used to operate blowers for activating the sewage.

Other cities were not slow to follow the example set by Plainfield. One of the first was Antigo, Wis. That plant, built in 1927, included a covered digestion tank with heating coils, a gas boiler, a pump for circulating hot water, a gas meter, and other auxiliary devices. For a few months before the gas boiler was installed, the gas was used in a coal-burning steam heater.

At present, water heaters and steam boilers are both used to generate heat for maintaining satisfactory temperatures in sludge tanks. Boiler installations must include an auxiliary water heater, because steam is not used directly in digestion tank coils. Heating the water directly by gas is slightly more efficient, but because of the low temperature at which the water heaters operate, their burners are frequently corroded by the action of the hydrogen sulfide in the sludge gas. This difficulty is not encountered in the boilers.

Methods for transferring the heat to the sludge include:

(1) Circulation of heated water by means of a closed system through coils of pipe in the digestion tank.

(2) Discharge of heated water or heated sludge liquor directly into the contents of the tank.

TABLE 10.—DATA RELATING TO GAS HOODS FOR SEPARATE SLUDGE DIGESTION TANKS

Location	Type of gas hood	Scum grid	Separate water-seal	Size in plan	Area, in square feet	Number per tank	Inside area of tank, in square feet	Ratio of hood area to tank area	Size of gas pipe, in inches
Antigo, Wis.	Circular	No[a]	Yes	3 ft. 0 in.	7.1	1	1 964	0.0036	1½
Aurora, Ill.	Circular	No	Yes	4 ft. 0 in.	12.6	1	2 500	0.005	1½
Baltimore, Md.	Sheet-iron, rectangular	No	No	2 ft. 6 in. by 7 ft. 0 in.	17.5	1	1 134	0.015	1½
Charlotte, N. C. (Sugar Creek)	Cast-iron flanged pipe	Cast-iron grating	No	2 ft. 0 in.	3.1	4	....	....	2
Chicago, Ill. (Des Plaines)	Galvanized sheet iron	No	No	....	6.0	2	314	0.038	1
Chicago, Calif.	Circular	No	Yes	3 ft. 0 in.	7.1	1	1 257	0.006	1¼
De Kalb, Ill.	Circular	No	Yes	3 ft. 0 in.	7.1	1	1 257	0.006	1½
Elyria, Ohio	Floating cover	No	Yes	....	......	1	1 964	....	1¼
Essen-Frohnhausen, Germany	Sheet-iron, circular	No	Yes	....	10.8[b]	1	1 614[b]	0.007	1½
Essen-Rellinghausen, Germany	Sheet metal	No	No	2 ft. 7½ in. sq.[b]	6.9	.....	....	....	2[b]
Fond du Lac, Wis.	Circular	No	Yes	4 ft. 0 in.	12.6	1	2 500	0.005	2
Fort Worth, Tex.	Rectangular	No	Yes	2 ft. 6 in. by 6 ft. 8 in.	16.7	2	4 418	0.008	2
Grand Rapids, Mich.	Rectangular	No	Yes	3 ft. 0 in. by 5 ft. 0 in.	15.0	2	3 848	0.008	3
Greensboro, N. C.	Rectangular	No	Yes	3 ft. 0 in. by 6 ft. 0 in.	18.0	2	2 827	0.013	1½
Halle, Germany (Tafelwerder)	Cast-iron	Wood	No	....	......	2	....	....	....
High Point, N. C.	Cast-iron flanged pipe	No	....	4 ft. 0 in.	12.6	3	363	0.104	2
Los Angeles, Calif. (Exp. Station)	Circular	No	No	2 ft. 0 in.	3.1	1	314	0.010	1
Munich, Germany (Garosslappen)	Sheet-iron	Wood	No	....	......	1	....	....	4[b]
Plainfield, N. J.	Steel, circular	No	No	5 ft. 0 in.	19.6	1	1 300	0.015	1
Springfield, Ill.	Floating cover	No	....	....	......	1	1 257	....	....
Sturgis, Mich.	Circular	No	Yes	4 ft. 6 in.	15.9	1	1 257	0.013	1½
Toledo, Ohio (Bay View Park)	Rectangular	No	Yes	2 ft. 5 in. by 6 ft. 0 in.	15.0	2	5 675	0.005	3
Toronto, Ont., Canada	Concrete, circular with bolted iron cover	No	No	3 ft. 0 in.	7.1	1	1 257	0.006	4
Waco, Tex.	Rectangular	No	Yes	2 ft. 6 in. by 4 ft. 2 in.	10.4	2	4 418	0.005	3
Waukesha, Wis.	Circular	No	(Oil seal)	4 ft. 0 in.	12.6	1	1 964	0.006	6

[a] Used for a while but found to be unsatisfactory.
[b] Converted from metric units.

(3) Pre-heating of the sludge before its discharge into the digestion tank.

(4) Circulation of sludge through a separate tank equipped with heating coils.

Digestion tanks are sometimes covered with earth to reduce heat losses; the earth should be kept dry, for it loses about one-half its efficiency if it becomes saturated. In some cases special insulating materials may be found valuable.

Any heating system requires considerable mechanical equipment and many regulating devices which need frequent checking and servicing.

Gas-Burning Power Equipment

One of the largest generating plants in the world driven by sludge gas is that at Birmingham, England. The first power unit—a 25-bhp gas engine—was installed by Mr. John D. Watson in 1921. The engine was used to operate a 5-in. centrifugal sludge pump with a capacity of 540 gal per min against a head of 36 ft. It consumed, per horse-power hour, 20 cu ft of gas, with a methane content ranging from 60 to 77 per cent. The digestion tank that furnished the gas had a sludge capacity of 2 160 cu ft.

Although the experimental unit showed several inherent weaknesses, and the rate of gas production fell short of expectations—especially during the winter—the engine worked successfully and produced sufficient power to drive the pump. As a result, a much larger engine (Fig. 10) was installed in 1927. It was rated at 150-bhp and drove a 100-kw alternator. A second engine, with an output of 400 bhp, was added in 1929, and another unit of the same size was installed a year later, making a total capacity of 950 bhp. Gas used in these engines in 1930 totaled 46 750 000 cu ft, an average of 128 000 cu ft per day.

Among sewage treatment works in the United States utilizing gas for power is the Sugar Creek Plant, at Charlotte, N. C. The installation includes two gasoline engines of standard design, except that the carburetors have been removed and a gas pipe has been attached to the intake manifold. In case it should be necessary to use gasoline, the carburetors can be re-attached in a short time. One of the units is a 75-hp, 4-cylinder engine (Fig. 11), direct-connected to a centrifugal pump handling 3 000 gal of sewage per min, and the other is a 225-hp, 6-cylinder engine driving a centrifugal air-compressor with a capacity of 3 500 cu ft of air per min. It has been estimated that the saving in electric current would repay the cost of the two engines (approximately $10 500) in about three years.

IV.—COMPUTATIONS OF TANK CAPACITY AND HEATING REQUIREMENTS

Notation

The notation introduced in this report conforms essentially to the "American Tentative Standard Symbols for Heat and Thermodynamics"[7] approved by the American Standards Association in 1931.

A = area, in square feet; A_R = area of roof; A_W = area of walls; A_B = area of bottom of tank;

[7] A.S.A. — Z10c — 1931.

a_t = the fraction of volatile solids digested in time t; a_0 = the fraction of volatile solids digested in time t_0;
B = theoretical required capacity of a sludge tank, in cubic feet;
b = thickness, in inches b_R = thickness of roof; b_W = thickness of walls; b_B = thickness of the bottom of the tank;
C = calorific value of gas, in British thermal units per cubic foot;
e = percentage efficiency of utilization;
f = surface coefficient of transmittance per degree Fahrenheit; f_i = surface coefficient of heat transfer, to the inner surface of tank; f_o = surface coefficient of heat transfer from the outer surface of the tank;
g = cubic feet of gas produced per pound of volatile solids digested;
K = a ratio; K_l = ratio of the volume of liquid sludge added daily to the volume of sludge in the tank; K_v = ratio of weight of volatile solids added daily to the weight of volatile solids in the tank;
k = thermal conductance of a homogeneous material 1 in. thick, in British thermal units per square foot per hour of area, per inch of thickness, per degree (Fahrenheit) difference of temperature, between the two surfaces of the material;
N = tributary population;
Q = rate of sewage flow, in millions of gallons daily;
q = thermal transmission, in British thermal units per hour; q_R = heat transmitted through the roof; q_W = heat transmitted through walls; q_B = heat transmitted through the bottom of the tank;
S = dry solids added daily to sludge tank in parts per million of sewage;
T = temperature; T_i = temperature of the sludge in the tank; T_o = temperature of the outside air, or earth; T_s = temperature of incoming sludge;
t = time, in days; adopted period of digestion; t_0 = time required for substantially complete digestion (arbitrarily defined as the time required to reduce the volatile solids by 75%); t_1 = maximum interval between sludge drawings; t_2 = maximum time during which no digestion occurs;
U = over-all coefficient of heat transmittance, in British thermal units per square foot per hour, per degree (Fahrenheit) difference in temperature over-all; U_R = coefficient for heat transmittance through the roof; U_W = coefficient for heat transmittance through the walls; U_B = coefficient for heat transmittance through the bottom of the tank;
V = volatile fraction of solids;
W = weight, in pounds; W_d = weight of dry solids added daily;
w = percentage of water by weight; w_0 = percentage of water in entering sludge; w_1 = percentage of water in the withdrawn sludge; w_m = mean percentage of water in the sludge remaining in the tank $= \frac{w_0 + w_1}{2}$.

Tank Capacity Computations

General.—Capacities of from 1.5 to 3.5 cu ft per capita have been recommended, as the result of experience, for the sludge chambers of Imhoff tanks. In separate digestion tanks, however, less space is required, because the period of digestion is shortened by heating the sludge, and by thickening, agitation,

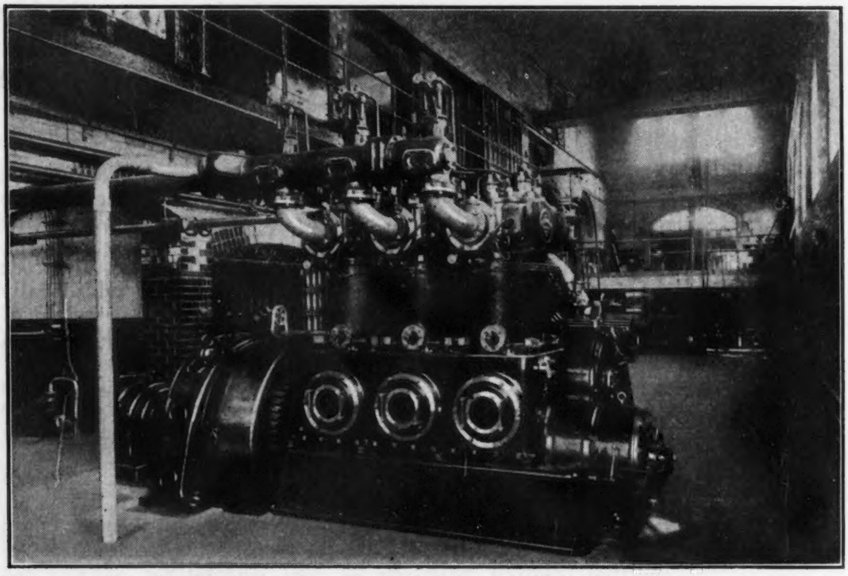


FIG. 10.—ENGINE USING SLUDGE GAS AS FUEL, BIRMINGHAM, ENGLAND

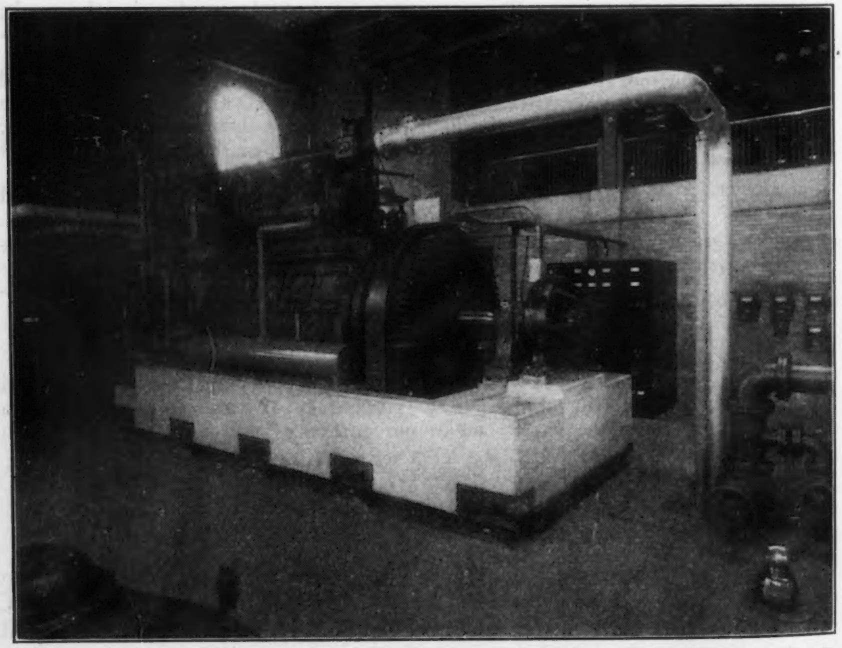


FIG. 11.—A 75-HORSE POWER ENGINE OPERATED BY SLUDGE GAS, CHARLOTTE, N. C.

or circulation. Moreover, there is no waste space such as that taken up by the sloping baffles in the lower story of the Imhoff tank. With the smaller, heated tanks, however, it must be borne in mind that the sludge must be drawn more frequently, and that the supernatant liquor is not cared for automatically.

It has been the practice to state the capacity of a sludge tank in cubic feet per capita, or cubic feet per million gallons daily of sewage. Neither of these units is altogether satisfactory, because of local differences in per capita quantities and in the strength of the sewage. The amount of solids in industrial wastes or in street wash from combined systems is a function of neither volume nor population. A better basis of design is the weight of solids in the sludge removed daily from the sewage. Consideration must also be given to the percentage of volatile matter in the solids, percentage digested, and, most important of all, the water content of the sludge.

Rational formulas have been developed for capacities of sludge tanks under several typical conditions, and diagrams have been prepared by the use of which capacities may be quickly obtained for widely varying conditions.

Case I.—Continuous Operation, with Daily Withdrawals.—Assume that a tank is in operation with a given weight of solids added daily and that an equal weight of solids disappears daily by digestion and withdrawal, so that the measurable sludge in the tank remains approximately at constant depth. Solids withdrawn include those passing out with the overflow of the supernatant liquor. The error in assuming a uniform rate of digestion from entry to withdrawal is on the side of safety.

The quantity of dry solids added on any one day will be reduced by digestion in the following period of t days by an amount, $a_t\ V\ W$ (see

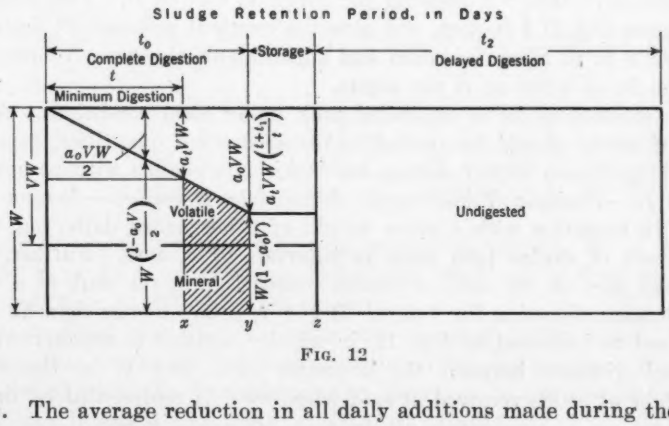


FIG. 12

Fig. 12). The average reduction in all daily additions made during the period will be $\frac{a_t\ V\ W}{2}$. The corresponding total reduction in weight will be $\frac{a_t\ V\ W\ t}{2}$, and the weight of solids remaining in the tank will be $W\ t\ \left(1 - \frac{a_t V}{2}\right)$.

In order that the quantity of solids in the tank may remain constant, the daily withdrawals (in pounds, dry basis) must be $W\left(1 - \frac{a_t V}{2}\right)$. Obviously, the weight of solids to be provided for in the tank is equal to t times the daily withdrawals. The theoretical total weight of solids and liquid is $\frac{W t \left(1 - \frac{a_t V}{2}\right)}{1 - w_m}$ which divided by the weight of a cubic foot of liquid (62.5 ± lb) gives the minimum theoretical required tank capacity, in cubic feet:

$$B = \frac{0.008\,(2 - a_t V)\,W t}{1 - w_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

The required capacity, in cubic feet per pound of dry solids added daily, is,

$$\frac{B}{W} = \frac{0.008\,(2 - a_t V)\,t}{1 - w_m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Capacity per capita of population may be expressed either as $\frac{B}{N}$, or as $\frac{S\,Q\,t\,(2 - a_t V)}{15 N\,(1 - w_m)}$.

Example 1.—Let $a_t = 0.50$; $V = \frac{2}{3}$; $t = 30$ days; $N = 100\,000$; $W = 10\,000$; $S = 120$; $Q = 10$; and $w_m = 0.925$ per cent. Then, by Equation (1), $\frac{B}{W} = 5.33$ cu ft per lb of solids added daily; and $B = 53\,000$ cu ft (net capacity). The net capacity per capita is 0.53 cu ft. Two tanks, 40 ft in diameter and 21.2 ft deep, will give the required volume. If the depth is increased 3 ft to allow for scum and supernatant, the gross volume will be 60 800 cu ft, or 0.608 cu ft per capita.

This solution is for a controlled tank under ideal conditions. A liberal factor of safety should be applied to the theoretical quantities, because the assumed conditions in any sewage are "subject to change without notice."

Case II.—Periodic Withdrawals, Incomplete Digestion.—Assume that a tank is in operation with a given weight of solids added daily, but that the withdrawals of sludge take place at intervals of t_1 days. Further, assume that every part of the mass of sludge removed from the tank at a drawing has undergone digestion for t days. If $t + t_1$ is not greater than t_0, the conditions are as indicated in Fig. 12, in which t_1 (max.) is represented by the horizontal distance between the ordinates at x and y, in the diagram. The weight of solids removed at said withdrawal is represented by the cross-hatched area and equals $W\ t_1\left[1 - \frac{a_t V}{2}\left(1 + \frac{t + t_1}{t}\right)\right]$. The maximum weight of solids in the tank before drawing, is $W\,(t + t_1)\left[1 - \frac{a_t V}{2}\left(\frac{t + t_1}{t}\right)\right]$.

The required volume is,

$$B = \frac{W\,(t + t_1)\left[1 - \frac{a_t V}{2}\left(\frac{t + t_1}{t}\right)\right]}{62.5\,(1 - w_m)} \quad \ldots\ldots\ldots\ldots\ldots(3)$$

Example 2.—Substitute the following numerical values: $W = 10\,000$; $t = 30$; $t_1 = 15$; $a_t = 0.50$; $V = \frac{2}{3}$; $w_m = 0.925$, in Equation (3); and $B = 72\,000$ cu ft, net capacity.

Again: If $t + t_1$ is greater than t_0, the conditions are as indicated in Fig. 12, in which t_1 (max.) is represented by the horizontal distance between the ordinates at x and z, in the diagram. The maximum weight of solids in the tank, before drawing is $W\left[t_0\left(1 + \frac{a_0 V}{2}\right) + (t + t_1 - t_0)\,(1 - a_0 V)\right]$, and the required volume is:

$$B = \frac{W}{62.5}\left[\frac{t_0\left(1 - \frac{a_0 V}{2}\right)}{1 - w_m} + \frac{(t_1 + t - t_0)\,(1 - a_0 V)}{1 - w_1}\right] \quad \ldots\ldots(4)$$

Example 3.—Substitute the following numerical values: $W = 10\,000$; $t = 30$; $t_1 = 30$; $t_0 = 45$; $a_0 = 0.75$; $V = \frac{2}{3}$; $w_m = -0.925$; $w_1 = 0.90$, in Equation (4); and $B = 84\,000$ cu ft, net capacity.

Case III.—Periodic Withdrawals, Complete Digestion.—The situation is shown in Fig. 12, in which t_1 is the horizontal distance between the ordinates at y and z, in the diagram. The maximum weight of solids in the tank before drawing is $W\left[t_0\left(1 - \frac{a_0 V}{2}\right) + t_1\,(1 - a_0 V)\right]$ and the required volume is:

$$B = \frac{W}{62.5}\left[\frac{t_0\left(1 - \frac{a_0 V}{2}\right)}{1 - w_m} + \frac{t_1\,(1 - a_0 V)}{1 - w_1}\right] \quad \ldots\ldots\ldots\ldots(5)$$

Example 4.—Substitute the following numerical values: $W = 10\,000$; $a_0 = 0.75$; $V = \frac{2}{3}$; $t_0 = 45$; $t_1 = 15$; $w_m = 0.925$; $w_1 = 0.90$, in Equation (5); and $B = 84\,000$ cu ft, net capacity.

It is to be noted that the identical results obtained in Examples (3) and (4) indicate that, under the assumed conditions, 75%, or complete, digestion may be reached when the intervals between drawings do not exceed 15 days, whereas 35-day intervals may result in only 50% digestion.

Case IV.—Unheated Tanks in Cold Climate.—In unheated tanks in a cold climate there will be periods during which digestion practically ceases. In such cases, capacity, in addition to that computed by Equation (4), will be required. If t_2 represents this period of suspended digestion, the addi-

tional capacity will amount to $\frac{W t_2}{62.5\,(1 - w_o)}$. Adding this to the right-hand member of Equation (4), a new expression is obtained for total capacity, as follows:

$$B = \frac{W}{62.5}\left[\frac{t_o\left(1 - \frac{a_o V}{2}\right)}{1 - w_m} + \frac{(t_1 + t - t_o)\,(1 - a_o V)}{1 - w_1} + \frac{t_2}{1 - w_o}\right] \quad ..(6)$$

The expression, $t_1 + t_2$, represents the maximum interval between drawings. The point where t_1 stops and t_2 begins is not sharply defined, but may be estimated, within reasonable limits, from the temperature of the sewage. Digestion may be assumed to cease when the temperature of the sludge has lowered to 50° F.

It has been observed that early drawings from Imhoff tanks, following cold weather, give sludge of greater density than is indicated by $\frac{t_2}{1 - w_o}$. This has led to the assumption that undigested sludge is densified by long storage and the volume correspondingly reduced. With increase in temperature, however, rising gas will expand the sludge mass and cause "foaming" and overflow if ample capacity is not provided.

Example 5.—Let the various numerical values be assumed as in Example 3 and let $t_3 = 60$. By substitution in Equation (6), $B = 276\,000$ cu ft.

Case V.—Tank Capacity as Determined by Permissible Ratio of Daily Additions of Volatile Solids to Volatile Solids Remaining in Tank.—Experience has demonstrated that the ratio, K_v, of volatile solids added, daily, to the volatile solids in the tank, should not exceed about 2%, for uncontrolled (unheated) tanks, nor 5%, for controlled tanks. Capacity determined by the preceding Equations (1), (3), or (4), should be checked to see that it meets this requirement. If digestion is assumed as progressing at a uniform rate, the ratio of weight of solids in the tank to their volatile content, may be expressed by $\frac{W\left(1 - \frac{a_t V}{2}\right)}{W V\left(1 - \frac{a_t}{2}\right)} = \frac{2 - a_t V}{V\,(2 - a_t)}$. In order not to exceed the ratio, K_v, the volatile solids in the tank at all times must be $\frac{V W}{K_v}$; hence, the total solids in the tank must be $\frac{V W\,(2 - a_t V)}{K_v V\,(2 - a_t)} = \frac{W\,(2 - a_t V)}{K_v\,(2 - a_t)}$. Taking into account the water content and dividing by 62.5, the required capacity is:

$$B = \frac{0.016\,W\,(2 - a_t V)}{(1 - w_m)\,K_v\,(2 - a_t)} \quad ....(7)$$

The tank should not be drawn lower than the capacity indicated by Equation (7). Expressing the permissible quantity of influent, daily, in terms

of its volumetric ratio to the quantity of sludge in the tank:

$$K_l = \frac{\text{Sludge added daily}}{\text{Sludge in the tank}} = \frac{0.016\,W}{1 - w_0} \div \frac{0.016\,W\,(2 - a_t\,V)}{(1 - w_m)\,K_v\,(2 - a_t)}$$

$$= K_v\,\frac{(1 - w_m)\,(2 - a_t)}{(1 - w_0)\,(2 - a_t\,V)} \qquad (8)$$

Example 6.—Substituting in Equation (7), the numerical values given in Example 1, $B = \dfrac{2\,370.37}{K_v}$. For an uncontrolled tank, $B = \dfrac{2\,370.37}{0.02}$ $= 118\,500$ cu ft, and for a controlled tank, $B = \dfrac{2\,370.37}{0.05} = 47\,400$ cu ft. Substituting numerical values in the last member of Equation (8), $K_l = 13.5\,K_v$. For uncontrolled tanks, $K_l = 1.35 \times 0.02 = 0.027$; and for controlled tanks, $K_l = 1.35 \times 0.05 = 0.0675$. $\left[\text{Check: } \dfrac{0.016 \times 10\,000}{0.05 \times 47\,400} = 0.0675.\right]$

It will be noted that the foregoing value of B, for the controlled tank (47 400 cu ft) is about one-ninth less than the value obtained in Case I, in which the period of digestion is assumed at thirty days. If the method presupposed in Case I were followed, namely, daily withdrawals, the former computation would stand and the capacity would be theoretically sufficient, providing the tank was never drawn down below a capacity of 47 400 cu ft, or eight-ninths of its computed net capacity.

In Case II, Equation (3), the net capacity was computed as 72 000 cu ft. In order always to leave 47 400 cu ft in the tank, it should never be drawn lower than 47 400 ÷ 72 000 = 65.8% of its net capacity. Similarly, in Equation (4) and in Case III, the limit of draw-down would be 47 400 ÷ 84 000 = 56.4% of the net capacity.

In Case IV, the unheated tank, during the period of delayed digestion, should not be drawn below 118 500 ÷ 276 000 = 17% of its net capacity.

It will be observed that the ratios, K_v and K_l, have two applications, useful to the plant operator: (1) The fixing of the limits of sludge withdrawal; and (2) the transformation of the ratio of volatile solids, K_v, into terms of relative volume, K_l. In the latter determination there are two principal factors which affect the value of K_l, namely, the change in water content of the sludge and the percentage reduction in volatile solids. For example, if the water in fresh sludge is reduced from 95% to 92.5% in the tank, the value of K_l will be increased 50 per cent. On the other hand, the greater the reduction in volatile solids, the lower the value of K_l.

Fig. 13 contains five diagrams: Fig. 13 (*a*) and (*b*) refer, respectively, to heated and unheated tanks. They are plotted from Equation (8) and exhibit the relations of K_v and K_l. Fig. 13 (*c*), (*d*), and (*e*) are based, respectively, on Equations (2), (4), and (6), from which, by using the constants employed, respectively, in Examples 1, 3, and 5, the following three abridged formulas have been deduced: $\dfrac{B}{W} = 3.2\ (2 - a_t\,V)$; $\dfrac{B}{W} = 7.2\left(\dfrac{5}{3} - a_0\,V\right)$;

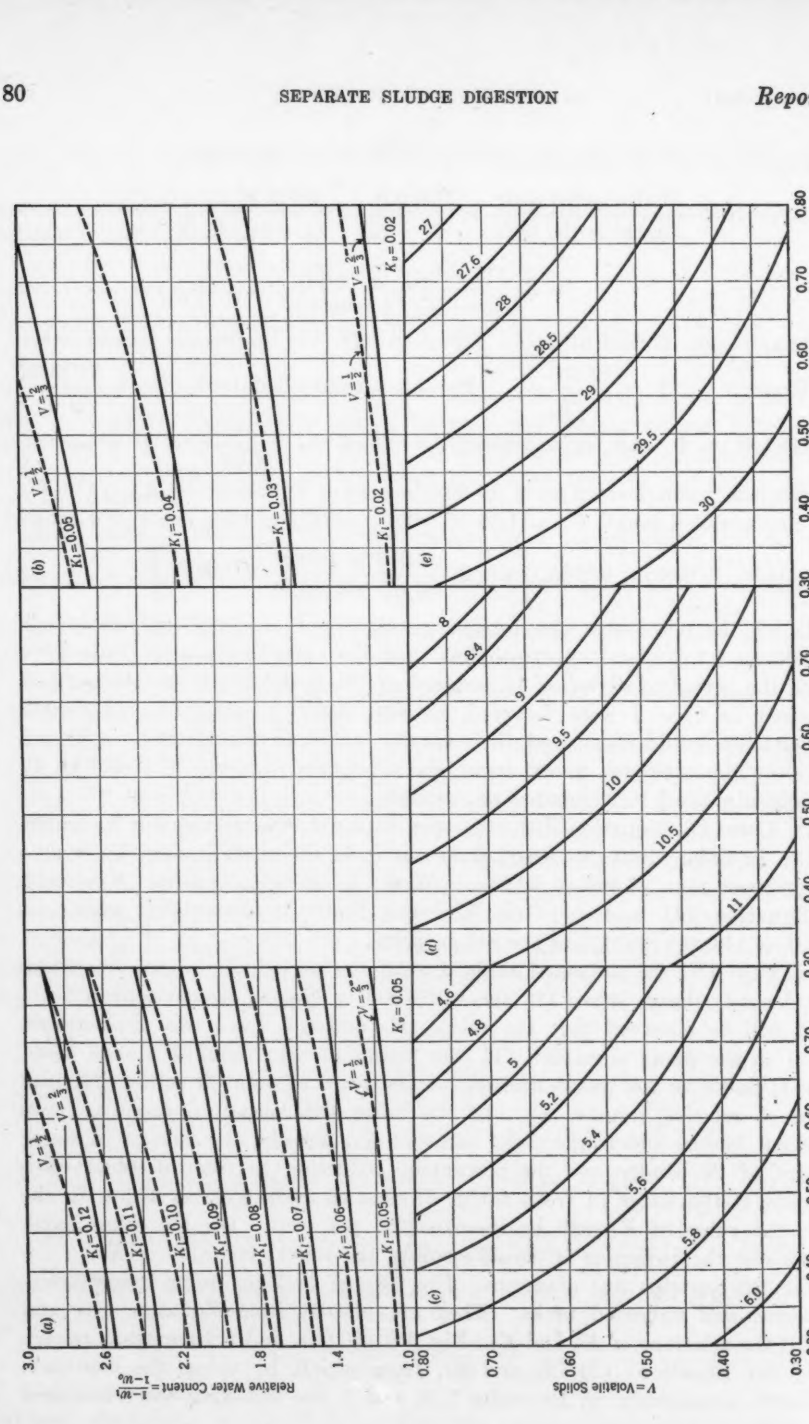


FIG. 13.—DIAGRAMS FOR DETERMINING THEORETICAL CAPACITY OF SLUDGE DIGESTION TANKS

and, $\frac{B}{W} = 7.2\left(\frac{13}{3} - a_0 V\right)$. Fig. 13 (*c*), (*d*), and (*e*) are plotted from these formulas with "Amount of Volatile Solids" and "Amount Digested" (both expressed in percentages) as variables. In each diagram, the quantity sought may be found on the curves or may be approximated closely by interpolation. For different assumptions of the time elements and water contents, variations may be made in the constants in the abridged formulas from which different, although similar, curves will result.

Heating Computations

General.—The quantity of gas produced by the digestion of sludge amounts to from 0.5 to 1.0 cu ft per capita per day, or from 5 to 19 cu ft (average, 7.8 cu ft) per lb of volatile solids added daily to the tank. A more definite unit for use in computation, however, is the amount produced per pound of volatile solids digested. The general average for this value is about 12 cu ft. Calorific values of sludge gas are discussed in Section I.

When a tank is first warmed, the temperature of the entire mass of sludge must be raised, and a relatively large amount of heat will be required; but after the operating temperature has been reached, it is only necessary to make up the losses due to conduction and radiation and to heat the relatively small quantity of incoming sludge.

Heat losses through the roof, walls, and bottom of the tank must all be taken into account. The formula commonly used for determining heat loss through a wall of homogeneous material with air or gas on one side, maintained at a given temperature, T_i, and with the other side open to the air, or in contact with earth at a lower temperature, T_o, is:

$$\text{British thermal units per hour} = A\,U = A\left(\frac{T_i - T_o}{\frac{1}{f_i} + \frac{b}{k} + \frac{1}{f_o}}\right) \quad \ldots\ldots(9)$$

If the wall is not homogeneous, but composed of several layers of material with different values of k, each material is considered separately; that is, the denominator of the complex fraction will contain more than one term, $\frac{b}{k}$. If liquid rather than air or gas is in contact with the wall on the inside, the fraction, $\frac{1}{f_i}$, is omitted. The surface coefficients on either side of air spaces less than $\frac{1}{2}$ in. in thickness are also omitted, assuming there are no convection currents.

Coefficients of thermal conductance and of surface transmittance for various materials are given[8] in Table 11. Most of these coefficients have been been determined in air for heating and ventilating purposes and from hot pipes buried in the ground. There is need for experiments to determine losses under the conditions actually obtaining in sewage practice; that is,

[8] See 1931 Guide, Am. Soc. of Heating and Ventilating Engrs.

from tanks of concrete, wood, and steel, containing sludge and gas at the range of temperatures met in sewage work. Coefficients of surface transmittance into the air from hollow tile, steel, wood, concrete, and earth surfaces, losses into surrounding wet or dry earth, and losses caused by flow of ground-water, should all be given further study.

TABLE 11.—COEFFICIENTS OF THERMAL CONDUCTANCE AND SURFACE TRANSMITTANCE

(*a*) CONDUCTANCE THROUGH HOMOGENEOUS MATERIALS, 1 INCH THICK, IN BRITISH THERMAL UNITS PER SQUARE FOOT PER HOUR.

Material	Coefficient	Material	Coefficient
White pine (across the grain)	0.78	Air spaces thicker than ¾ in.	1.10
Virginia pine	0.96	Gypsum plaster	2.32
Yellow pine	1.00	Dry brickwork	4.00
		Damp or wet brickwork	5.00
Maple	1.10 to 1.20	Cement mortar	8.00
		Cinder concrete	5.20
Wood (average value)	1.00	Stone concrete	8.30
Cellular gypsum	0.59		

(*b*) TRANSMITTANCE FROM SURFACES EXPOSED TO STILL AIR*, IN BRITISH THERMAL UNITS PER SQUARE FOOT PER HOUR.

Material	Coefficient*
Smooth plaster surfaces	0.93
Concrete	1.30
Wood	1.40
Brickwork	1.40
Average of many building materials	1.34
Concrete exposed to dry earth	1.00
Concrete exposed to water or wet earth	2.00
Hollow tile (2-in. furring)†	1.18
Tar and gravel roofing (⅜-in.)†	3.67

* For air motion at 15 miles per hr., these coefficients should be multiplied by 3.
† Over-all value for non-homogeneous combinations of materials.

Example 7.—Let $W = 4\,000$ lb; $V = \frac{2}{3}$; $a_o = 0.75$; $w_o = 0.95$; $g = 12$ cu ft; $C = 700$ Btu per cu ft; $e = 60\%$; $T_i = 82°$ F; $T_s = 56°$ F (mean annual); T_o in air, $= 50°$ F (mean annual); T_o, in earth, $= 60°$ F (mean annual) and T_o, for ground-water, $= 54°$ F. The velocity of the air is assumed to be 15 miles per hr.

The dimensions of the digestion tank are as follows: Diameter, 40 ft; height, 24 ft, of which all but 3 ft, is banked with dry earth; roof, 8-in. concrete; walls 18-in. concrete; and floor, 12-in. concrete, located below the water-table. Gas is present between the sludge surface and the roof of the tank.

The heat available per hour is:

$$\frac{W\,V\,a_0\,g\,C\,e}{24} = \frac{4\,000 \times \frac{2}{3} \times 0.75 \times 12 \times 700 \times 0.60}{24} = 420\,000 \text{ Btu}$$

The heat required to bring the incoming sludge to the specified digestion temperature is:

$$\frac{W\,(T_i - T_s)}{24\,(1 - w_0)} = \frac{4\,000\,(82 - 56)}{24\,(1 - 0.95)} = 86\,666 \text{ Btu per hr}$$

The losses are found by substituting known values in Equation (9): $A_R\,U_R = 20\,600$ Btu per hr. and,

$$A_w\,U_w = (3 \times 40\pi\,U_1) + (21 \times 40\pi\,U_2) = 120\pi\,U_1 + 7\;U_2)$$

in which U_1 refers to the part of the wall above the ground and U_2 to that below. Then,

$$A_w\,U_w = 377\left[\frac{82-50}{\dfrac{18}{8.3}+\dfrac{1}{4.02}}+\frac{7\,(82-60)}{\dfrac{18}{8.3}+\dfrac{1}{1}}\right] = 23\,300 \text{ Btu per hr}$$

and,

$$A_w\,U_w = \frac{1\,257\,(82-54)}{\dfrac{12}{8.3}+\dfrac{1}{2}} = 18\,100 \text{ Btu per hr}$$

The total heat loss is $A_R\,U_R + A_w\,U_w + A_R\,U_R$, or 62 000 Btu per hr. Adding to this the amount required to heat the incoming sludge, the total heat required is found to be 148 667 Btu per hr, or 35.4% of the heat available from the gases of digestion.

These computations, it will be noted, were based on assumed mean annual temperatures for incoming sludge, air, earth, and ground-water. More heat will be required per hour in the winter, and less in the summer. A complete investigation, of course, would include computation of the requirements for both extremes

Although the cost of construction rather than the question of heat loss will usually dictate structural dimensions, it is of interest to note the effect of changes in those dimensions on the amount of heat lost. For example, a decrease of 6 in. in the thickness of the tank wall would raise the value of A_wU_w to 28 900 Btu per hr—an increase of 24 per cent. On the other hand, if the thickness of the roof were doubled, $A_R\,U_R$ would become 13 380 Btu per hr, or 65% of the original value. It should be noted also that if in Example 7 the sludge had been in contact with the roof, the fraction, $\frac{1}{1.34}$, would have been omitted from the denominator in the expression for $A_R\,U_R$ and the heat loss would have been increased to 33 800 Btu per hr.

V.—METHODS OF SLUDGE DISPOSAL

Regardless of whether or not disposal is preceded by digestion, the problem of getting rid of sludge is an important one both to the designer and to the operator. Local conditions will determine the extent of the provisions necessary, and, together with the method of sewage treatment in use, must be taken into account in selecting the methods to be used. In general, the methods may be divided into (1) methods for disposing of liquid sludge; and (2) methods for disposing of dewatered sludge.

Liquid Sludge Disposal

The five main methods of disposing of liquid sludge are: (*a*) Transportation by scow to some suitably isolated point in deep water, in a lake, sea, or ocean; (*b*) transportation by tank wagon or truck to agricultural lands where

the solid portions are eventually plowed under and become a source of plant food; (*c*) "lagooning," or intermittently flooding the sludge upon a favorably located tract of land, until deposited solids completely fill the area thus used; (*d*) broad irrigation; and (*e*), discharging into the flood waters of a stream, or by pipe line into the deep water of a lake or sea.

Method (*a*) has a limited applicability. Outstanding examples are large seaport cities. The City of London has used this method, and New York City will dispose of some undigested sludge from the Wards Island plant in this manner.

As the fertilizing value of liquid sludge is extremely small in proportion to its bulk, Method (*b*) is not promising unless local conditions are peculiarly favorable.

Method (*c*) is not final unless the deposited solids constitute a permanent fill of suitable character. The dried sludge may be hauled away by farmers for soil treatment, or by others for use on lawns, golf courses, parks, etc., so that the capacity of the lagooned area will be prolonged indefinitely Undigested sludge is lagooned in this manner at Indianapolis, Ind.

The use of Method (*d*), also, is possible in only a few localities, where the soil and the climate conditions are both favorable.

Method (*e*) has a limited application and its use is not likely to meet the approval of either laymen or public health officials.

Disposal of Dried Sludge

In a large majority of sewage works, the liquid tank sludge is dewatered before being finally disposed of. This may be done on drying beds or by several types of pressing or vacuum filtration equipment. In general, the drying beds produce a cake from 3 to 5 in. thick, and the filter presses and vacuum filtration equipment produce a cake less than 0.5 in. thick. Either cake can be transported readily.

There are six main methods of disposing of dried or partly dried sludge: (*a*) Sale or free disposal for use as fertilizer; (*b*) wasting; (*c*) use as fill; (*d*) burying; (*e*) incineration; and (*f*) transportation to sea by scow.

Method (a).—Sale or Free Disposal for Use as Fertilizer.—In the Municipal Index for 1931, twenty-seven cities and villages in the United States, representing a population of 639 000, reported that all their sewage sludge was sold as fertilizer, and thirteen additional cities, representing a population of 664 000, reported the sale of quantities varying from 10 to 95% of the total production. Data on the sale of sludge by some of these cities are given[9] in Table 12.

The most conspicuous example of the disposal of sewage sludge as fertilizer is Milwaukee, Wis. (not included in Table 12) where during 1928, 1929, and 1930, 91 800 tons of sludge (Milorganite) was sold for $1 720 000—an average price of $18.75 per ton and an average production of approximately 1.0 ton of fertilizer per million gallons of sewage treated. The Milwaukee works include grit chambers, coarse screens, fine screens, activated sludge treatment, clarification, and sludge conditioning, filtration, and drying.

[9] See Municipal Index, 1931.

TABLE 12.—DATA ON SALE OF SLUDGE FOR FERTILIZER

City	1930 population	Type of plant	Percentage of total sludge sold	Price per ton
Arkansas:				
DeQueen	2 900	....	100	....
California:				
Los Angeles County Sanitary Districts	200 000	Activated sludge	100	$2.00
South Pasadena	13 724	Activated sludge	100	36.00
Alhambra	29 500	Activated sludge	100	25.25
Pasadena	123 000	Activated sludge	100	20.00
Pomona	20 804	Imhoff tanks and areation	100	6.00
San Bernardino	37 500	Imhoff tanks: Trickling filters	100	1 000 *
Indiana:				
Brazil	8 744	Imhoff tanks: Trickling filters	100	20.00 *
Huntington	13 420	Imhoff tanks	....	1.50
Kentucky:				
Fort Thomas	10 039	Imhoff tanks: Trickling filters	100	2.00
Michigan:				
Harbor Beach	1 892	Imhoff tanks: Sand filters	75	1.50
Pontiac	64 928	Imhoff tanks: Trickling filters	100	0.50
Missouri:				
Moberly	13 772	Activated sludge	60	1.50
Springfield	57 527	Imhoff tanks: Trickling filters	50	2.00 †
New Jersey:				
Audubon	8 904	Imhoff tanks: Trickling filters	Some	1.00
Sea Isle	850	Plain sedimentation	50	....
Trenton	123 356	Imhoff tanks	10	0.75
New York:				
East Rochester	6 627	Imhoff tanks: Trickling filters	50	0.50
Mt. Morris	3 238	Imhoff tanks	100	....
Rochester	328 132	Imhoff tanks: Trickling filters	95	0.25
Schenectady	95 692	Imhoff tanks: Trickling filters	100	0.25 ‡
North Carolina:				
Charlotte	82 675	Activated sludge	40	....
Winston-Salem	75 274	Sedimentation	100	0.00
Ohio:				
Celina	4 664	Imhoff tanks	10	....
Chardon	1 800	Imhoff tanks	50	....
Columbiana	2 484	Imhoff tanks: Trickling filters	100	1.00 ‡
Pennsylvania:				
Elizabethtown	3 940	Imhoff tanks: Trickling filters	100	1.00
South Carolina:				
Greenville	29 154	Septic tanks	16	....
Wisconsin:				
Madison	57 899	Imhoff tanks: Trickling filters	100	1.00
Monroe	5 013	Septic tanks: Sand filters	100	0.25

* Price per year. † Price per cubic yard. ‡ Price per load.

The major element contributing to the value of sludge as a fertilizer is nitrogen. Schaetzle[10] reported the nitrogen content (dry-residue basis) in Baltimore, Md., sludge to be as follows:

	Percentage
Imhoff tank sludge	2.75
Digestion tank sludge	2.45
Raw sludge	2.64
Final settling basin sludge	3.19

Keefer[11] reported the results of sludge-drying at Baltimore extending over a period from August 1, 1921, to June 1, 1924, during which time 7 448 tons of wet sludge containing 68% moisture were dried to 2 949 tons of a moisture content varying from 9.4% to 23.6% and having a content of nitrogen as ammonia of from 2.05% to 2.60% on the dry basis.

In general, it may be said that the sludge produced in an activated sludge plant is considerably richer in nitrogen than that produced in clarification-

[10] *Engineering News-Record,* July 14, and 21, 1921.
[11] *Loc. cit.,* February 11, 1926.

the solid portions are eventually plowed under and become a source of plant food; (*c*) "lagooning," or intermittently flooding the sludge upon a favorably located tract of land, until deposited solids completely fill the area thus used; (*d*) broad irrigation; and (*e*), discharging into the flood waters of a stream, or by pipe line into the deep water of a lake or sea.

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Celina	4 664	Imhoff tanks	10	......
Chardon	1 800	Imhoff tanks	50	......
Columbiana	2 484	Imhoff tanks: Trickling filters	100	1.00 ‡
Pennsylvania:				
Elizabethtown	3 940	Imhoff tanks: Trickling filters	100	1.00
South Carolina:				
Greenville	29 154	Septic tanks	16	......
Wisconsin:				
Madison	57 899	Imhoff tanks: Trickling filters	100	1.00
Monroe	5 013	Septic tanks: Sand filters	100	0.25

* Price per year. † Price per cubic yard. ‡ Price per load.

The major element contributing to the value of sludge as a fertilizer is nitrogen. Schaetzle[10] reported the nitrogen content (dry-residue basis) in Baltimore, Md., sludge to be as follows:

	Percentage
Imhoff tank sludge	2.75
Digestion tank sludge	2.45
Raw sludge	2.64
Final settling basin sludge	3.19

Keefer[11] reported the results of sludge-drying at Baltimore extending over a period from August 1, 1921, to June 1, 1924, during which time 7 448 tons of wet sludge containing 68% moisture were dried to 2 949 tons of a moisture content varying from 9.4% to 23.6% and having a content of nitrogen as ammonia of from 2.05% to 2.60% on the dry basis.

In general, it may be said that the sludge produced in an activated sludge plant is considerably richer in nitrogen than that produced in clarification-

[10] *Engineering News-Record,* July 14, and 21, 1921.
[11] *Loc. cit.,* February 11, 1926.

sludge digestion plants. The Milwaukee sludge, for example, averaged about 6.5% nitrogen as ammonia during 1928, 1929, and 1930.

Phosphoric acid and potash are usually present in sewage sludge in small quantities (less than 1%). To the present, the relatively low market price of these ingredients has removed them from serious consideration; however, in the future, they may be of value.

Fig. 14 shows the selling price of nitrogen as ammonia for the twenty-two years, 1910–1931, inclusive, based on the sale of garbage tankage at Chicago,

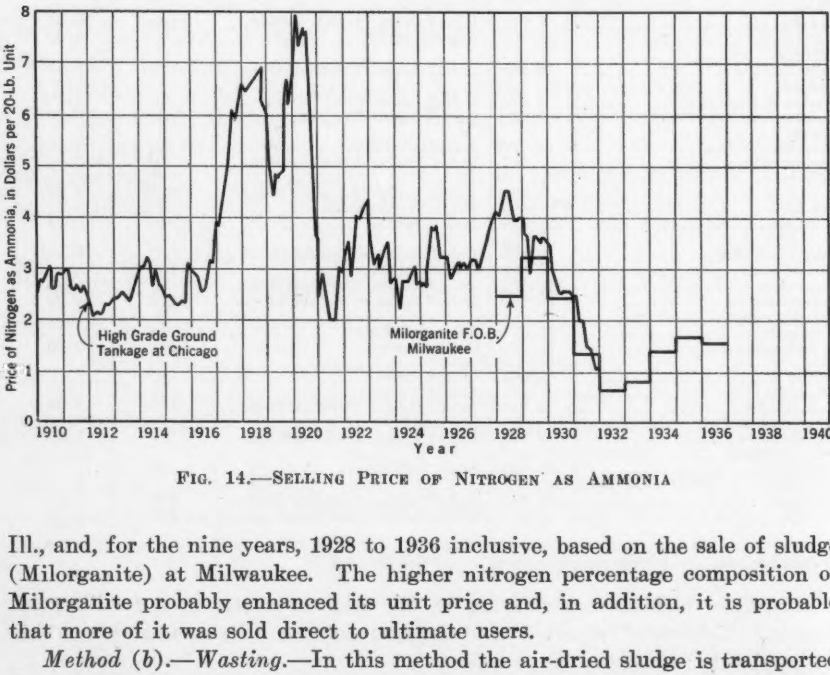

FIG. 14.—SELLING PRICE OF NITROGEN AS AMMONIA

Ill., and, for the nine years, 1928 to 1936 inclusive, based on the sale of sludge (Milorganite) at Milwaukee. The higher nitrogen percentage composition of Milorganite probably enhanced its unit price and, in addition, it is probable that more of it was sold direct to ultimate users.

Method (b).—Wasting.—In this method the air-dried sludge is transported from the drying beds to some convenient place where it is allowed to accumulate; no plan is prepared for its ultimate disposal.

Method (c).—Use as Fill.—In this case the air-dried sludge is transported to a borrow-pit, or some other excavated area. This method of disposal is limited by the capacity of the area which can be used economically for this purpose.

Method (d).—Burying.—In this method the dried sludge is thrown into trenches and covered with earth. This method is expensive, and is warranted only where a lack of isolation makes it desirable to remove all possible source of nuisance.

Method (e).—Incineration.—The sludge is burned in specially adapted furnaces or in open piles. If air-dried sludge is properly incinerated, nothing remains but the ash, which is inert and not a source of nuisance. In

some cases, the sludge from a drying bed is collected in a pile on the bed itself and burned, leaving only the ash to be removed.

Method (f).—Transportation to Sea by Scow.—This method of disposal is limited to seaport cities, and it is a question of economy as to whether the extra cost of drying and the smaller bulk to be transported is preferable to the disposal of the larger bulk of liquid sludge in the same manner.

Summary.—In many instances, it will be found that in actual practice, two or more of these methods will be in use at the same place. For example, where sludge is wasted or where it is used as fill, portions of the accumulated material may be hauled away by private individuals for use as fertilizer.

The Municipal Index for 1931 reports the methods of sludge disposal of 398 cities and villages in the United States, as follows:

(1) Sale or use as fertilizer	210
(2) Wasting	68
(3) Use as fill	38
(4) Burying	23
(5) Discharge into a body of water	18
(6) Part used as fertilizer, remainder wasted	9
(7) Part used as fertilizer, remainder used as fill	9
(8) Incineration	7
(9) Lagooning	6
(10) Transportation by scow for disposal in ocean	3
(11) Part used as fertilizer, remainder incinerated	3
(12) Broad irrigation	1
(13) Part used as fertilizer, remainder lagooned	1
(14) Part used as fertilizer, remainder buried	1
(15) Part incinerated, remainder used for fill	1
Total reporting	398

In the foregoing fifteen methods of disposal, the first five comprise 90% of the total, as indicated in the following percentage comparison:

(1) Sale or use as fertilizer	52.8
(2) Wasting	17.1
(3) Use as fill	9.5
(4) Burying	5.8
(5) Discharge into body of water	4.5
Total	89.7

It is probable that some who reported that sludge was incinerated or buried had in mind screenings instead of sludge, and that others who reported it as discharged into a body of water were confusing it with plant effluent.

VI.—OPERATING ROUTINE OF SLUDGE DIGESTION

The entire process of sludge digestion should be carried on without creating objectionable odors; should leave the solid and liquid by-products in

a favorable condition for further treatment or final disposal; should provide for recovery of by-products if desirable; and, finally, should be economical and dependable.

The basic design of two-story digestion tanks has been standardized to a large extent. As a consequence, the operating technique is also more or less standard. This has not been the case with separate digestion tanks, in which variations in basic features of design, and differences in the quality of sludge handled, have resulted in a wide variety of operating practice. Some of the following statements, therefore, may be subject to considerable modification as future data are accumulated and co-ordinated.

Placing Tanks in Operation

Experience has shown that digestion tanks can be put into successful operation without the use of seeding material, but the time required for building up an alkaline sludge will undoubtedly be longer, and more careful supervision of operation may be necessary, than if the tank is seeded initially. The quantity of seeding material introduced usually ranges from 20% to 40% of the tank capacity. Seeding is especially desirable if a tank is being put into operation in the winter, although even then it is not absolutely necessary for either the two-story or the separate digestion tank.

Introduction of Sludge

After tanks are in operation the problem of maintaining a proper balance between the digested, partly digested, and fresh solids, is encountered. The quantity of seeding material in the digestion compartment should always be sufficient to counterbalance the acid produced by the incoming fresh solids. It is advisable to add fresh solids continuously, or at frequent intervals, and at rates not exceeding 3% to 5% of the solids in the tank. This proportion should probably be based upon volatile matter content rather than on a total solids basis (see Section IV).

Use of Lime

The optimum pH-value, or reaction, for any particular plant must be determined by experimentation. Reaction control should not be necessary if the tanks are properly operated, although it may be necessary in cases where the unusual character of sewage or improper operation of tanks has destroyed the alkaline balance of the sludge.

The re-agent generally used in reaction control is lime hydrate, which not only increases alkalinity, but also coagulates the colloidal organic matter. The results obtained depend to a large extent upon the quantities of re-agent added, and the method of application or mixing.

Stirring of Sludge

There is considerable difference of opinion as to the advantages of stirring sludge. It does seem necessary or desirable, however, that some stirring be done, at least during intervals when raw sludge is being introduced into the

tank. The amount of stirring that should accompany this process remains open to question. There is apparently little necessity for stirring in a 2-story tank, although some agitation may be necessary when sludge is being withdrawn. Stirring in separate digestion tanks has been practiced to varying degrees in many installations. The advantages usually claimed are that it sets free the entrained gas at a uniform rate, and promotes digestion by helping to maintain a favorable reaction, mixing any chemical re-agents that may be added, and assisting in distributing raw sludge through the tank.

Collection, Measurement, and Storage of Gas

The operation of this phase of the sludge digestion process may be summarized, as follows:

1.—Individual and master meter readings should be recorded daily. Meters should be checked occasionally for accuracy.

2.—Gas lines, domes, etc., should be checked frequently and kept clear and free from obstructions at all times. Usually, ample sizes of pipes will insure against troubles of this nature.

3.—Pressures in the system should be checked and recorded daily, or more often.

4.—Water in seals should be maintained at proper levels, and kept clear of sludge and scum. Outside seals should be filled with oil or similar material during winter months to prevent freezing.

5.—Lights and fires should be kept away from digestion tanks and from the gas-collection system.

6.—Gas masks should be kept available for use in case of emergency.

Removal of Sludge

The removal of sludge depends upon the degree of digestion attained. In unheated tanks it is advisable to hold sludge in the tanks as long as possible in the spring in order to take care of the winter accumulation of partly digested sludge.

In the process of drawing, the sludge should be allowed to flow at a slow regular rate, in order to prevent the formation of channels and the withdrawal of partly digested sludge. Failure of sludge to flow may usually be remedied by agitation with water-jets, or by mechanical means.

Removal of Scum

The accumulation of large quantities of scum, which consists of fibrous material, such as leaves, sticks, matches, and seeds, in the gas vents of 2-story tanks, or on the surface of separate digestion tanks, may cause odors, prevent the even and free discharge of gas, or clog the pipe lines. Heavy layers of scum may form only during the first few months of operation, or may persist for long periods without necessarily causing a reduction in tank efficiency.

When necessary, various methods are used for preventing excessive formations of scum, such as the installation of submerging racks, or the mechanical

breaking up of the layer by hand or by stirring equipment with revolving arms, thus causing the scum to re-settle. In some cases, it has been found successful to flush with water or sludge liquor so that the liquid scum can be drawn off through pipe lines. Ordinarily, it will be necessary to remove scum only once or twice a year, if at all.

Removal of Supernatant Sludge Liquor

The satisfactory disposal of the supernatant liquor which separates from the solids in separate digestion tanks, presents a difficult problem. This liquor is never clear, has a high oxygen demand, and is relatively high both in suspended and dissolved solids.

Draw-off lines for supernatant sludge liquor should be provided at different elevations in the tank. The various methods of disposing of the liquor include its return to the inlet of sedimentation tanks or to the digestion compartment of 2-story tanks, or its discharge to drying beds.

Control of Foaming

The term, "foaming", is applied to a condition which may exist in either a 2-story tank, or a separate digestion tank, when large quantities of light, gas-lifted sludge rise in the gas vents or to the surface of the tanks. This condition is not only unsightly in appearance and productive of foul odors, but it interferes seriously with the efficiency of the plant.

Various methods have been suggested for the control of foaming, such as (*a*) drawing sludge; (*b*) hosing gas-vent areas by water under pressure; (*c*) resting the tank by putting it out of service; (*d*) circulation of supernatant liquor on to the surface of the material; (*e*) adding hydrated lime either to the raw sewage or, preferably, to sludge as it is pumped back into the gas vents; (*f*) treatment of the raw sewage with chlorine; and (*g*) the removal of all the sludge from the tank and the building up of a new sludge. The accepted practice seems to be the withdrawal of sludge from the tank and the maintenance of proper quantities of good seeding material.

Control of Odors

The entire elimination of odors about a sewage treatment plant is impossible, but a thorough knowledge of the treatment processes and careful operation of the plant will usually make it possible to avoid odors about which justifiable complaint could be made. It is more effective to avoid conditions that produce odors than to control the odors themselves.

One of the most important single factors having a direct bearing upon the control of odors is that of general cleanliness. All accumulations of sewage solids and grease on the walks or walls of the tanks, or about the grounds, should be removed immediately. Except for well-digested sludge drawn upon the beds, or otherwise disposed of, no sludge should be left exposed to the atmosphere. The hydrogen sulfide odors which sometimes develop in sedimentation and sludge digestion tanks may be controlled rather effectively by the use of liquid chlorine.

Tools Required in Routine Operation

General Tools.—These include tools that may be used elsewhere about the plant, such as wheel-barrows, shovels, hoes, brooms, ladders, and valve keys.

Special Tools.—Tools required for 2-story or separate sludge digestion tanks include water hose and nozzle for flushing, rubber squeegees, or chain drag, for cleaning flow compartment walls, perforated metal or wire skimmer for removal of scum, pitcher pump and hose for determining sludge level, and sampling buckets for composite samples, and long-handled dippers for securing the small amounts.

Analyses and Records

Laboratory control and the keeping of records of laboratory analyses are important as a guide to proper operation. The records of solids introduced and removed from tanks are important because the efficiency of digestion processes is largely measured by the reduction of volatile water. Gas quantities and temperature records are important in studying rates and progress of digestion.

Every care should be taken to have the results of the analyses represent a true portrayal of what has been accomplished during the process of changing (fresh) solids to digested sludge[12]. Data that should be obtained in connection with sludge digestion tanks may be summarized as follows:

(1) General Data (recorded daily): Temperature of atmosphere, in degrees Fahrenheit; hot water circulated, in gallons per day; temperature of circulating water (incoming and outgoing); and temperature of sludge in tank.

(2) Incoming Sludge: Time of introducing sludge, in hours; volume added, in gallons; temperature of sludge, in degrees Fahrenheit; and analysis of sludge, daily, or for each volume added: (*a*) Percentage of total and dissolved solids; (*b*) percentage of ash and volatile matter; (*c*) pH-value and alkalinity; and (*d*) specific gravity or actual weight per gallon.

(3) Sludge Withdrawn: Volume withdrawn, in gallons; and analysis of sludge, daily[14] or for each volume withdrawn (same as for incoming sludge).

(4) Supernatant Sludge Liquor: Volume withdrawn, in gallons; analysis of supernatant, daily[13], or for each volume withdrawn: (*a*) Percentage of total and dissolved solids; (*b*) percentage of ash and volatile matter; and (*c*) pH-value and alkalinity.

(5) Scum (Recorded Monthly): Thickness, in inches; volume removed, in gallons; analysis of scum: (*a*) Percentage of moisture; (*b*) percentage of ash and volatile matter; (*c*) pH-value and alkalinity; and (*d*) specific gravity.

(6) Gas: Volume, in cubic feet (recorded daily); analysis of gas (weekly): (*a*) Heat value, in British thermal units; and (*b*) percentage of CO_2, N_2, CH_4, O_2, and H_2.

In sampling digested sludge, composite samples should be collected proportionate to the quantity of sludge pumped. The fresh solids should be sampled by suspending small buckets in the flow compartments of sedimentation tanks.

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[12] "Standard Methods for the Examination of Water and Sewage," pub. by the Am. Public Health Assoc.

[13] Daily or sterilized weekly composites from daily samples.

Analyses of sludge removed from primary settling tanks and sludge digestion tanks should include observations as to color, odor, and streaking on a glass plate; and determination of volume, percentage of moisture, percentage of volatile matter, pH-value, alkalinity, specific gravity, and occasionally grease and total nitrogen, and possibly bio-chemical oxygen demand.

It is of great importance, in the operation of digestion tanks of both types, to measure and record the depth of sludge in the tank, as well as the quantities added and removed. In the case of a 2-story tank, the information is important in order to keep the sludge line well below the slot.

Sludge may be measured with any of the following types of equipment:

1.—Pitcher pump with flexible hose. The suction end of the hose can be raised or lowered until the sludge line is found.

2.—Flat tray of fine wire mesh or perforated plate, attached to a chain.

3.—Bottles (with removable stoppers) which can be lowered by means of a pole to obtain samples at various depths.

All of the aforementioned methods are fairly reliable, but the pitcher pump and hose is the equipment most widely used.

ACKNOWLEDGMENTS

The Committee wishes to acknowledge the assistance in the preparation of this report to the members of a special Sub-Committee, as follows: Messrs. W. S. Mahlie, F. W. Mohlman, T. C. Schaetzle, and W. F. Shepard, and R. A. Allton, H. E. Babbitt, R. H. Gould, Glen D. Holmes, H. E. Miller, and W. Rudolfs, Members, Am. Soc. C. E., W. D. Hatfield and R. T. Regester, Assoc. Members, Am. Soc. C. E., and C. K. Calvert, and W. R. Copeland, Affiliates, Am. Soc. C. E.

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APPENDIX I

OPERATING DATA FROM GRAND RAPIDS, MICHIGAN, APRIL, 1931, THROUGH JUNE, 1932

Through the courtesy of the City Officials at Grand Rapids, Mich., and of James R. Rumsey, M. Am. Soc. C. E., Superintendent of the Sewage Treatment Works, and N. G. Damoose, Chemist, the Committee was given the opportunity of making observations and collecting specific information relat-

ing to the operation of the sludge digestion tanks at Grand Rapids, from April 1, 1931, through June, 1932.

The sewage treatment plant at Grand Rapids comprises screening, sedimentation, and separate sludge digestion, and includes square settling tanks and traction-type equipment for removing sludge. There are four sludge digestion tanks and two sludge storage tanks. The digestion tanks are 70 ft in diameter and 24.5 ft deep (liquid side depth), and are equipped with tight covers, stirring devices, and heating coils. Three pumps are provided for handling raw sludge and two for digested sludge. Interconnections permit its transfer from one tank to another, although during the observation period there was practically no re-circulation of sludge between the tanks, and no seeding.

The rated capacity of the plant is 32.5 mgd, and the average flow during the period of observation was 24.4 mgd. Thus, the digestion tanks were not heavily loaded. The estimated connected population upon which all the following per capita computations are based is 168 000. The population equivalent, based on a 5-day bio-chemical oxygen demand of 0.168 lb per capita, is 180 000.

The City of Grand Rapids has a combined sewerage system. The sewage is relatively fresh, and the average flow is about 140 gal per capita per day. Although there are no unusual industrial sewages tributary to the treatment plant, the per capita quantity of suspended matter is relatively high, amounting to 0.230 lb per capita per day. A summary of typical analyses for the observation period is given in Table 13.

TABLE 13.—CHARACTERISTICS OF SEWAGE AT GRAND RAPIDS, MICHIGAN, DURING PERIOD OF OBSERVATION BY COMMITTEE

Period	Sewage flow, in million gallons daily	Sewage temperature, in degrees Fahrenheit	Total Solids			Suspended Solids			5-day bio-chemical oxygen demand	pH
			Total	Volatile, in parts per million	Percentage volatile	Total, in parts per million	Volatile, in parts per million	Percentage volatile		
1931:										
April	22.2	....	1 064	539	49.6	252	172	71.2	199	7.05
May	20.7	....	1 092	620	56.6	154	115	78.8	172	7.06
June	26.2	....	1 509	916	60.1	175	135	77.1	149	6.98
July	23.4	69.5	1 635	882	54.1	172	122	72.8	132	6.98
August	24.3	70.2	1 785	1 095	61.2	233	156	66.6	140	7.24
September	22.0	70.5	1 632	1 136	69.5	248	178	74.7	152	7.20
October	21.7	66.0	1 385	907	65.0	198	149	75.4	129	7.28
November	21.8	60.6	995	534	53.3	175	122	71.9	124	7.21
December	25.0	55.4	957	562	58.2	154	129	83.0	189	6.88
1932:										
January	23.1	50.2	933	467	50.4	148	91	65.4	188	6.97
February	25.5	50.2	1 098	569	51.8	213	139	65.3	203	7.33
March	27.6	49.0	1 095	643	58.8	187	136	72.7	264	7.32
April	26.8	52.2	1 049	569	54.1	184	129	73.3	151	7.25
May	29.2	59.3	1 105	619	56.0	168	116	68.9	184	7.04
June	26.3	67.3	1 366	875	64.1	206	142	69.1	168	7.12
Average	24.4	....	1 246	728	57.5	191	135	72.4	169	7.12

OPERATING ROUTINE

The sludge-collecting equipment in the primary settling tanks is operated continuously. At 8-hr intervals the valves on the sludge lines are opened

and fresh sludge is pumped to each digestion tank, for a period of 10 to 30 min. This routine of operation results in fresh sludge with an average solids content of 4.9 per cent.

Space for the incoming sludge was made available either by the discharge of sludge supernatant, or by the withdrawal of digested sludge. The ordinary routine was to allow sludge supernatant to flow from the digestion tanks whenever fresh sludge was introduced, but to withdraw digested sludge only once in 24 hr. The supernatant was returned to the inlet of the primary settling tanks. There was no difficulty in maintaining a practically uniform temperature in the digestion tanks, the departure from the average during the delivery of sludge being negligible.

There is a sampling cock on the discharge of each of the five pumps. During pumping, a 1.5-pt sample was taken every 10 min and composited. The length of the pumping period was determined by the thinning of sludge at the sampling cock.

To facilitate uniform operation, a ratio of sludge withdrawn to sludge introduced was determined from time to time, in order to maintain the proper sludge depth in the tanks. Ratios ranging from 17.5% to 50% were applied.

Data Secured

During the period of observation, an effort was made to secure a complete record of the operation and behavior of the sludge digestion tanks. All the necessary analyses relating to incoming sludge, withdrawn sludge, sludge supernatant, scum, and gas were included and were made by the Laboratory Staff of the Grand Rapids treatment works on daily composites with few omissions. In addition, several sets of determinations were made to ascertain the characteristics of the sludge at depth intervals of 5 ft. It was impossible to get more than a few analyses of the scum and the sludge supernatant.

The data were averaged from daily analyses. A summary is given in Table 14. The percentage of volatile matter and of solids in the fresh and withdrawn sludges for each of the four tanks is shown on Fig. 15.

The sampling and analysis of sludge are subject to some error and, therefore, the results cannot be considered as precise. As a measure of accuracy, a computation of a solids balance was made.

All determinations of the solids content of sludge were made by evaporation, and the results, therefore, are in total solids rather than in suspended solids. The difference between the two is only a matter of 2% or 3%, depending upon the moisture content of the sludge.

For the computation of the solids balance, a period between June 15, 1931, and May 31, 1932, was taken, as these were the first and last dates on which analyses were made of the total solids content of the digestion tanks.

During the observation period there was but one unusual condition of any importance. Difficulty has been experienced in Grand Rapids with the disposal of lime sludge from the water-softening plant. During April and August, 1931, some of it was discharged into the sewers. This sludge was almost entirely calcium carbonate, with a small quantity of magnesium

TABLE 14.—SUMMARY OF DATA COLLECTED AT GRAND RAPIDS, MICHIGAN, SEWAGE TREATMENT PLANT

Period	Incoming Sludge				Sludge Withdrawn				Sludge Supernatant			
	Total Solids		Volatile Solids		Total Solids		Volatile Solids		Total Solids		Volatile Solids	
	Percentage	Im thousands of pounds	Percentage	In thousands of pounds	Percentage	In thousands of pounds	Percentage	In thousands of pounds	Percentage	In thousands of pounds	Percentage	In thousands of pounds
1931:												
April	8.8	817	59.0	482	14.6	534	44.1	247	0.5	74	61.0	39
May	6.3	455	65.8	300	13.0	330	49.5	164	0.5	50	61.0	30
June*	4.9	260	66.1	172	11.9	126	48.7	61	0.5	28	61.0	17
June†	....	368	71.9	265	10.4	104	48.3	50	0.5	27	61.0	16
July	4.9	608	65.7	400	9.0	160	47.9	77	0.5	49	61.1	30
August	7.6	1 005	58.7	590	14.5	508	46.6	235	0.5	60	61.1	36
September	5.5	949	67.9	645	18.8	430	46.1	198	0.5	38	61.1	23
October	4.7	667	70.4	470	19.8	611	45.8	280	0.5	39	63.8	25
November	4.0	518	70.7	366	15.3	840	47.8	402	0.5	27	57.7	16
December	3.6	470	74.5	350	16.1	308	47.5	146	0.5	37	57.7	21
1932:												
January	3.5	446	68.2	304	13.9	795	46.7	371	0.35	13	57.0	7
February	3.5	777	71.9	559	10.8	361	48.1	173	0.35	21	57.0	12
March	3.2	732	74.2	544	8.8	124	49.1	61	0.34	35	59.4	21
April	3.26	675	75.1	507	9.0	150	49.9	75	0.35	31	57.7	18
May	4.0	680	66.2	450	9.3	179	48.8	88	0.51	41	57.7	24
June	5.2	798	66.8	533	9.6	178	48.1	86	0.27	25	57.0	14
Totals‡	....	10 225	....	6 937	....	5 738	....	2 714	....	595	....	349
Average‡	4.9	....	67.7	....	12.9	....	47.3	....	0.44	...	58.6	...
Totals§	....	7 895	....	5 450	....	4 570	....	....	....	418	....	...
Average§	....	....	....	....	....	....	....	....	....	...	....	...

Period	Gas					Solids Balance						
	Volume, in thousands of cubic feet (corrected to 30 inches at 60° F)	In thousands of pounds ‖	Cubic feet per capita per day (168 000 population)	Cubic feet per pound of incoming volatile matter	Percentage of incoming volatile matter reduced to gas	Total solids discharged, in thousands of pounds	Accumulation of total solids, in thousands of pounds	Volatile solids discharged in sludge and sludge supernatant, in thousands of pounds	Reduction in Volatile Solids: In thousands of pounds	Reduction in Volatile Solids: Percentage	Total volatile solids discharged, in thousands of pounds	Accumulation of volatile solids, in thousands of pounds
1931:												
April	2 879	188	0.57	6.0	39	796	+21	286	196	41	474	+8
May	4 520	296	0.87	15.1	99	676	−221	194	106	35	490	−190
June*	2 114	138	0.79	11.0	80	292	−32	78	96	56	216	−44
June†	2 014	132	0.86	7.6	50	263	+105	66	199	75	198	+67
July	4 159	272	0.80	10.4	68	481	+127	107	293	73	379	+21
August	4 715	308	0.91	8.0	52	876	+139	271	319	54	579	+11
September	4 176	273	0.83	6.5	42	741	+208	221	424	66	494	+151
October	3 898	250	0.77	8.3	53	900	−233	305	165	35	555	−85
November	4 114	267	0.82	11.2	73	1 134	−616	418	−52	−14	685	−319
December	3 577	232	0.69	10.2	66	577	−107	168	182	52	400	−50
1932:												
January	2 348	153	0.45	7.7	50	961	−515	378	−74	−24	531	−227
February	2 960	192	0.61	5.3	34	574	+203	185	374	67	377	+182
March	4 519	294	0.87	8.3	54	453	+279	82	462	85	376	+168
April	4 198	273	0.83	8.3	54	454	+221	93	414	82	366	+141
May	4 134	269	0.80	9.2	60	489	+191	112	338	75	381	+69
June	4 454	290	0.88	8.4	54	493	+305	100	433	81	390	+143
Totals‡	58 779	3 827	....	....	..	10 160	+65	3 064	3 875	..	6 891	+46
Average‡	......	.....	0.77	8.5	55	.....	.....	....	.....	56	.....	.....
Totals§	......	2 915	....	....	..	7 903	−8	....	.....	..	5 321	+129
Average§	......	.....	....	....	..	.....	.....	....	.....	..	.....	.....

*June 1 to 16, inclusive. † June 17 to 30, inclusive. ‡ Entire 15-month period. §June 17, 1931, through May 31, 1932.
‖ Assuming 65 lb. per 1 000 cu. ft.

hydrate. It had no caustic alkalinity. About 44% of the sludge was volatile. During the first period, the dry weight of lime sludge was about three times, and during the second period, about twice, the dry weight of suspended sew-

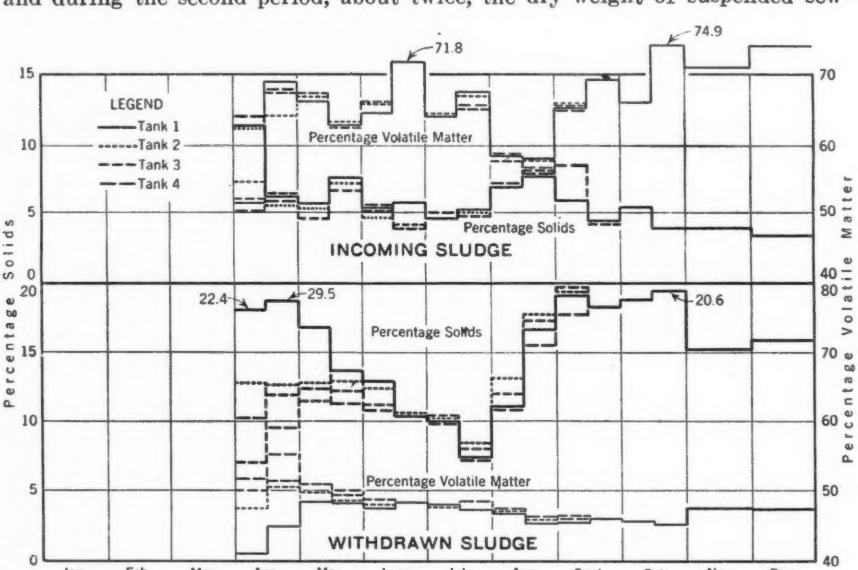

FIG. 15.—CHARACTERISTICS OF INCOMING SLUDGE AND WITHDRAWN SLUDGE FOR EACH OF THE FOUR DIGESTION TANKS AT GRAND RAPIDS, MICH.

age solids. After the first period (ending on April 13, 1931) it required 36 days for the lime to work out of the plant. After the second period (ending on September 3, 1931) practically all the lime added during August had disappeared from the raw sludge by September 4.

CONDITION OF SLUDGE IN TANKS

Fig. 16 shows the percentage of solids in the sludge in each of the four tanks at various depths on the days when samples were taken and analyzed.

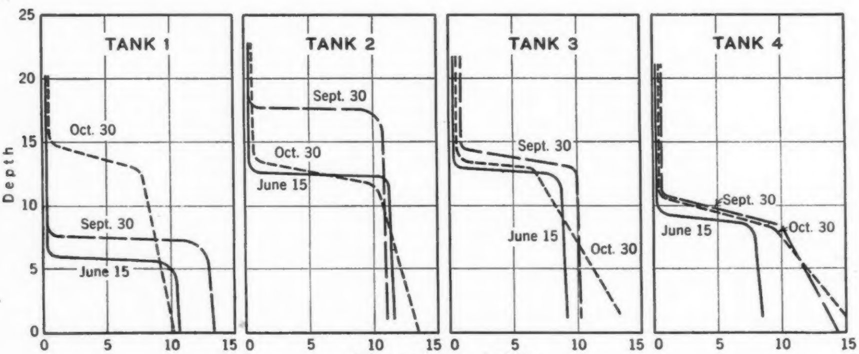

FIG. 16.—PERCENTAGE OF SOLIDS IN DIGESTION TANKS AT GRAND RAPIDS, MICH.

The average percentage of solids for all four tanks was 4.7. The reason for this abnormally low value is that the sludge line itself was low—about at mid-depth in each tank. As indicated in Fig. 16, the tanks were nearly half full of sludge so that the average moisture content was abnormally high. This line was relatively definite, and the average percentage of solids below it, for the four tanks, was 8.9. The average solids content of the sludge as withdrawn, was 12.9.

There was not much variation in the pH-value between the top and bottom of the tanks, the change in general being from 7.0 at the top to 7.6 at and near the bottom. The volatile content of the tank solids decreased appreciably from top to bottom, as shown in Table 15.

TABLE 15.—VOLATILE CONTENT OF SOLIDS IN DIGESTION TANKS AT GRAND RAPIDS, MICHIGAN, AT VARIOUS DISTANCES FROM BOTTOM OF TANK

Distance from bottom, in feet	Volatile Content of Solids, in Percentages					
	Oct. 30, 1931	Feb. 4, 1932	Mar. 1, 1932	Apr. 14, 1932	May 4, 1932	June 1, 1932
Tank No. 1						
2.5	47.4	48.4	46.5	49.6	48.4	49.4
7.5	48.3	49.3	47.6	49.3	51.2	49.8
12.5	49.3	55.4	57.3	50.9	52.2	53.8
17.5	66.7	60.3	58.2	61.5	60.4	54.7
Tank No. 2						
2.5	47.0	*	47.4	48.5	50.2	49.4
7.5	47.2	*	49.1	48.2	50.1	49.2
12.5	49.6	*	57.5	56.3	50.0	50.9
17.5	74.7	*	58.8	55.6	59.7	57.0
Tank No. 3						
2.5	47.4	47.1	48.6	48.9	50.0	46.7
7.5	47.8	47.2	55.0	48.7	49.3	46.8
12.5	49.8	55.0	57.6	54.0	51.5	52.8
17.5	62.4	58.3	55.2	53.4	56.4	52.9
Tank No. 4						
2.5	46.2	44.6	47.6	49.8	50.7	46.8
7.5	46.3	45.2	54.0	51.1	49.4	46.4
12.5	57.5	53.4	55.6	57.8	50.2	49.1
17.5	60.6	54.1	51.2	57.8	55.8	55.7

*Empty

SLUDGE SUPERNATANT

The quantity of sludge supernatant was determined by computing the difference between the quantity of fresh sludge and the quantity withdrawn, as obtained from Venturi meter readings.

In general, the supernatant carried from 2 700 to 5 000 ppm of total suspended solids. The solids discharged from the tanks in sludge supernatant for the entire period of observation amounted to about 5.8% of the total solids pumped into the tanks during the same period.

SCUM

There was a typical, black, somewhat fibrous scum, relatively thick, under the roof of each digestion tank during the observation period. The scum varied in thickness up to a maximum of about 3 ft. It gave no trouble, and no effort was made to remove it or to break it up. Moisture content and other pertinent data are given in Table 16. When two of the tanks were emptied (in January and May, 1932), it was found that the scum layer, upon reaching the bottom of the tank, could not be pumped out even after strong hosing. It was 1.5 to 2.0 ft thick and was of a heavy clay-like consistency matted with hair and sticks.

TABLE 16.—CHARACTERISTICS OF SCUM ON DIGESTION TANKS AT GRAND RAPIDS, MICHIGAN

Date	Tank No.	Moisture content	Percentage volatile	pH	Remarks
Floating Scum Layer					
September, 1931	1	93.3	96.2	7.45	Average of four analyses
September, 1931	2	93.2	96.2	7.47	Average of four analyses
September, 1931	3	94.2	95.8	7.47	Average of four analyses
September, 1931	4	93.2	96.2	7.46	Average of four analyses
October, 1931	1	95.37	95.3	7.0	....
Scum Remaining After Tank Was Emptied					
January, 1932	1	57.6	83.1	......	Composite of four samples
May, 1932	2	66.2	80.3	......	Composite of four samples

TEMPERATURE

The temperature of the digestion tanks was observed continuously by recording thermometers of the Bristol type. An average temperature was secured by placing a temperature element across each tank, rising from a point near the bottom at one corner to a point near the top at the corner diagonally opposite. Relatively uniform temperatures prevailed, as shown by Table 17.

TABLE 17.—DIGESTION TANK TEMPERATURES AT GRAND RAPIDS, MICHIGAN

Month	Tank No. 1			Tank No. 2			Tank No. 3			Tank No. 4		
	Maximum	Minimum	Average	Maximum	Minimum	Average	Maximum	Minimum	Average	Maximum	Minimum	Average
1931:												
April	100	61	80	87	75	84	88	73	83	87	70	80
May	86	82	84	87	84	85	86	82	84	86	83	84
June	85	82	84	88	85	86	86	84	85	86	85	85
July	87	85	86	88	85	87	87	84	86	88	85	86
August	86	86	86	90	85	86	88	84	86	88	85	87
September	90	85	87	88	84	86	86	84	85	88	84	86
October	90	83	86	89	83	85	87	82	85	89	83	86
November	87	85	86	87	85	86	87	85	86	87	85	86
December	91	83	86	91	83	86	91	83	86	91	83	86
1932:												
January	90	78	86	90	78	86	90	78	86	90	78	86
February	84	72	79	84	72	79	84	72	79	84	72	79
March	87	81	85	87	81	85	87	81	85	87	81	85
April	94	85	88	94	85	88	94	85	88	94	85	88
May	91	88	90	91	88	90	91	88	90	91	88	90
June	93	86	90	93	86	90	93	86	90	93	86	90

GAS PRODUCTION

The metered quantities of gas are considered to be somewhat lower than the amount actually produced, because of occasional blowing of the gas seals and some clogging of the lines. The seals have recently been increased in depth, and changes have been made in the domes to eliminate some of these difficul-

ties. Subsequent meter records indicate somewhat higher rates of production. For the purposes of this study, however, the records as obtained have been used—proper consideration, of course, being given to barometric pressure, moisture, and temperature. The quantitative results are shown in Table 14, and the composition and calorific value as indicated by various analyses throughout the period are shown in Table 18.

This gas has been used in the boilers for digestion-tank heating, without any difficulty, and in the laboratory, and for general heating about the plant.

TABLE 18.—CHARACTERISTICS OF GAS PRODUCED AT GRAND RAPIDS, MICHIGAN, SEWAGE TREATMENT PLANT

Period	British thermal unit, V_1	Percentages						
		CO_2	N_2	Illuminants	CH_4	O_2	CO	H_2
1931:								
April 1–16	661	29.4	6.45	0.18	59.2	0.11	0.05	4.7
16–30	705	31.1	2.40	0.30	65.3	0.07	0.0	1.0
May 1–16	692	31.8	2.3	0.30	63.6	0.10	0.0	2.0
16–31	735	31.7	0.6	0.30	67.4	0.10	0.0	0.0
June 1–16	714	30.1	3.1	0.20	66.0	0.60	0.0	0.0
16–30	686	30.4	4.3	0.30	65.0	0.00	0.0	0.0
July 1–16	683	30.8	4.3	0.20	63.2	0.10	0.0	1.5
16–31	683	29.9	5.9	0.10	62.2	0.10	0.0	2.0
August 1–16	682	28.5	7.4	0.10	63.7	0.10	0.0	0.2
16–31	653	28.9	6.8	0.10	59.7	0.10	0.0	4.4
September 1–16	654	29.8	1.8	0.40	57.1	0.20	0.0	10.7
16–30	664	31.0	1.9	0.60	58.8	0.10	0.2	7.4
October 1–16	733	31.3	0.6	0.20	68.6	0.10	0.0	0.0
16–30	727	31.7	0.0	0.10	68.1	0.20	0.0	0.0
November	725	31.7	0.4	0.2	67.6	0.2	0.0	0.0
December	729	31.6	0.1	0.2	68.0	0.2	0.0	0.0
1932:								
January	731	31.8	0.1	0.3	68.0	0.2	0.0	0.0
February	706	32.4	0.2	0.4	65.6	0.3	0.0	0.0
March	706	33.8	0.3	0.2	66.1	0.2	0.0	0.0
April	725	32.0	0.1	0.2	67.6	0.2	0.2	0.0
May	714	33.2	0.1	0.4	66.3	0.4	0.0	0.0
June	706	33.2	0.5	0.3	65.6	0.4	0.0	0.0

SOLIDS BALANCE

The data for the computation of solids balance were taken between June 16, 1931, and May 31, 1932, as these are the first and last dates on which analyses were made giving the total solids content of the digestion tanks. A summary of the solids balance follows:

(*A*) Total weight of solids in digestion tanks on June 16, 1931 (as computed from the data in Fig. 15), in pounds	1 010 000
Total solids introduced as fresh sludge from the primary settling tanks between June 16, 1931, and May 31, 1932, in pounds	7 895 000
Total solids to be accounted for, in pounds	8 905 000

(*B*) Total weight of solids in digestion tanks on June 1, 1932 (as computed from the data in Fig 15), in pounds	1 302 000
Total solids removed from digestion tanks as sludge, in pounds	4 570 000
Total solids withdrawn from tanks with sludge supernatant, in pounds	418 000
Equivalent weight of gas as reported by the gas meters with a correction of 1.0% added for gas lost through water-seals (and representing the weight of solids volatilized), in pounds	2 944 000
Total weight of solids accounted for, in pounds	9 234 000

In the foregoing list, the solids accounted for are greater than those to be accounted for, by 329 000 lb, or approximately 3.7 per cent. This is a close balance, considering the difficulties of sampling, of measuring sludge in meters, and of determining the percentage of volatile matter and the specific gravity of the sludge.

Computed Digestion Time

A digestion time was computed as the displacement period of solids withdrawn, by dividing the total weight of solids in all four tanks by the daily weight of all solids removed therefrom, whether as sludge, as sludge supernatant, or as gas. Table 19 gives these quantities and the displacement periods.

TABLE 19.—Computation of Digestion Time

Date of analysis	Weight of dry solids in four tanks, in pounds	Weight of dry solids removed per day*, in pounds	Displacement period, in days
1931:			
June 15	1 010 000	16 500	61
October 30	1 363 000	28 100	49
1932:			
March 1	701 000	14 900	47
May 4	863 000	16 100	54

* Average for the two months following the date of analysis of tank content.

All the sludge was well digested, the volatile matter being consistently less than 50 per cent. The time used for digestion, as indicated by these observations, is not more than 50 days and may be somewhat less.

APPENDIX II

Operating Data from Elyria, Ohio, April 1, 1931, Through September 30, 1932

For an 18-month period extending from April 1, 1931, to September 30, 1932, arrangements similar to those described in Appendix I were made for collecting information relative to the operation of separate sludge digestion

tanks at Elyria, Ohio. The opportunity was afforded through the courtesy of the Hon. James A. Hewitt, Mayor, and the Hon. J. N. Eidt, Safety-Service Director, and the work was done through the faithful co-operation of James R. Collier, Superintendent and Chemist at the sewage treatment works.

The sewage works at Elyria consists of bar gratings, grit chambers, preliminary sedimentation tanks of the 2-story type, aeration tanks, activated-sludge settling tanks, separate sludge digestion tanks, and covered sludge-drying beds.

The two 2-story tanks, each 62 ft in length, 31½ ft in width, and with a water depth of 27 ft, provide a normal detention period in the sedimentation compartment of 60 min. They are equipped for gas collection.

There are two separate sludge-digestion tanks, each 50 ft in diameter, with conical bottoms, the total water depth being 34 ft, measured from the wall coping. Both tanks are equipped with floating covers with provision for gas collection. Each tank is heated by water circulated by a centrifugal pump through galvanized iron pipe, hung around the circumference of the tank 18 ft below its coping. A continuous record of the temperature of incoming and outgoing heating water is kept by recording thermometers. Sludge temperatures are measured, as desired, by thermometers suspended in the sludge. All gas is metered and two gas boilers are provided for heating the water used in the circulating system.

Between the two digestion tanks are a sludge-mixing and control chamber and a room housing the water-seals, a master gas meter, sludge-control valves, and an automatic gas-pressure regulator and excess gas escape. There is no provision for gas storage except in the digestion tank. Sludge is transferred from the activated-sludge settling tanks by gravity, and may be discharged either to the separate digestion tanks, the sludge-drying beds, or to the sludge suction well (whence it can be returned to the primary tanks, or to the digenstion tanks). Sludge mixing may be accomplished in the digestion tanks by drawing off at the bottom and pumping back to the top of the tank.

Sludge supernatant from each tank can be drawn off at either of two elevations and discharged directly to the sludge-drying beds, or pumped to the sewage as it enters the 2-story tanks, or to the sludge compartments of the 2-story tanks.

The plant is designed to serve a population of 36 000, with an average sewage flow of 100 gal per capita daily, although the estimated connected population at the time of the observations was 21 000. The design capacity was based upon a 60-day period of digestion, with heating for maintaining a temperature of 77° F.

The sewerage system, with the exception of that serving the older and more densely populated section of the city lying between the east and west forks of the Black River, is of the separate type. The sewage is relatively fresh and of normal strength, although there is a small percentage of pickling liquor wastes. The average volatile content of the suspended solids in the crude sewage is 64.2% and the percentage of volatile matter in the partly digested 2-story tank sludge introduced into the sludge-digestion tanks is 61.7

per cent. Summaries of sewage analyses and of the characteristics of the sludge removed from the 2-story tanks are contained in Tables 20 and 21.

TABLE 20.—CHARACTERISTICS OF RAW SEWAGE AT ELYRIA, OHIO, DURING PERIOD OF OBSERVATION BY COMMITTEE

Date	Sewage flow, in million gallons daily	Temperature of air, in degrees Fahrenheit	Temperature of sewage, in degrees Fahrenheit	Total Solids		Suspended Solids		Total iron as Fe, in parts per million	5-day biochemical oxygen demand	pH
				In parts per million	Percentage volatile	In parts per million	Percentage volatile			
1931:										
April	3 289	48	51	760	37.6	211	61.1	8.9	173	7.2
May	2 300	59	57	843	38.5	241	79.0	19.0	208	7.0
June	1 960	71	64	805	44.9	231	71.5	11.0	209	7.0
July	1 675	78	69	841	49.3	241	76.3	4.4	215	6.9
August	1 793	73	69	775	47.6	205	55.1	3.5	208	6.9
September	1 857	69	71	774	51.2	218	52.3	3.8	224	6.9
October	1 803	56	68	748	51.1	198	58.1	3.7	227	7.0
November	1 798	49	65	839	56.7	216	71.8	3.2	251	6.9
December	2 355	39	58	725	46.7	186	62.9	3.7	187	7.3
1932:										
January	3 837	39	53	717	44.5	155	61.3	5.4	119	7.1
February	2 839	36	50	785	44.2	194	64.9	8.8	190	7.2
March	2 660	32	49	819	45.3	221	61.5	4.8	192	7.2
April	3 066	46	49	815	41.7	180	66.7	8.1	200	7.2
May	2 618	62	53	806	42.8	197	61.4	7.5	262	7.1
June	1 770	72	63	911	50.2	305	64.3	6.4	320	7.1
July	1 851	74	68	740	48.9	194	62.9	6.6	285	7.0
August	1 558	72	70	688	58.0	200	60.5	4.6	281	6.9
September	1 695	65	71	681	62.8	212	65.6	4.1	251	6.9
Average	2 263	....	....	782	47.8	212	64.2	6.5	222	7.03

TABLE 21.—CHARACTERISTICS OF SLUDGE ADDED TO SLUDGE-DIGESTION TANKS AT ELYRIA, OHIO

(Sludge Drawn from Preliminary Sedimentation-Activated Sludge Settling Tanks)

Date	Volume added, in gallons	Percentage			pH	Alkalinity	Specific gravity
		Moisture	Ash	Volatile			
1931:							
April	433 160	96.80	35.56	64.44	6.2	1 100	1.010
May	275 184	95,38	40,04	59,96	6.2	1 350	1.011
June	275 184	94.70	37.49	62.51	6.2	1 510	1.008
July	323 596	95.31	39.32	60.68	6.2	1 630	1.010
August	363 090	96.18	43.61	56.39	6.2	1 210	1.008
September	397 488	96.85	39.07	60.93	6.3	970	1.007
October	229 312	97.50	38.06	61.94	6.4	837	1.008
November	305 760	97.27	37.02	62.98	6.4	1 066	1.008
December	295 528	96.52	34.68	65.32	6.3	1 275	1.008
1932:							
January	229 320	95.16	36.18	63.82	6.2	1 325	1.008
February	183 456	94.65	37.22	62.78	6.2	1 600	1.015
March	257 348	94.14	33.16	66.84	6.2	1 283	1.018
April	168 168	93.87	34.31	65.69	6.2	1 616	1.013
May	183 456	93.60	36.60	63.40	6.2	1 635	1.017
June	275 184	94.13	38.10	61.90	6.3	1 587	1.014
July	259 896	96.02	40.20	59.80	6.4	1 397	1.012
August	447 174	97.03	42.70	57.30	6.5	873	1.012
September	391 118	97.04	41.50	58.50	6.3	653	1:011

OPERATING ROUTINE

The 2-story settling tanks are operated in parallel and the direction of flow is reversed regularly at the beginning of each month. Scum is drawn

from beneath the gas-collection domes not oftener than twice each month. Sludge removed from the activated sludge-settling tanks is introduced continuously into the incoming sewage at the entrance to the 2-story tanks.

TABLE 22.—Summary of Data Collected at Elyria, Ohio, Sewage Treatment Plant

Period	Sludge to Digestion Tanks				Sludge Withdrawn			
	Total Solids		Volatile Solids		Total Solids		Volatile Solids	
	Percentage	In pounds	Per centage	In pounds	Per centage	In pounds	Per centage	In pounds
1931:								
April	3.2	117 270	64.5	75 470	4.4	17 300	47.5	8 340
May	4.6	107 310	60.0	64 350	4.7	9 730	42.4	4 130
June	5.3	112 610	62.5	70 460	5.2	25 690	46.7	11 600
July	4.7	127 950	60.7	77 650	5.6	39 010	46.6	18 170
August	3.8	116 190	56.4	65 380	5.5	19 150	46.2	8 850
September	3.0	98 470	60.9	60 580	6.0	43 290	47.8	20 650
October	2.5	48 140	61.9	29 818	5.9	31 141	47.1	14 877
November	2.7	70 126	63.0	44 168	5.8	15 034	47.4	7 121
December	3.5	86 356	65.3	56 411	5.6	15 778	48.8	7 693
1932:								
January	4.8	93 281	63.8	59 536	4.3	3 962	49.4	1 957
February	5.4	82 999	62.8	52 109	4.6	11 959	47.6	5 694
March	5.9	127 848	66.8	85 458	5.6	24 773	47.9	11 875
April	6.1	87 057	65.7	57 190	5.2	20 892	48.9	10 216
May	6.4	99 475	63.4	63 071	5.3	32 978	47.7	15 732
June	5.9	136 402	61.9	84 437	5.7	24 521	47.0	11 527
July	4.0	87 336	59.8	52 226	6.1	31 824	48.4	15 391
August	3.0	111 999	57.3	64 173	5.8	34 882	48.2	16 821
September	3.0	97 614	58.5	57 106	5.9	36 317	46.4	16 862
Totals	...	1 808 433	....	1 119 593	...	438 231	....	207 306
Average	4.3	....	61.7	....	5.4	....	47.2	....

Period	Sludge Supernatant				Gas				
	Total Solids		Volatile Solids		Volatile corrected to 30-in. Baro.-60°F, in cubic feet	In pounds	Cubic feet per capita per day	Cubic feet per pound, incoming volatile matter	Percentage of incoming volatile matter reduced to gas
	Percentage	In pounds	Percentage	In pounds					
1931:									
April	0.3	5 900	48.2	2 840	363 620	23 590	0.57	4.99	32.5
May	0.4	7 070	48.4	3 410	320 190	21 000	0.49	5.15	33.7
June	0.2	2 660	47.8	1 270	323 960	20 940	0.52	5.76	37.3
July	0.9	17 580	47.9	8 360	339 200	21 910	0.53	4.39	28.3
August	0.8	14 230	48.5	6 910	334 020	21 490	0.51	5.45	35.2
September	0.2	9 070	43.8	4 210	326 060	20 940	0.52	8.59	55.2
October	0.3	7 294	45.7	3 332	340 593	21 764	0.52	11.42	73.0
November	0.9	16 605	46.0	7 639	295 681	18 983	0.47	6.69	43.0
December	0.5	11 030	46.9	5 173	308 449	19 772	0.47	5.47	35.0
1932:									
January	1.8	37 541	48.2	18 093	366 214	23 877	0.56	6.15	40.1
February	2.9	26 246	48.9	12 823	326 503	21 321	0.54	6.27	40.9
March	2.4	47 149	46.9	22 108	377 181	24 441	0.58	4.41	28.6
April	1.8	14 375	48.5	6 978	319 263	20 848	0.51	5.58	36.5
May	3.1	53 986	49.0	26 444	267 562	17 258	0.41	4.24	27.4
June	3.3	31 712	46.4	14 699	316 971	20 223	0.50	3.75	24.0
July	2.2	27 942	46.8	13 079	268 143	17 376	0.41	5.13	33.3
August	1.5	52 361	49.2	25 751	300 495	19 292	0.46	4.68	30.1
September	0.2	3 313	50.2	1 664	215 122	14 047	0.34	3.77	24.6
Totals	...	386 064	....	184 783	5 709 227	369 072	....	....	....
Average	1.3	.....	47.5	.....	....	.....	0.50	5.65	36.5

Once each week sludge supernatant is drawn from the separate digestion tanks and discharged into the sludge compartments of the 2-story tanks at points just above the top of the hoppers. Following the supernatant draw-off, if desired, ripe sludge is applied to the sludge-drying beds, and regularly each week, a charge of primary tank sludge is admitted to each digestion tank. After this, if mixing is desired, it is accomplished by drawing ripe sludge from the bottom of the tanks and pumping it into the fresh solids which have been charged at the top.

The total solids content of the digested sludge during the 18-month period of observation averaged only 5.3 per cent. This could undoubtedly be increased by a longer detention period in the digestion tanks, but due to the demand for dried sludge (which is sold at a fixed price per bed), sludge was withdrawn as soon as it was satisfactory for drying.

Data Secured

A summary of the data collected during the 18-month period of observation is given in Table 22.

Measurements of sludge added to, and removed from, the digestion tanks were made by noting the actual displacement in the tanks. The volume of sludge removed was also checked by observations of wet sludge applied to the sludge-drying beds.

Sludge Supernatant

The sludge supernatant from the digestion tanks contained from slightly less than 5 000 to 33 000 ppm of total solids, with a volatile matter content of 47.5 per cent. The total solids discharged in the sludge supernatant amounted to about 21% of the total solids admitted to the digestion tanks.

Analyses of the supernatant were made at frequent intervals, with results shown in Table 23.

TABLE 23.—Characteristics of Sludge Supernatant at Elyra, Ohio

Date	Percentage			pH	Alkalinity	Specific gravity
	Moisture	Ash	Volatile			
Sept. 30, 1931:						
East tank	99.75	59.4	40.6	7.3	3 340	1.004
West tank	99.82	63.2	36.8	7.4	3 140	1.003
Oct. 15, 1931:						
East tank	99.58	51.7	48.3	7.4	2 860	1.005
West tank	99.47	50.2	49.8	7.3	2 600	1.006
Oct. 27, 1931:						
East tank	99.71	61.2	38.8	7.4	2 720	1.004
West tank	99.77	58.8	41.2	7.5	2 680	1.003
October	99.65	54.3	45.7	7.4	2 710	1.004
November	99.12	54.0	46.0	7.2	2 620	1.004
December	99.46	53.1	46.9	7.2	2 760	1.006

Date	Percentage			pH	Alkalinity	Specific gravity
	Moisture	Ash	Volatile			
1932:						
January	98.16	51.8	48.2	7.4	2 980	1.009
February	97.09	51.1	48.9	7.3	3 600	1.014
March	97.55	53.1	46.9	7.5	3 870	1.014
April	98.22	51.5	48.5	7.5	4 290	1.007
May	96.90	51.0	49.0	7.5	4 560	1 009
June	96.72	53.6	46.4	7.4	4 730	1.007
July	97.75	53.2	46.8	7.4	4 630	1.006
August	98.46	50.8	49.2	7.4	3 280	1.007
September	99.81	49.8	50.2	7.3	2 350	1.003

Gas

The gas from the separate sludge digestion tanks, as well as that collected from the gas vents of the 2-story tanks, is metered through separate gas meters and is used for heating the water circulated in the digestion tanks. At present, all excess gas is burned and wasted. During the first part of the period considerable gas escaped through the water-seals before the covers had been finally adjusted to maintain a level position. It is estimated that the loss amounted to at least 10% of the total volume metered, and, therefore, the quantities have been corrected to include this allowance. (In addition, a quantity of gas amounting to about 10% of these volumes was collected from the primary settling tanks.) The characteristics of the gas are shown in Table 24.

TABLE 24.—CHARACTERISTICS OF GAS PRODUCED AT ELYRIA, OHIO, SEWAGE TREATMENT PLANT

Period	British thermal units per cubic foot	Percentage			Period	British thermal units per cubic foot	Percentage			Period	British thermal units per cubic foot	Percentage		
		CO_2	CH_4	O_2			CO_2	CH_4	O_2			CO_2	CH_4	O_2
1931					1931 (*Continued*)					1932 (*Continued*)				
April	738	30.2	69.2	0.6	November	750	29.4	70.0	0.6	May	742	29.9	69.7	0.4
May	730	31.1	68.2	0.7	December	764	29.2	70.3	0.5	June	756	28.8	70.7	0.5
June	748	30.0	69.5	0.5						July	738	30.2	69.3	0.5
July	764	29.9	69.6	0.5	1932					August	745	29.5	70.0	0.5
August	744	29.6	69.9	0.5						September	744	30.8	68.6	0.6
September	758	29.4	70.0	0.6	January	732	30.8	68.7	0.5	....	...	....	....	...
October	752	28.9	70.6	0.5	February	731	30.8	68.6	0.6	....	...	....	....	...
					March	746	30.2	69.3	0.5	....	...	....	....	...
					April	730	30.9	68.6	0.5	....	...	....	....	...

The number of cubic feet of gas per pound of incoming volatile matter was undoubtedly lower than could have been obtained, as sludge was drawn to the drying beds sooner than would be the case ordinarily. The gas has an average calorific value of about 750 Btu per cu ft, and burns readily under the boilers. No trouble has been experienced with clogging of the gas lines and the discharge of gas has been fairly uniform.

Scum

No attempt was made during the period of observation to remove or to break up the layer of scum in the digestion tanks. The thickness of the layer varied from 10 in. to a maximum of 24 in. An average of several analyses showed it to have the following characteristics:

Moisture content	86.4%
Ash content	34.2%
Volatile matter	65.8%
pH-value	6.9%
Ether soluble (dry solids)	27.2%

Temperature

The temperature of the sludge in the digestion tank was maintained at an average of 78° F, and varied from a minimum of 74° to a maximum of 82 degrees. The average temperatures of the incoming and outgoing circulating water were 107° and 96° F, respectively.

Solids Balance

No determination of the total solids content of the tanks was made at the end of the period of observation; hence, the solids balance can only show the difference between total incoming and discharged solids for the 18-month period. This computation is as follows:

Total weight of solids added to tanks........		1 808 433
Total weight of solids removed as sludge	438 231	
Total weight of solids withdrawn with supernatant	386 064	
Equivalent weight of gas, including allowance for loss..........	394 019	1 218 314
Solids unaccounted for.....................		590 119

The solids unaccounted for, without adjustment for a possible difference in the amounts of solids in the tanks at the beginning and at the end of the test period, amount to 32.6% of the total solids added to the tanks. Undoubtedly, a large part of this unaccounted for loss is attributable to liquefaction during digestion.

Period of Digestion

If it is assumed that the period of digestion is represented by the actual time of displacement of the sludge in the digestion tanks, the average number of days required for digestion during the period of observation at Elyria would be: $548 \div \dfrac{5\ 293\ 422}{2\ (300\ 540)} = 62.$

It will be noted that the volatile matter in the sludge removed from the digestion tanks was at all times less than 50 per cent. The sludge was odorless, of good quality, and thoroughly satisfactory for drying and final disposal.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SEDIMENTATION IN QUIESCENT AND TURBULENT BASINS

Discussion

BY J. J. SLADE, JR., ESQ.

J. J. SLADE, JR.,[14] ESQ. (by letter).[14a]—In the analysis presented in this paper the writer was prompted by the realization that the problem of the settling of solids in suspension may be separated completely from consideration of the very complex relations that exist between the particles and the liquid. Homogeneity of material does not imply uniformity of settling, and it is this fact that led the writer to introduce a distribution function into the analysis. Since this distribution function is a function only of the hydraulic properties of the material, the homogeneity or heterogeneity of the material is irrelevant to the phenomenon of sedimentation considered only from the point of view of bulk that settles out in a given time.

That objections to the writer's development were made in the discussion on the grounds that coagulation takes place is, therefore, very surprising to the writer since it is precisely coagulation that his distribution function takes into account. It appears desirable, therefore, to explain this point in greater detail.

Consider, for instance, a clay suspension so homogeneous that all the particles may be taken as identical in size, shape, and density. These particles will not fall at a uniform rate through the liquid. The statistical-mechanical picture of subsidence will be somewhat as follows: Two particles falling freely will come close together and, because of the ionization potentials surrounding them, they will adhere. After that, they fall as a single particle with a different velocity. Farther down a third particle will adhere to these two and the three will fall together, with a still different velocity. These nuclei may grow to fairly large masses depending on the physico-chemical character and the concentration of the solids. These nuclei are not stable, however. As they go down a collision may disrupt them instead of adding more particles to the mass. This interchange is always going on, but

NOTE.—The paper by J. J. Slade, Jr., Esq., was published in December, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1936, by Thomas R. Camp, M. Am. Soc. C. E.; and May, 1936, by Messrs. Harry H. Hatch, Harry H. Moseley, and George J. Schroepfer.

[14] Associate Prof. of Eng. Mechanics, Rutgers Univ., New Brunswick, N. J.

[14a] Received by the Secretary November 10, 1936.

in a large enough sample a statistical equilibrium is established, so that a certain percentage of the particles at any time will be falling singly, a certain percentage will be in pairs, etc. Each one of these groups is characterized by its individual hydraulic value, and it is this distribution of hydraulic values that is given by the writer's distribution function. Obviously, no matter how homogeneous or how complex the material may be, it will always be characterized by some distribution function of its hydraulic values.

The method given by the writer to determine the distribution function takes into account only the hydraulic behavior of the sub-groups into which the material arranges itself and, consequently, it leads to a statement only of the bulk that is removed in a given time; that is, it takes coagulation fully into account.

Professor Camp offers two severe criticisms: First, that it is unfortunate that the theory should be based on Mr. Hazen's work, because his formulas are known to fail; and, second, that in introducing the distribution function the basic assumptions on which the theory is established are completely violated.

These two objections may be answered as one. In the first place, the writer's development is scarcely a theory; it is more in the nature of a truism. In its more general form it may be stated thus: If the quantity of material which has hydraulic values between v and $v + dv$ is $\phi(t)\,dt$, and, if at the end of time, a, the quantity remaining in suspension is a fraction, $f(a, t)$ of the amount initially there, then for this small range in v the quantity remaining in suspension is $\phi(t)\,f(a, t)\,dt$, and the total quantity is $\int \phi(t)\,f(a, t)\,dt$. The integration extends, of course, over the complete range of values of t. Thus far, this is independent of Mr. Hazen's logic, and it is merely a truism stated in compact symbolic form. This statement is useful or useless, depending on whether or not $\phi(t)$ and $f(a, t)$ can be evaluated.

Consider, then, a particle (or globule of particles cohering as a single body) that is falling freely through the liquid. In this case the function, $f(a, t)$, takes the form of Equation (4). This is simply Hazen's statement of Stokes' law, which is known to be correct well within the errors of experiment. For material that has attained statistical equilibrium of coagulation, Equation (6) is true. It remains only to determine $\phi(t)$ and the writer has shown in detail how this may be done. Therefore this equation is a determinate, complete, and correct statement of sedimentation in a still liquid which takes fully into account, coagulation and every factor pertinent to the problem.

For the condition described as critical turbulence the form of $f(a, t)$ is also known. It is simply $f(a, t) = 1$. This is the condition in which turbulence is so great that the material does not settle out. From no turbulence to critical turbulence there is a steady transition of $f(a, t)$ from $\left(1 - \frac{a}{t}\right)$ to 1. This transition can be determined only through experiment. The writer has suggested the form, Equation (19), but this is really quite general because $k(t)$ must be determined experimentally. There is one particular condition of turbulence, which the writer has called complete, in which he has evalu-

ated the function, $f(a, t)$. This evaluation is given in Equation (27) and in that case explicit use was made of Hazen's theory. Whether or not there does exist a condition of turbulence in which $f(a, t) = e^{-\frac{a}{t}}$, the writer is not qualified to state, but if under some condition of turbulence the suspended material is observed to have a fairly uniform vertical distribution, then this form is quite nearly correct. That this form is not a bad approximation to certain conditions of turbulence is shown by the results obtained by applying it to the experiments of Dilling and Pearse[4]. In view of the explanation that has been given of the distribution function it is seen that the choice of these experiments for illustration is not so unfortunate as Professor Camp considers it. However, except for the form of this particular evaluation, the writer's development is in no way dependent on Hazen's definition of turbulence.

Professor Camp's further criticism of the writer's use of this particular function is completely invalidated by the manner in which he has obtained his Equation (38). In so far as the variable of integration is concerned, a definite integral is a function only of its limits. Professor Camp's squaring of a part of the integrand[14b] follows from nothing that the writer has included in his paper.

It must be explicitly stated that the writer's development as presented herein holds only for the case in which statistical equilibrium of coagulation occurs in a time that is short compared to the period of retention. This may by no means be the case in practise. As Mr. Schroepfer points out, sewage solids change in character hourly, daily, and seasonally. Formally, this is quite easily taken into account by allowing the distribution function to be dependent both on a and t; practically, however, no advantage would result from this extension. The theory holds only where statistical equilibrium is obtained throughout the period of retention.

It is far from the writer's intention to offer an exercise in curve fitting in presenting this paper. He quite agrees with Mr. Hatch that if a short formula will do as well as a long one, the short formula is much the more desirable of the two.

In evaluating the constants of Equation (36) the writer desired merely to show that Equation (31) does represent the process accurately. Curve fitting after the process is complete is merely a waste of time. However, if the curve for the performance of a tank can be constructed accurately beforehand, then even a long computation is justified. For instance, if through experiments (such as will be indicated under Items (1) to (4) subsequently) it is possible to assign to a given tank a certain degree of turbulence for some given rate of discharge, and if, from a test-tube sample of the sludge, the distribution function is determined by the process represented in Figs. 4 and 5, the writer's contention is that Fig. 9 may be drawn before the tank is in

[4] Rept. on Industrial Wastes from the Stockyards and Packingtown in Chicago, by A. W. Dilling, Assoc. Am. Soc. C. E., and Langdon Pearse, M. Am. Soc. C. E., Vol. II, January, 1921 (The San. Dist. of Chicago), pp. 140–141.

[14b] Corrections for *Transactions:* Delete entire paragraph, including Equations (37) and (38), at bottom of p. 283 and top of p. 284, *Proceedings,* Am. Soc. C. E., February, 1936, as well as the first full sentence at the top of p. 285.

operation; in other words, that it is possible to predict the performance of a tank for any rate of discharge and for any material that may be put through it.

If this is true, it is certainly possible to design a tank to operate in a given manner for a particular type of sludge, and to predict its operation for other types that may be put through it.

Mr. Moseley is quite right in stating that there already exists a vast amount of information regarding the operation of settling tanks. However, it is not the amount of information that is lacking, but the kind of experiments required to evaluate the constants presented in this paper.

Since the principal outcome of this investigation is the suggestion of a unified experimental procedure and standardization of terms, and since it is not likely that the writer will have the opportunity in the near future to undertake the experiments himself, he should like, in closing, to suggest an outline of what he thinks could be done to correlate this theory with practise:

(1) A standard set of materials should be selected for experimental purposes. Each material should be selected for the uniformity of its hydraulic behavior, and as many materials should be in this group as will give a fair range of hydraulic values.

(2) A standard cylinder should be adopted for these experiments. These cylinders should be large enough so that the walls will not appreciably modify the hydraulic behavior of the suspended solids and so that a sample will be able to attain the statistical equilibrium it reaches in a full-sized tank. These cylinders should be equipped with mechanical rotors. By means of these rotors it is possible to give turbulence precise values. For instance, the value, 1, may be assigned to the turbulence produced by a standard rotor in a standard cylinder of water when it rotates at the rate of 1 rpm, the value, 2, when it rotates at the rate of 2 rpm, etc.

(3) The hydraulic behavior of the standard materials of Item (1) may then be studied when subjected to various standard degrees of turbulence and, in this manner, the variations of the function, $f(a, t)$, may be determined for each.

(4) By the use of some suitable material of Item (1) a tank of standard design may be calibrated. Thus, for a given rate of discharge, the material will settle out in one or the other of the various ways determined in Item (3), so that, for that particular rate of discharge, the tank will have assigned to it one of the degrees of turbulence standardized in Item (2). Interpolations, of course, will be necessary.

When data such as Items (1) to (4) have been collected, it will be possible to make use of the theory presented in this paper. This is certainly a formidable set of experiments which requires a fine technique, but, in the writer's opinion, this suggestion represents only a small fraction of the work that has been done in the past and is now in progress in the field; and it has the advantage of a reasonable theory to unify the results. The much more difficult problem of taking into account slow time changes in the material may then be considered.

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Founded November 5, 1852

DISCUSSIONS

COMPARISON OF SLUICE-GATE DISCHARGE IN MODEL AND PROTOTYPE

Discussion

BY FRED WILLIAM BLAISDELL, JUN. AM. SOC. C. E.

FRED WILLIAM BLAISDELL,[9] JUN. AM. SOC. C. E. (by letter).[9a]—It has been suggested by Mr. Boucher that the statement, "the coefficient, c_d, is not equal for all openings or heads", be changed to "the coefficient, c_d, varies with different gate-openings and different heads." In Table 1 the coefficient, c_d, varies with different gate-openings but is constant for each gate-opening. In Table 2, where $n = 0.5$, the coefficient, c_d, varies with both gate-opening and head.

At present, the writer is unable to find a good reason for the erratic value of c_d in Table 1 when the gate-opening is 4.0 ft. It will be noticed in Fig. 2, however, that the experimental points obtained for the model gate-opening of 4.0 ft do not follow the curve as well as the points for other gate-openings.

The discussion by Mr. Hurst was of the type which the writer had hoped his paper would inspire. The accuracy of the results of tests made under the direction of Mr. Hurst indicates what can be expected from carefully constructed and carefully tested models. The fact that water is distributed in Egypt on the basis of model experiments indicates the confidence placed in these tests.

Equation (2), given by Mr. Hurst, applies only to free flow into air or to free-flow conditions. In this equation, F is usually assumed to be one-half the gate-opening, or $H - F$ is the head to the center of the gate-opening. In one of the publications mentioned by Mr. Hurst values of F are given for the sluices of the Assuan Dam[10]. These values of F range from 73 to 95% of the opening, which indicates that for the Assuan Dam the head should not be measured from the center of the gate-opening, but from some point above the center.

NOTE.—The paper by Fred William Blaisdell, Jun. Am. Soc. C. E., was published in January, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings* as follows: May, 1936, by Messrs. Raymond Boucher, and H. E. Hurst; and November, 1936, by G. H. Hickox, Assoc. M. Am. Soc. C. E.

[9] Junior Soil Conservationist, Hydr. Laboratory, National Bureau of Standards, Washington, D. C.

[9a] Received by the Secretary November 30, 1936.

[10] *Physical Dept. Paper No. 24*, Ministry of Public Works, Egypt, p. 5, Table 1.

Mr. Hurst states[11] that:

"This 'Free Condition' with the down-stream level above the level of the sill of the sluice is always marked by the occurrence of a standing wave in the culvert downstream of the sluice-gate"; and, "Observation of the flow shows a well-marked standing wave when the flow is similar to free into air. As the upstream level is lowered to the point where free-flow conditions cease, the wave gradually works back along the culvert to the sluice-gate. While the standing wave exists the flow is steady. When the standing wave begins to disappear the flow is pulsating and the wave oscillates between the sluice-gate and a point a few centimeters down stream of it, appearing and disappearing."

In the model sluices of the Tremont gates, a hydraulic jump was noticed under some conditions. If the standing wave mentioned in the quotation can be called a hydraulic jump, then free-flow conditions, as defined by Mr. Hurst, undoubtedly existed at times in the Tremont gates model.

Equation (3) corresponds to the writer's Equation (1). Submerged flow is the type of flow that existed most of the time in the Tremont model, although it is possible that intermediate flow is present at times. An attempt was made to construct a plot for the Tremont gates experimental results similar to Fig. 5 but there were too few points to define the curves properly.

The derivation of a value for the roughness factor, n, for use in the Manning equation, by Mr. Hickox, is of interest. In this model of the Tremont gates the roughness factor does not assume major importance because of the relatively rough prototype, the large scale ratio, and the ease of building a model having a roughness factor of about 0.108 as computed by Mr. Hickox. In many models, however, the effect of roughness can not be disposed of so easily.

[11] *Physical Dept. Paper No. 25*, Ministry of Public Works, Egypt, pp. 4 and 14.

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DISCUSSIONS

BEHAVIOR OF STATIONARY WIRE ROPES IN TENSION AND BENDING

Discussion

BY DOUGLAS M. STEWART, JUN. AM. SOC. C. E.

DOUGLAS M. STEWART,[14] JUN. AM. SOC. C. E. (by letter).[14a]—Various discussers of this paper have stated certain of the limitations in the testing and analysis of stationary wire ropes in bending over sheaves. For these contributions the writer is especially grateful.

Mr. Meals has assisted in clearing up the chronology of the various formulas for bending stress listed. His previous conclusion that the maximum bending stress does not necessarily lie in the outer wires of the strand was supported in the paper. Equations (14) and (16) give values much more consistent with the test results obtained than those considered in the paper, and indicate that for sheaves of a diameter, such as that normally used in practice, Equation (14) can be used safely and will give fairly accurate predictions of strength.

Loadings of the tensile specimens too close to 50% of the ultimate strength, in determining the modulus of elasticity, were admittedly abnormally high when considered in the light of conventional factors of safety; and yet, as pointed out by Professor Boomsliter, overloads of this amount are frequently experienced in ordinary hoisting ropes, and when pre-stressing suspender cables for bridges to raise the modulus, such loads are commonly used. As shown by Set No. 11, in the ordinary range of working loads, a value of $E_r = 10\,000\,000$ lb per sq in. may be expected after a few loadings, and it is a matter for further investigation to determine whether such a rope in normal service will attain modulus values comparable with those tested.

The general consistency of test results for each specimen, and the agreement between measurements made on opposite sides of the specimen, indicate that a 10-in. gage length was adequate, although a greater length is undoubt-

NOTE.—The paper by Douglas M. Stewart, Jun. Am. Soc. C. E., was published in February, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1936, by Messers. C. D. Meals, and G. P. Boomsliter; and August, 1936, by Ingvald E. Madsen, Jun. Am. Soc. C. E.

[14] Sales Engr., Ingersoll-Rand Co., New York, N. Y.

[14a] Received by the Secretary November 19, 1936.

edly to be preferred when physical limitations permit. On the specimens on which six 10-in gage lengths were used, a difference of only 3% in modulus between those nearest the sockets and those at the center shows this consistency, and the writer believed that accuracy of this order was within the experimental limits of such testing. Special swivel clamps would no doubt have been of assistance in eliminating twisting of the measuring apparatus, but corrections for twist using the transit and scale were of very small magnitude except for loads near the breaking point. The use of ropes with hemp centers for these tests was dictated by practical considerations, and the question of whether this type, or that with an independent wire-rope center, is the more typical seems debatable. An extended research into the properties of such ropes would clarify many questionable points in the analysis.

Professor Boomsliter has stated clearly one of the chief causes of variation in any mathematical analysis of stresses in a wire rope. There can be no question that forces of the nature described actually exist in a rope, but since so many assumptions must be made in considering them, such frictional stresses are usually neglected. That these forces may have an appreciable effect on the strength of a rope, in certain cases, is shown in the numerical example considered. Variation in the quantity of lubricant used in the ropes was not attempted; all tests were made on new commercial ropes and represent current practice in this respect. Mr. Madsen makes mention of such stresses in the last paragraph of his discussion, but the very fact that strengths in both bending and tension may be predicted with fair accuracy by the formulas given, whether empirical or otherwise, indicates that too careful consideration is not justified.

Mr. Madsen has had the opportunity of carrying on the writer's work on various other types of wire rope specimens, and his conclusions merit careful consideration. It is to be regretted that complete results of his tests were not available for his discussion.

Specifically referring to Mr. Madsen's discussion, it is stated that the usual stress theories, as represnted by Equations (1) to (7), inclusive, may not be far from wrong below the elastic limit. These formulas actually indicate that the elastic limit (if a wire rope may be said to possess such a property) is reached long before the point shown by actual tests. Furthermore, breaking of stationary ropes in service under low loads almost never occurs, although no one will deny that fatigue will cause breakage in moving ropes at values of load far less than that given by any of these formulas. Since the bending stress is constant regardless of the load on the rope, it is obvious that if exceptionally large stresses are indicated by such formulas at the breaking load, the error will be even greater proportionally at lower values of the direct stress.

The nicking effect of one wire on another has been emphasized clearly, and the reduction in area (and, hence, in ultimate load) is appreciable. Although care must be taken in interpreting tests on specimens of this nature after being stressed almost to failure, it is apparent that the loss in the

strength of the rope must be affected by this nicking of certain of the outer wires. In any cross-section, however, the number of such wires is not large, being limited to those in the outer layer of a strand where it touches another strand. The use of a stress-strain curve for a wire that has been loaded practically to failure, over the entire range of a tensile test, does not seem to be justified. From Fig. 20, the percentage loss in strength of the wire in a tensile test is very close to zero, which means that, at failure, the wire should show an ultimate strength of about 219 000 lb per sq in. This strength is very close to that obtained in Curve E of Fig. 22, whereas Curve C shows a stress of only about 185 000 lb per sq in. at failure, or about 15% lower strength. Fig. 21 cannot be based on wires taken from a tension specimen and still be consistent with Fig. 20, and values taken from such a curve based on nicked wires from a bending specimen (which themselves are not typical of conditions prevailing over the cross-section) are of little help in correcting Fig. 22.

The correct stress-strain curve of the wire to use would be one based on a gradual nicking, such as takes place in the rope, but lacking this, the original stress-strain curve seems to give the best results. The difference in length between the chord length measured in a $\frac{1}{2}$-in. gage length on the wire and the actual center-line length is so small that it may safely be neglected. That the stress in the inner wires is greater than the average over the cross-section, is appreciated by wire rope manufacturers, who make the inner wires slightly larger to withstand this effect.

The need for more extensive research in this field, to explain the apparent inconsistencies of various experiments has been mentioned previously, and is emphasized by the discussions of this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MODERN CONCEPTIONS OF THE MECHANICS OF FLUID TURBULENCE

Discussion

By Hunter Rouse, Assoc. M. Am. Soc. C. E.

Hunter Rouse,[46] Assoc. M. Am. Soc. C. E. (by letter).[46a]—Offered primarily as an interpretative review for hydraulic engineers of present-day knowledge of fluid turbulence, the paper was at the same time an attempt to emphasize basic methods of attack that have proved invaluable in the applied science of fluid mechanics. The interest that readers have shown for this subject through printed discussion, correspondence, and verbal comment is indeed encouraging. In this closing discussion, therefore, the writer seeks to clarify certain aspects of the problem that were questioned in the discussions, introducing as further illustration of the basic thesis several related phases having direct bearing upon the contents of the paper.

Professor Posey's statistical approach to the study of turbulent fluctuations is well adapted to research of a fundamental nature, but it would seem to the writer to be of questionable practical value. Evidently the behavior of the three components of fluctuation is a sufficient measure of the degree of turbulence and hence of all phenomena depending upon the mixing process. Although leading surely to a more thorough understanding of the turbulence mechanism, such analysis, divorced as it is from the basic pattern of mean flow, would scarcely tend to relate the degree of turbulence to flow characteristics readily measurable. The fact must be emphasized that the success of von Kármán's method lies in proving the existence of correlation at every point and in recognizing the similarity of the turbulent pattern from point to point, but above all in determining the degree of mixing as a universal function of the mean velocity gradient.

Note.—The paper by Hunter Rouse, Assoc. M. Am. Soc. C. E., was published in January, 1936, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: April, 1936, by Chesley J. Posey, Jun. Am. Soc. C. E.; May, 1936 by Messrs. S. Franz Yasines, Benjamin Miller, and Ralph W. Powell; August, 1936, by Joe W. Johnson, Jun. Am. Soc. C. E.; and November, 1936, by Messrs. Warren E. Wilson, and Theodore von Kármán.

[46] Asst. Prof. of Fluid Mechanics, California Inst. of Technology; Soil Conservation Service, Pasadena, Calif.

[46a] Received by the Secretary November 17, 1936.

Although it is not altogether likely to provide a simple mathematical statement of commercial pipe resistance, the analysis of roughness suggested by Professor Posey would surely help to clarify the basic action of surface irregularities. Systematic experimentation on controlled artificial roughness of an elementary nature, to which this type of analysis is admirably suited, may well replace such hit-and-miss investigations of internally threaded pipe as have been conducted in past decades. The roughness parameters used by Professor Posey—ϵ, ϵ', and $i_{av}\,\epsilon''$—are excellent criteria on which to base such research.

For the present, however, it would seem not only necessary but practicable to rely upon experimental determination of a single characteristic length parameter to designate the effective absolute roughness of a surface, whether referred to the sand grains used by Nikuradse or to some standard otherwise determined. Nikuradse was the first to make systematic studies of artificial roughness covering a sufficient range to be generally significant and, therefore, his studies would seem to merit use as a reference for comparative rating of other roughness types. Nevertheless, such adoption would be regrettable if only for the fact that the sand grains in his analysis, and his method of applying them to the surface, do not lend themselves to ease in reproduction. Sand of a given mean grain diameter undeniably varies greatly in shape and hence in roughness action; and even a change in the grade of lacquer used as binder might alter the absolute roughness appreciably. It would be better for one to select arbitrarily, as a standard reference, a numerical relative roughness yielding some convenient resistance coefficient; for instance, fix $\frac{r_0}{\epsilon}$ at 500 for a value of $f = 0.02$. This point lies close enough to Nikuradse's values to provide ease in comparison, but its arbitrary selection renders it forever independent of a standard sand, lacquer, and technique of application. This value is then readily convertible to a single length parameter, ϵ, designating the effective absolute roughness of commercial surfaces; the sole drawback lies in the unavoidable fact that this length parameter is not dimensionless, and hence tables of effective absolute roughness of different materials must necessarily vary among the several dimensional systems.

In this connction, a paper by H. Schlichting[47] that appeared almost simultaneously with the writer's paper, describes further research on artificial roughness which merits notice at this point. In an endeavor to develop a means of testing actual samples of commercial surfaces for effective absolute roughness, Schlichting modified Nikuradse's apparatus to include a wide, shallow, rectangular test section of considerable length, one side of which was composed of the surface to be investigated, the remaining three sides being very smooth. Piezometer openings at frequent intervals were located along only the wide smooth side, and the test specimen was of sufficient length to permit full establishment of flow before the sections of pressure measurement were reached; Pitot tube traverses were possible at several locations.

[47] "Experimentelle Untersuchungen zum Rauhigkeitsproblem", *Ingenieur-Archiv*, Band VII, Heft 1, 1936.

Beginning, for purposes of comparison, with sand surfaces similar to those of Nikuradse, Schlichting thereby proved that combined measurements of longitudinal pressure gradient and transverse velocity gradient were sufficient to determine the intensity of shear along the rough surface. This was accomplished through use of the expression for the universal velocity gradient near a smooth wall (Equation (55)), and the relationship:

$$(\tau_0)_s + (\tau_0)_r = -b\,\frac{dp}{dL} \quad \ldots\ldots\ldots\ldots\ldots\ldots (93)$$

in which b is the distance between the smooth wall and the rough test specimen, and $(\tau_0)_s$ and $(\tau_0)_r$ designate the intensity of shear at the smooth and rough walls, respectively.

Referring to the paper it will be recalled that the universal velocity distribution curves for both smooth and rough walls were based upon the fact that the entire pattern of turbulent flow in the vicinity of the wall is a function of distance from the wall and hence is completely independent of any effect of the wall on the other side of the center line. Thus, even if the two opposite walls are of a different nature, it is still to be expected that only in the centermost regions of flow will the influence of both walls be felt. Fig. 29(*b*),

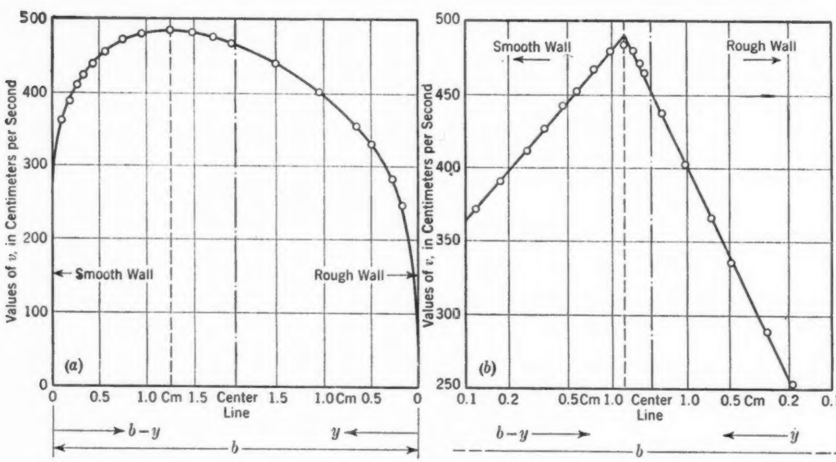

FIG. 29.—VELOCITY DISTRIBUTION BETWEEN SMOOTH AND ROUGH WALLS.

taken from Schlichting's paper, is excellent vindication of this reasoning, the velocity distribution on either side of the maximum following a logarithmic function corresponding to the universal plots of Figs. 17 and 18. Furthermore, the constants of the several distribution curves for sanded surfaces determined in this way checked Nikuradse's measured values within a few per cent.

Use of such an apparatus for practical purposes (in Schlichting's case primarily for investigation of the roughness of ship hulls) is straighforward and simple: The test specimen in the form of a long flat plate is mounted in

the rectangular conduit; measurements are made of pressure gradient and velocity distribution at several different flow rates (only one, however, being necessary theoretically). From the semi-logarithmic distribution curves Equation (55) enables the direct determination of $(\tau_0)_s$, and $(\tau_0)_r$ is then found at once from Equation (93). Equation (56), finally, provides a means of determining the effective absolute roughness referred to the sand used by Nikuradse; it is evident that to standardize some arbitrary value of ϵ requires simply a change in the constant, 5.85, of Equation (56). It should also be apparent that this procedure is correct only for that type of roughness which results in a resistance coefficient that is independent of the Reynolds number.

Other salient features of Schlichting's paper are equally deserving of notice in this review. Proceeding beyond the roughness studied by Nikuradse, Schlichting prepared test plates in which the projections consisted, in turn, of staggered rows of small balls, hemispheres, cones, and structural angles soldered carefully to the plate. With unchanging "relative" roughness (for instance, with balls of constant diameter), the spacing was varied systematically to yield curves of resistance as a function of the longitudinally projected area of the protrusions. As was to be expected, with increasing concentration of the roughness elements the resistance increased to a maximum value, finally dropping off again as the elements became crowded together.

Two essential conclusions were drawn by Schlichting from his results: First, the form and extent of linear projection of the elements from the wall are insufficient data for the determination of absolute roughness without knowledge of the concentration of the elements per unit wall area; these three variables might also be used in Equation (69), although they differ somewhat from those suggested by Professor Posey. Second, the effect of the roughness elements is equivalent to the formation of a wake during flow past an immersed body because the limiting resistance of each individual element investigated was of practically the same magnitude as that of a similar body moving through a fluid with a velocity equal to that at the outer edge of the roughness element (by "limiting resistance" is meant that of a single element as the concentration approached a minimum). As the elements become more crowded, it is clear that their effectiveness as individuals should decrease, the over-all effect still increasing until mutual interference finally results in a gradual reduction of the total resistance. Obviously, the limit of crowding is reached when the elements are packed so closely as to produce, in effect, a smooth wall.

The relative ease of experimentation, once such an apparatus for commercial testing is made available, and the evident dependability of results for widely different types of roughness, lead the writer to believe that such a method may lend itself to other purposes than simply the testing of commercial plate material. For instance, since the effect of roughness on turbulent resistance is identical for both closed and open conduits, such technique is easily adapted to the study of canal linings, whether of timber, concrete, or brick. Simply by building a test section of the material in question, and sealing the apparatus on top of it in inverted position, a conduit is formed

exactly as in Schlichting's original tests. This would necessitate rebuilding the Chezy equation into a form such as that of Equation (16) or Equation (17) since the Kutter and Manning n used to determine the Chezy C is not a length parameter equivalent to the effective absolute roughness, ϵ. The idea has also occurred to the writer that similar methods might possibly be extended to the case of movable beds, eliminating the free surface by means of a smooth upper boundary. This might well permit investigation of the true shear and mean velocity distribution in the vicinity of the bottom, for through Schlichting's method both may be determined by indirect measurement.

Mr. Miller's comments on empirical *versus* analytical investigation must be examined more closely, for there are actually four, rather than two, methods of arriving at expressions for fluid motion. One must distinguish, first, between experimental measurement (which the writer by no means scorns) and empirical formulation of working rules for design—two processes which Mr. Miller seems to bulk as one. Experimental measurement is the mechanical determination of one or more flow characteristics for a given state of motion. Pure empiricism, on the other hand, includes the effort to develop a practical generalization of the results of experimental measurement for a number of different conditions; it may provide simple working formulas for design within the range of available data—but, although the data may be accurate, there is no guaranty whatever for the physical truth of the empirical statement, for only by chance are natural laws discovered in this way. Were this still the only recourse of the hydraulician, the subject of pipe resistance would probably be in the same jumbled state as it was before Blasius published his analysis of Schoder's experimental data.

On a considerably higher plane is the second method, that of Blasius and actually that which Mr. Miller is upholding—one which is partly analytical in character since it is based upon physically sound dimensional analysis; although experimental measurements are still necessary to determine the type of function and the numerical constants relating the several dimensionless parameters, the investigator is well beyond the chance of pure empiricism when he interprets his results. Still further advanced, it would seem to the writer, is the method of fluid mechanics presented in the paper, because it combines with dimensional analysis a reasonable and closely approximate physical analysis, thus determining the probable form of the function and leaving only the numerical constants to experimental investigation. The fourth method—complete rational analysis—leaves nothing to be found experimentally; it is the ultimate goal of every field of science, but as yet only in isolated cases has it been fully a success.

Although Mr. Miller looks forward to the attainment of this goal, he seems to frown upon present efforts in that direction. He must remember that the "law of turbulent flow" which he uses to emphasize the advantages of "empirical" relationships, and which "might" have been discovered earlier from experimental data, actually was far more the result of careful physical analysis than of empirical luck. Mr. Miller evidently attributes the law

to Nikuradse's experimental work published in 1932[15], whereas the latter was an outgrowth of von Kármán's original analysis published in 1930[19, 48]. As Mr. Miller finally remarks, an "infinite velocity at the center" would indeed be interesting, but the writer can recall no mention of such an assumption. The infinite velocity gradient at the wall, on the other hand, was not assumed to portray wall conditions, but to provide the necessary mathematical limit for a curve of which the characteristics are used only in the central regions of the flow. If ever found, a single analytic function to include both the laminar film and the zone of highly turbulent mixing would very likely prove too complex to serve any practical purpose.

As Mr. Yasines looks upon the application of dimensional analysis with such apparent skepticism, further discussion to supplement the original statements involving Equations (8) to (17) and (46) to (48) may possibly prevent future misunderstanding of this basic factor in the investigation of fluid motion. Mr. Yasines asks what means the investigator has of knowing which characteristics to include in the original expression. Briefly stated, the only variables belonging in any correct statement of flow conditions at constant temperature are as follows: The several linear and angular dimensions describing the geometry of the flow; two flow characteristics, a pressure gradient (or a shearing stress) and a velocity; and the fluid properties of density, specific weight, absolute viscosity, surface tension, and elastic modulus. Of these, either pressure gradient or velocity is usually considered the dependent variable. Use of the Π-theorem will group these variables in a number of dimensionless terms, each including four variables, three of which appear in every Π-term. It is most reasonable to select velocity, density, and a significant length as the three repeating variables, since this grouping yields as Π-terms the Froude, Reynolds, Weber, and Cauchy numbers.

If the flow is completely enclosed, specific weight and surface tension (and, hence, the Froude and Weber numbers) will disappear from the relationship. For cases of constant density, the modulus of elasticity (and, hence, the Cauchy number) may also be dropped. It is evident that use of wall shear instead of pressure gradient is perfectly sound, since the two are definitely related through Equation (15); although pressure is more easily measured, intensity of shear is often more significant. In open-channel flow the pressure gradient must be replaced by either boundary shear or surface slope. Little experience is necessary to reach these conclusions. However, if experience is completely lacking, the infallible method is to include every one of these variables in the basic statement, afterward eliminating from the equation those Π-terms which, through experimental trial, prove to have no effect upon the dependent variable.

It must be remarked at this point that the Froude and the Reynolds numbers cannot "fail to perform their functions", despite Mr. Yasines' remark to the contrary. Only when they are misused will the investigator be disap-

[15] "Gesetzmässigkeiten der turbulenten Strömung in glatten Rohren", von J. Nikuradse, V. D. I. *Forschungsheft*, 356, 1932.

[19] "Mechanische Aehnlichkeit und Turbulenz", von Theodor von Kármán, M. Am. Soc. C. E., *Göttinger Nachrichten*, Math. Phs. Kl., 1930; see also, by the same author, "Turbulence and Skin Friction", *Journal of the Aerautical Sciences*, January, 1934.

[48] "Mechanische Aehnlichkeit und Turbulenz", Verhandlungen des III. Internationalen Kongresses für technische Mechanik, Stockholm, 1930.

pointed, and then only he is to blame. It is futile, for instance, to expect the Froude number to bear the brunt of boundary roughness and viscous action without any assistance whatever from roughness and viscosity parameters, a condition too often encountered in model studies. "'Scale effect" and "limitations of similitude" are expressions often used to imply that correction factors are necessary to compensate for inherent weakness in theory. Such is by no means true; scale effect is fully covered by the several dimensionless parameters, and the limitations are not analytical, but practical.

Although the foregoing choice of the three repeating variables is the most logical, in that it separates the fluid force-characteristics (specific weight, viscosity, surface tension, and elastic modulus) and thus leads to the most fundamental dimensionless parameters, it is sometimes expedient to use other groupings for the purpose of emphasizing some significant functional relationship not otherwise apparent. For example, the study of turbulence in smooth pipes involves only the measurable variables, V, D, ρ, $\frac{dp}{dL}$, and μ. Customary grouping leads to the expression,

$$\phi\left[\frac{\rho V^2}{D\frac{dp}{dL}}, \frac{V D}{\frac{\mu}{\rho}}\right] = 0 \qquad (94)$$

which yields the familiar equation,

$$\frac{dp}{dL} = \phi' (\mathbf{R}) \frac{\rho V^2}{D} \qquad (95)$$

Mr. Miller has shown, implicitly, that by choosing D, ρ, and $\frac{dP}{dL}$ as the three repeating characteristics there will result,

$$\phi\left[\frac{\rho^{\frac{1}{2}} V}{D^{\frac{1}{2}}\left(\frac{dp}{dL}\right)^{\frac{1}{2}}}, \frac{D^{\frac{3}{2}} \rho^{\frac{1}{2}}\left(\frac{dp}{dL}\right)^{\frac{1}{2}}}{\mu}\right] = 0 \qquad (96)$$

or, using more familiar parameters (see Equation (16)),

$$\phi\left(\frac{1}{\sqrt{f}}, \mathbf{R}\sqrt{f}\right) = 0 \qquad (97)$$

A plot of $\frac{1}{\sqrt{f}}$ against log $\mathbf{R}\sqrt{f}$ yields a straight line, thereby disclosing the apparent fact that this line may be extrapolated without probable error. The writer cannot refrain from pointing out, however, that a logarithmic plot of Equation (95), first published by Blasius, also led to a straight line, and only after further research was it proved that extrapolation would introduce serious error. Assurance that Equation (75) may safely be extrapolated beyond the range of experimental data is not the outcome of Mr. Miller's

dimensional analysis but of Professor von Kármán's analytical determination of the fact that the functional relationship actually is linear as long as viscous shear has a negligible effect upon the turbulent pattern. Since Equation (75) submitted by Mr. Miller states the same relationship as Equation (68), it does not eliminate the drawback mentioned in the text preceding Fig. 19—one can conveniently solve for **R** (or Q), with $f \left(\text{or } \frac{dp}{dL}\right)$ given, but not *vice versa.*

Mr. Yasines' use of the Navier-Stokes equations to illustrate his remarks on the dubious significance of the Reynolds number surely warrants further investigation. These equations are as follows:

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} = -\frac{1}{\rho} \frac{\partial}{\partial x} (p + \gamma h) + \frac{\mu}{\rho} \nabla^2 v_x \ldots (98)$$

$$\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} = -\frac{1}{\rho} \frac{\partial}{\partial y} (p + \gamma h) + \frac{\mu}{\rho} \nabla^2 v_y \ldots (99)$$

and,

$$\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{\rho} \frac{\partial}{\partial z} \left((p + \gamma h)\right) + \frac{\mu}{\rho} \nabla^2 v_z . (100)$$

in which v_x, v_y, and v_z, are the components of the instantaneous velocity vector, and,

$$\nabla^2 v_x = \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}, \text{ etc.} \ldots\ldots\ldots\ldots (101)$$

The left side of each equation denotes the component of acceleration at any point with time and with distance in three-dimensional space; the right side represents the sum of all force components per unit mass which produce this acceleration. If the first term at the left of each equation is zero, the flow is steady (that is, unchanging with time); if the second, third, and fourth terms are zero, the flow is uniform (that is, unchanging with distance in the direction of motion). If the flow is both steady and uniform, there is no acceleration; the terms at the left disappear, and the terms at the right then indicate the equilibrium of pressure, weight, and viscous shear.

Obviously, only in this last case can the term for fluid density be dropped. Hence, Mr. Yasines' statement that in viscous flow the density plays no part holds true only if the flow is both uniform and steady. Under these conditions the density should not be included in the original group of variables for dimensional analysis; since there are then only four variables (pressure gradient, velocity, diameter, and viscosity), there will be only one Π-term, as seen from Equation (29); only the numerical constant, 32, cannot be found by this method. Equation (30) is derived by introducing the density at two compensating points, which produces a useful equation without altering the fundamental expression. Laminar flow, however, is uniform only if the stream lines are parallel, and it must be remembered that any boundary curvature or rapid change of flow section will involve mass acceleration, under which conditions appreciable velocity will make the density of decided importance.

The Navier-Stokes equations, contrary to popular opinion, apply just as accurately to turbulent flow as to laminar, for the velocity components used in the equations refer to the actual instantaneous velocity at any point in question. Whether the flow is laminar or turbulent, viscous action is the only means of reducing the total energy of flow, through conversion into heat. Although it is possible for laminar flow to be both uniform and steady, turbulent flow is always unsteady and non-uniform in the strict sense of the word, for the velocity fluctuations vary at a given point with time, and at a given instant with distance in any direction. Therefore, it is evident that, however correct the Navier-Stokes equations may be, it is futile to use them for a practical study of turbulence as long as they remain in their present form.

Statistically speaking, it is reasonable to consider each velocity component to be composed of a temporal mean value and a secondary fluctuating value:

$$v_x = \overline{v_x} + v'_x \quad \text{.....................} (102a)$$

$$v_y = \overline{v_y} + v'_y \quad \text{.....................} (102b)$$

and,

$$v_z = \overline{v_z} + v'_z \quad \text{.....................} (102c)$$

Turbulent flow is then considered both steady and uniform if the values of the mean components do not vary with either time or space. By substituting these values for the velocity components in the original Navier-Stokes equations, one obtains parallel expressions for the Newtonian acceleration-force-mass equilibrium. Remembering that, while the temporal mean of any component of fluctuation is zero, the product of any two components (owing to the existence of correlation) will have a temporal average of finite magnitude, these parallel expressions may be written in terms of the mean velocity components and mean products of the components of fluctuation, as follows:[49]

$$\frac{\partial \overline{v_x}}{\partial t} + \overline{v_x}\frac{\partial \overline{v_x}}{\partial x} + \overline{v_y}\frac{\partial \overline{v_x}}{\partial y} + \overline{v_z}\frac{\partial \overline{v_x}}{\partial z}$$
$$= -\frac{1}{\rho}\frac{\partial}{\partial x}(p + \gamma h) + \frac{\mu}{\rho}\nabla^2 \overline{v_x} - \left(\frac{\partial \overline{(v'_x)^2}}{\partial x} + \frac{\partial \overline{v'_x v'_y}}{\partial y} + \frac{\partial \overline{v'_x v'_z}}{\partial z}\right) \quad ..(103)$$

$$\frac{\partial \overline{v_y}}{\partial t} + \overline{v_x}\frac{\partial \overline{v_y}}{\partial x} + \overline{v_y}\frac{\partial \overline{v_y}}{\partial y} + \overline{v_z}\frac{\partial \overline{v_y}}{\partial z}$$
$$= -\frac{1}{\rho}\frac{\partial}{\partial y}(p + \gamma h) + \frac{\mu}{\rho}\nabla^2 \overline{v_y} - \left(\frac{\partial \overline{(v'_y)^2}}{\partial y} + \frac{\partial \overline{v'_y v'_z}}{\partial z} + \frac{\partial \overline{v'_y v'_x}}{\partial x}\right) \quad ..(104)$$

and,

$$\frac{\partial \overline{v_z}}{\partial t} + \overline{v_x}\frac{\partial \overline{v_z}}{\partial x} + \overline{v_y}\frac{\partial \overline{v_z}}{\partial y} + \overline{v_z}\frac{\partial \overline{v_z}}{\partial z}$$
$$= -\frac{1}{\rho}\frac{\partial}{\partial t}(p + \gamma h) + \frac{\mu}{\rho}\nabla^2 \overline{v_z} - \left(\frac{\partial \overline{(v'_z)^2}}{\partial z} + \frac{\partial \overline{v'_z v'_x}}{\partial x} + \frac{\partial \overline{v'_z v'_y}}{\partial y}\right) \quad ..(105)$$

[49] "Turbulente Strömungen", by W. Tollmien, Handbuch der Experimentalphysik, Band IV, Teil 1, Leipzig, 1931.

Under these circumstances, the differential equation for steady uniform turbulent flow in a pipe will become, from Equation (103),

$$\frac{\partial(p + \gamma\, h)}{\partial x} = \mu\, \frac{\partial^2 \bar{v}}{\partial y^2} - \rho\, \frac{\partial \overline{v'_x v'_y}}{\partial y} \quad \ldots\ldots\ldots\ldots\ldots (106)$$

Mr. Yasines will note that, contrary to steady uniform laminar flow, there is now no possible way of eliminating the fluid density. In other words, once turbulent flow is described in terms of the mean, rather than instantaneous, velocity components (similar to the mean velocity, V, in the Reynolds number), then density must play a very essential rôle in any correct expression. (It is to be remarked that, for simplicity, the bar denoting temporal mean velocity was consistently omitted in the paper.)

From these considerations, Mr. Yasines' query about the writer's "rate of passage of momentum" should find a satisfactory answer. Since the components of fluctuation vary directly with the average velocity of flow, the rate of passage of momentum is a convenient means of representing, in measurable quantities, the magnitude of the momentum transport in the mixing process; and since the effect of viscous shear in terms of the mean velocity gradient varies inversely with the average velocity of flow, the Reynolds number for turbulent flow in pipes really denotes the ratio between the shearing intensity due to momentum transport, on the one hand, and that due to viscous shear in terms of the mean velocity gradient, on the other.

Similar conclusions will result from the following reasoning: In the Navier-Stokes equations the three components of the shearing stress, τ, are simply those due to viscous action, regardless of whether the flow is laminar or turbulent:

$$\tau_{xy} = \tau_{yx} = \mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \quad \ldots\ldots\ldots\ldots\ldots (107)$$

$$\tau_{yz} = \tau_{zy} = \mu \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right) \quad \ldots\ldots\ldots\ldots\ldots (108)$$

and,

$$\tau_{zx} = \tau_{xz} = \mu \left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right) \quad \ldots\ldots\ldots\ldots\ldots (109)$$

Once the instantantous velocity gradient is replaced by the mean velocity gradient, if the flow is turbulent there must exist additional terms to express the high viscous stresses in the turbulent eddies—that is, the effective shear of the momentum transport; introducing Boussinesq's turbulence coefficient, η, these become:

$$\bar{\tau}_{xy} = \bar{\tau}_{yx} = \mu \left(\frac{\partial \bar{v}_x}{\partial y} + \frac{\partial \bar{v}_y}{\partial x} \right) - \rho\, \overline{v'\, v'_x\, v'_y}\, (\mu + \eta) = \left(\frac{\partial \bar{v}_x}{\partial y} + \frac{\partial \bar{v}_y}{\partial x} \right) \quad \ldots (110)$$

$$\bar{\tau}_{yz} = \bar{\tau}_{zy} = \mu \left(\frac{\partial \bar{v}_y}{\partial z} + \frac{\partial \bar{v}_z}{\partial y} \right) - \rho\, \overline{v'_y\, v'_z} = (\mu + \eta) \left(\frac{\partial \bar{v}_y}{\partial z} + \frac{\partial \bar{v}_z}{\partial y} \right) \quad \ldots (111)$$

and,

$$\bar{\tau}_{zx} = \bar{\tau}_{xz} = \mu \left(\frac{\partial \bar{v}_z}{\partial x} + \frac{\partial \bar{v}_x}{\partial z} \right) - \rho\, \overline{v'_z v'_x} = (\mu + \eta) \left(\frac{\partial \bar{v}_z}{\partial x} + \frac{\partial \bar{v}_x}{\partial z} \right) \quad \text{..(112)}$$

It must be realized that Equations (107) to (109) and (110) to (112) are merely different ways of stating the same basic effect of viscous shear.

Reducing Equation (110) to the case of flow in a pipe:

$$\bar{\tau} = \mu \frac{d\bar{v}}{dy} - \rho\, \overline{v'_x v'_y} = (\mu + \eta) \frac{d\bar{v}}{dy} \quad \text{............(113)}$$

In the paper it was shown that with increasing Reynolds number the magnitude of η becomes increasingly greater than that of μ, and that above the approximate Reynolds number of 100 000, the magnitude of μ is comparatively insignificant. In the light of the foregoing discussion, the Reynolds number might also be considered to denote the relative magnitude of η and μ, the former being averaged over the flow section.

Since in highly turbulent motion $\mu \frac{d\bar{v}}{dy}$ is generally negligible when compared with $\eta \frac{d\bar{v}}{dy}$, Equation (113) may then be written in the form,

$$\bar{\tau} = -\rho\, \overline{v'_x v'_y} = \eta \frac{d\bar{v}}{dy} \quad \text{.................(114)}$$

Introducing the expression, $| v'_x | = l \frac{d\bar{v}}{dy}$ (refer to the development of Equation (42)):

$$\frac{\eta}{\rho} = | v'_y |\, l \quad \text{.......................(115)}$$

in which the vertical bars denote mean absolute magnitude, regardless of sign.

Equation (115) is of very basic importance, for it not only lends added clarity to Boussinesq's coefficient of turbulence, but is a direct measure of the transporting power of the mixing process. The writer stressed only the transport of momentum by this means, but the fact must be appreciated that it is the same mechanism which results in a transportation of heat, salinity, or suspended matter from one region of turbulent flow to another. The rate of transport of momentum has been shown to depend upon only the factor, η, and the mean velocity gradient, thus producing an effective shearing stress:

$$\bar{\tau} = \rho\, | v'_y |\, l \frac{d\bar{v}}{dy} \quad \text{....................(116)}$$

The problem of suspended load in a stream may be attacked in similar fashion. If at any depth of flow there are n particles per unit volume of fluid, the concentration gradient being written $\frac{dn}{dy}$, then, due to the transverse

fluctuations, fluid masses bearing n particles per unit volume will be carried the mean distance, l, across the flow to regions where the concentration differs by the amount, $l \dfrac{dn}{dy}$. Thus, the temporal rate of passage of sediment per unit area, N, will be the product of the rate per unit area of transverse flow, $| v'_y |$, and the difference between the sediment concentration in the traveling fluid mass and that of the region into which it comes, $- l \dfrac{dn}{dy}$ —in other words, the product of $\dfrac{\eta}{\rho}$ and the sediment gradient:

$$N = - | v'_y | \, l \, \frac{dn}{dy} \qquad \text{.....................(117)}$$

Identical reasoning applies to the transportation of salinity, and similar considerations show that the rate of heat transfer per unit area, q, will depend only upon $\dfrac{\eta}{\rho}$, the specific heat, c, and the mean temperature gradient across the flow, $\dfrac{d\,\theta}{dy}$:

$$q = - | v'_y | \, l \, c \, \frac{d\,\theta}{dy} \qquad \text{.....................(118)}$$

Such heat transfer is purely convective, and must be distinguished from the process of conduction in the laminar film.

The parallel development and structure of Equations (116) to (118) was shown to excellent advantage by von Kármán in a résumé of turbulence theories[50]. Since the problem of suspended load in a stream is of paramount interest in river hydraulics, amplification of this method of treatment should be of value at this time. If the sediment transportation of the stream as a whole is in a uniform state of equilibrium, the rate of upward transfer by the mixing process must equal the downward movement due to settling. Designating the settling velocity of a given particle size by w, then N, the temporal rate of transportation per unit area, must equal the number of particles per unit volume multiplied by their rate of fall:

$$N = w \, n = - | v'_y | \, l \, \frac{dn}{dy} \qquad \text{.....................(119)}$$

This expression may be integrated in the form,

$$\log_e \frac{n}{n_a} = - w \int_a^y \frac{dy}{|v'_y| \, l} \qquad \text{.....................(120)}$$

in which $\dfrac{n}{n_a}$ is the relative concentration at any point, referred to some arbitrary elevation, a, above the bottom. The evaluation of the last term in

[50] "Some Aspects of the Turbulence Problem", *Proceedings,* Fourth International Congress for Applied Mechanics, Cambridge, England, 1934.

the expression depends only upon knowledge of the kinematic turbulence factor, $\frac{\eta}{\rho} = |v'_y|\, l$, as a function of the depth.

As shown by von Kármán, introduction of Equation (116) yields an integrable expression,

$$\frac{dy}{|v'_y|\, l} = \frac{dy}{\dfrac{\tau}{l\dfrac{d\bar{v}}{dy}}} = \frac{l\dfrac{d\bar{v}}{dy}}{\tau_0\left(1 - \dfrac{y}{d}\right)}\, dy \qquad (121)$$

in terms of the mean velocity gradient. If the latter is of the universal logarithmic type, then from Equation (59),

$$\frac{d\bar{v}}{dy} = \frac{1}{\kappa}\sqrt{\frac{\tau_0}{\rho}}\,\frac{1}{y} \qquad (122)$$

and Equation (120) becomes:

$$\frac{n}{n_a} = \left[\frac{1 - \dfrac{y}{d}}{\dfrac{y}{d}} \times \frac{\dfrac{a}{d}}{1 - \dfrac{a}{d}}\right]^z = \left[\frac{1 - \dfrac{y'}{d'}}{1 + \dfrac{y'}{a}}\right]^z \qquad (123)$$

in which $z = \dfrac{w}{\kappa\sqrt{\dfrac{\tau_0}{\rho}}}$. Two pertinent facts are to be noted in Equation (123): The relative sediment distribution, for a given particle size (that is, for a given settling velocity), is a function of relative depth only. It cannot, however, be extended to the channel bottom, but only to some arbitrary depth, $d' = d - a$, and then only if the concentration at that depth is not so great as to change either the density of the fluid mixture or the assumed velocity distribution. If the bottom is smooth, the logarithmic velocity curve does not extend to the bottom, and there is no mixing in the final layer of fluid at the boundary. If the bottom is rough, knowledge of the total suspended load would permit complete solution of the sediment distribution (provided the logarithmic law could be extended to the free surface), but this expression in no way permits determination of the total transporting capacity of the stream. The latter probably depends upon conditions in the transition region between movable bed load and completely suspended matter, a problem still awaiting a satisfactory physical analysis.

On the assumption that the logarithmic velocity curve actually extends to the free surface of a very wide stream, Equation (123) enables one to construct a plot of sediment distribution curves (Fig. 30) for various values of the suspended load parameter, $z = \dfrac{w}{\kappa\sqrt{\dfrac{\tau_0}{\rho}}}$, indicating, for a given state

of flow, the relative distribution of the different particle sizes based upon their settling velocities. Such a diagram was first developed by Arthur T. Ippen, Jun. Am. Soc. C. E., at the suggestion of Professor von Kármán. An interesting comparison may be made with a group of measured curves published by L. G. Straub[51], Assoc. M. Am. Soc. C. E.

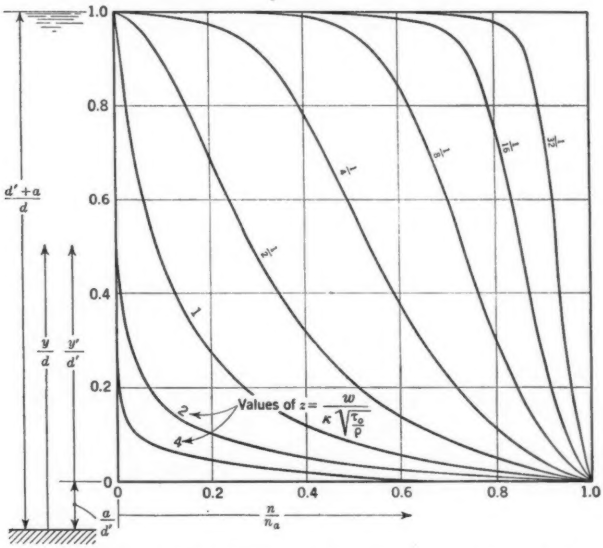

FIG. 30.—DIMENSIONLESS PLOT OF SUSPENDED LOAD DISTRIBUTION.

Professor Powell's remarks on the continuity of the f- versus $\mathbf{R}$-function and on the actual curvature of the velocity gradient in the laminar film are quite true. It is not only a probability, but a physical certainty, that there is no sudden change of law at about $\mathbf{R} = 100\,000$. Viscous shear in terms of the mean velocity gradient diminishes gradually from the critical region on, but never disappears. It is a simple matter to construct a smooth curve through the measured points, somewhat less simple to express its variation in a brief empirical formula, and, to date, quite impossible to determine this function analytically. Neither is there any abrupt limit to the laminar film—despite the obvious convenience of designating an arbitrary point of juncture between turbulent and laminar regions. As Professor Powell will have noticed, Equation (36) was offered to give a preliminary, "rough idea" of boundary layer thickness "when the Reynolds number is high." To paraphrase his remark, it is not necessary to think of the velocity in the boundary layer as varying non-uniformly in this region, since the departure from uniform variation is quite insignificant—particularly when compared with the obviously arbitrary magnitude of v_w. Equation (62) completely avoids this latter discrepancy, and still introduces no appreciable error. Obviously, however, it does not apply to the region chosen by Professor Powell for illus-

[51] "Transportation of Sediment in Suspension", *Civil Engineering,* May, 1936.

tration (R = 3 000), for as yet no successful analysis has been made of the Reynolds number range from 2 000 to 100 000.

Were the mechanism of turbulence basically different in the various phenomena in which it plays a leading rôle, Professor Powell's criticism that the title of the paper is too broad would be more fully justified. The writer's endeavor was not to present a complete text upon the subject, but to make available to hydraulic engineers the essential features in compact and comprehensible form, with pipe flow the most pertinent kind of illustration possible. It is only to be regretted that hydraulicians have previously had to rely upon aeronautical literature for such material, for the objectives of the two professions are by no means identical. On the assumption that criticism of this nature indicates imperfect success on the writer's part, a further effort was made in the foregoing relation of sediment transportation to the turbulent mechanism. It still remains to point out more explicitly the significance of the boundary layer to turbulence in general, particularly as the latter really includes the phenomenon of flow in pipes. However, while pipe flow is ordinarily a case of established uniform motion, primary interest in the boundary layer has to do with its rate of growth in the direction of flow; that is, the basic problem is essentially one of non-uniform movement.

Consider a smooth, thin plate moved at a steady rate through a fluid originally at rest. Since the fluid in immediate contact with the plate will have the same velocity as the plate itself, the resulting shearing stresses must cause movement of the surrounding fluid at a rate decreasing with distance away from the plate and increasing with distance back from the leading edge. Although this movement, mathematically speaking, extends an infinite distance away from the plate, it is convenient to designate that region in which the motion is appreciable as the boundary layer; in this sense, the boundary layer begins with a zero thickness at the leading edge and increases gradually from section to section. It is possible, analytically[52], to compute the thickness at any section as a function of the relative velocity of plate and undisturbed fluid, the kinematic viscosity, and the distance from the leading edge, proceeding on the assumption that the velocity distribution curve is a similar function at all sections; this assumption has been vindicated experimentally except in the immediate vicinity of the leading edge. (It should be evident that the kinematic, rather than the absolute, viscosity must be used, since non-uniform flow is a case of mass acceleration even when the motion is laminar.) Such analysis as this has led to basic expressions for the shearing stress and the velocity at any point in the fluid near the moving plate.

Since the fluid was originally at rest and the surface of the plate very smooth, the resulting movement is necessarily a laminar one; but as the boundary layer becomes increasingly thicker the farther back it extends, some distance from the leading edge an unstable condition is reached (marked by a fairly definite critical Reynolds number, R_δ, composed of the boundary layer thickness, the relative velocity, and the kinematic viscosity) at which move-

[52] "Grenzschichten in Flüssigkeiten mit kleiner Reibung", von H. Blasius, *Zeitschrift für Mathematik und Physik,* Band 60, 1912.

ment within the boundary layer abruptly becomes turbulent. From this point, there exists a turbulent boundary layer distinguished by a different type of velocity distribution and a more pronounced shearing stress; but since along the surface of the smooth plate the velocity of the fluid must be identical with that of the plate, there must still be a thin laminar region directly at the boundary, in which no turbulence exists; this region is called the laminar sub-layer. The flow pattern in the turbulent boundary layer may be determined as a function either of a Reynolds number composed of the relative velocity, the kinematic viscosity, and the distance from the leading edge, or of another Reynolds number in which the thickness of the boundary layer replaces the distance from the leading edge[10]. A simple proportionality always exists between these two Reynolds numbers in the turbulent region.

Had the leading edge of the plate, or the plate surface, been sufficiently rough, the boundary layer would have been a turbulent one from the outset. The result would be similar if the fluid were not originally at rest but slightly disturbed, since only a small degree of instability is sufficient to bring about fully developed turbulence at a very early stage of boundary layer growth.

The relation between this phase of boundary layer study and the case of flow in pipes should be quite evident. At the beginning of a pipe line, if the entrance is well rounded and the supply of fluid completely at rest, a laminar boundary layer will begin to form at the entrance around the entire inside of the pipe, growing in thickness with distance from the entrance. The limit of growth is attained when the effective thickness of the layer is equal to the pipe radius (that is, when the layer has penetrated to the center line of the flow), beyond which section the flow is fully established.

If the critical Reynolds number of the boundary layer is reached before the flow is thus established, this layer will become turbulent and then continue to grow as such until it reaches the central regions of the flow. Under these conditions the initial onset of turbulence will obviously not be in the central, undisturbed core of fluid. If, on the other hand, the supply of fluid is not in a state of rest, or if the pipe is rough or the entrance poor, the boundary layer will be turbulent from the outset, and its growth will soon bring it to the center line of flow. From these facts it will be clear that what is popularly called the boundary layer in pipe flow is actually the laminar sub-layer, whereas the true boundary layer extends to the very middle of the pipe.

Although the equations of boundary layer growth are at present primarily of value to aeronautical engineers and naval architects for use in determining skin resistance of air and water craft, of interest to the hydraulic engineer should be the fact that the equations for established flow in pipes have almost identical counterparts for boundary layer phenomena, since the fundamental mechanism of flow is the same. The coefficient of resistance, f, has its parallel in the local coefficient of resistance for the boundary layer, c_f, which necessarily varies with τ_0 from section to section:

$$c_f = \frac{\tau_0}{\rho \frac{V^2}{2}} \qquad (124)$$

If τ_0 is integrated over the distance, x, from the leading edge to yield the total resisting force, F_x, up to that section, then a mean coefficient of resistance may be used, having the form,

$$C_f = \frac{F_x}{b\,x\,\rho\,\frac{V^2}{2}} \quad \text{.......................(125)}$$

In the range of the laminar boundary layer, Equations (124) and (125) were shown analytically by Prandtl and Blasius to equal,

$$c_f = \frac{0.664}{\left(\frac{V x}{\nu}\right)^{\frac{1}{2}}} \quad \text{......................(126}a\text{)}$$

and,

$$C_f = 2\,c_f = \frac{1.328}{\left(\frac{V x}{\nu}\right)^{\frac{1}{2}}} \quad \text{...................(126}b\text{)}$$

Over a limited range past the critical point, corresponding to the Blasius range for pipe flow, Prandtl and von Kármán have shown the functional relationship to be closely exponential, just as in the case of pipes:

$$c_f = \frac{0.059}{\left(\frac{V x}{\nu}\right)^{\frac{1}{5}}} \quad \text{......................(127}a\text{)}$$

and,

$$C_f = \frac{0.074}{\left(\frac{V x}{\nu}\right)^{\frac{1}{5}}} \quad \text{.....................(127}b\text{)}$$

For larger Reynolds numbers, however, at which viscous action need no longer be considered in the mixing process, the universal relationship for smooth pipe resistance (refer to Equation (68)) has a close double in von Kármán's universal equation for the boundary layer:

$$\frac{1}{\sqrt{c_f}} = 1.7 + 4.15 \log_{10} (\mathsf{R}_x\, c_f) \quad \text{..............(128}a\text{)}$$

and,

$$\frac{1}{\sqrt{C_f}} = 4.15 \log_{10} (\mathsf{R}_x\, C_f) \quad \text{.................(128}b\text{)}$$

in which R_x is the Reynolds number based upon distance from the leading edge. The three regions are shown in the logarithmic plot of Fig. 31, bearing striking similarity to Figs. 6 and 7.

Boundary layer growth has not yet begun to claim the attention of hydraulic engineers, in part because their interests, as opposed to those

of æronautical and naval designers, are still focused on problems of uniform flow; but uniform flow is by no means the sole type with which hydraulicians must deal, and such instances as siphons, draft-tubes, turbines, spillways, and

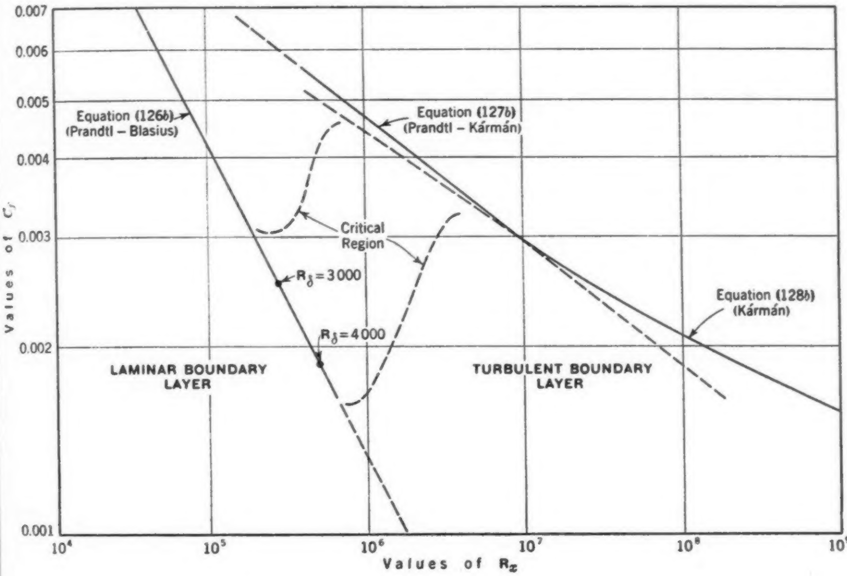

FIG. 31.—RESISTANCE CURVES FOR THE BOUNDARY LAYER.

even open channels, are only a few of the many cases in which occurrences within the boundary layer may have a decided influence upon the flow pattern. It has long been known that acceleration tends to make the velocity distribution more uniform, whereas deceleration (whether due to curvature or to local or general increase of flow section) not only hastens the spread of the boundary layer, but may even bring about a reversal of flow at the boundary surface. Under such conditions separation will occur, with a high rate of loss in the accompanying turbulent wake. Needless to say, unless the engineer remains content with rule-of-thumb empiricism, a clear understanding of boundary layer phenomena may soon become essential to healthy progress in hydraulic design.

Mr. Johnson has the writer's full appreciation for his excellent endeavor to extend the theories of universal velocity distribution to the difficult case of open channel flow—a very apt contribution at this time. Although, for many years, the velocity distribution has been held by certain engineers to be logarithmic[53], complexities resulting from the existence of a free surface, nonuniformity of the flow section, secondary currents, variable shear over the

[53] See, for instance, R. Jasmund, *Zeitschrift für Bauwesen*, 43, 1893; "Die Quergeschwindigkeitskurve bei turbulenter Strömung", von H. Krey, *Zeitschrift für angewandte Mathematik und Mechanik*, 7, 1927; and "Hydraulik", von P. Forchheimer, B. G. Teubner, Leipzig und Berlin, 1930, S. 179.

boundary, and the presence of bed and suspended loads have continued to prove obstacles to a general rational analysis.

It is evident that at the free surface of a stream the mixing length cannot have its maximum value as in the case of pipe flow. In the latter instance, however, the discontinuity in the universal velocity distribution along the center line of the pipe, owing to lack of correlation in the fluctuations in this region, still does not introduce serious error in this relationship. It is possible that discontinuity at the free surface is no more serious, as indicated by Mr. Johnson's measurements. The question of secondary currents, however, cannot be ignored unless the channel width is extremely great. Prandtl has explained this secondary motion in closed conduits as the resultant action of the radial components of fluctuation at points of maximum curvature of the isovels. Although further analysis may be necessary in applying this reasoning to the case of a free surface, one fact is clear: Since secondary currents depress the region of maximum velocity farther and farther below the surface as the width-depth ratio decreases, it is evident that such motion cannot be ignored in deriving a universal velocity relationship. Introduction of the hydraulic radius to average the different intensities of boundary shear (resulting, as do the secondary currents, from the shape and variable roughness of the cross-section) is still to be proved fully justifiable in a general expression; any given hydraulic radius may apply to a wide range of channel proportions, and surely the change in geometrical cross-section will influence the flow pattern. Mr. Johnson's empirical expression, $T = k\,v^2$, depends entirely upon judicious choice of k—which is at best a variable quantity Needless to state, movable-bed studies must continue to retain their empirical nature as long as uniform open channel flow itself has not been mastered.

Mr. Wilson will note that he has omitted an essential variable in Equation (92)—the slope, S, properly included by Lindquist[44]. However, it is questionable whether Equation (92) even then is complete (or exponential, as assumed), except for small, smooth-walled channels of approximately similar form. In the light of the foregoing discussion, uniform open channel flow with fixed bottom should be completely described by a relationship of the form,

$$\phi\left(\frac{V^2}{d_0\dfrac{\gamma}{\rho}},\ \frac{V\,d_0}{\dfrac{\mu}{\rho}},\ S,\ \frac{\epsilon}{d_0},\ \beta\right)=0 \qquad \text{.............(129)}$$

in which the terms are, respectively, the Froude number, the Reynolds number (if $d_0 < b$), the relative roughness, and some dimensionless shape factor describing the proportions of the cross-section of flow.

Either S or $\dfrac{V^2}{d_0\dfrac{\gamma}{\rho}}$ might be selected as the dependent variable, but it is evident that the resulting relationship, although physically correct, is too cumbersome to be practicable. However, by a development parallel to that

leading to Equations (13) and (15), introduction of the mean wall shear, $(\tau_0)_m$, will lead to the expression,

$$(\tau_0)_m = \gamma R S = \frac{f}{4} \rho \frac{V^2}{2} \qquad (130)$$

or,

$$f = 4 \frac{\gamma R S}{\rho \dfrac{V^2}{2}} = 4 \frac{R S}{\dfrac{V^2}{2 g}} \qquad (131)$$

whence, in the Chezy form,

$$V = C_c \sqrt{R S} = \sqrt{\frac{8 g}{f}} \sqrt{R S} \qquad (132)$$

The Chezy coefficient, C_c, is different numerically and dimensionally from the C in Equation (92).

Assuming that the hydraulic radius, R, is a suitable length to be used in the Reynolds number and the term for relative roughness,

$$f = \phi' \left(\frac{V R}{\nu}, \frac{\epsilon}{R}, \beta \right) \qquad (133)$$

These three independent parameters will vary considerably in their relative influence on the resistance coefficient, f. The Reynolds number probably has marked significance only in small channels (such as are encountered in model studies) or those with very smooth boundaries, being subordinate to $\frac{\epsilon}{R}$ in appreciably rough channels, just as in the case of pipes. Of course, β becomes of no importance only as the width-depth ratio becomes great, unless the side slopes extend far into the central regions of flow. Some appropriate factor similar to the width-depth ratio, such as $\frac{d_0}{R}$, might conveniently replace β, as long as the channel remains of a given general form (that is, elliptical, trapezoidal, rectangular, etc.), but this evidently cannot define the relative proportions of all such forms at one and the same time.

Once non-uniform flow is considered, the problem becomes even more difficult. This phase has been approximately solved, as far as the immediate needs of practical design are concerned, by a combination of empirical data on resistance with rational analysis, through the use of certain simplifying assumptions. The empirical data involve the magnitude of the Chezy C as a function of channel roughness, hydraulic radius, and mean velocity, and were largely determined for cases of uniform flow. It is generally realized that non-uniform flow—whether accelerative or decelerative—will result in a rate of energy loss (and, hence, boundary shear) differing from that in uniform flow at the same depth by more than a negligible amount.

The writer stresses these difficulties not from any sense of discouragement, but in order that full understanding of the obstacles to be surmounted may prevent investigators from plowing blindly into a field so demanding of careful cultivation. It is hoped, therefore, that the combination of the basic thesis as presented in the paper, the discussion, and the closure may prove a stimulus to further research in a subject that is most essential to hydraulic engineering. To Professor von Kármán the writer expresses his appreciation, not only for contributing to the discussion of the paper, but for critically examining the contents of this closure.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

SIMULTANEOUS EQUATIONS IN MECHANICS SOLVED BY ITERATION

Discussion

By Messrs. Marvin A. Gray, and John E. Goldberg

Marvin A. Gray,[18] Esq. (by letter).[18a]—Equation (1) of this paper is a conception of the theorem of three moments which, in itself, requires additional explanation for teaching; in a typically popular book of applied mechanics the theorem of three moments is given nine times (probably many more times in other texts and reference books) as:

$$M_1 L_1 + 2\,M_2\,(L_1 + L_2) + M_3\,L_2 = -\frac{1}{4}\,w_1\,L^3_1 - \frac{1}{4}\,w_2\,L^3_2 \quad \ldots\ldots(63)$$

The symbol, Q, is used in only one special problem, whereas there are forty to fifty problems in which the last mentioned form (Equation (63)) is used; and U_x is not mentioned in the entire book. It might be of interest to note that this particular problem of moments, in which Q was used, is not even mentioned in this paper. The development of the author's expression of the three-moment theorem comes from the following algebraic maneuvers:

$$M_1 L_1 + 2\,M_2\,(L_1 + L_2) + M_3\,L_2 = -\frac{1}{4}\,w_1\,L^3_1 - \frac{1}{4}\,w_2\,L^3_2 \quad \ldots(64a)$$

and,

$$M_1\,\frac{L_1}{I_1} + 2\,M_2\left(\frac{L_1}{I_1} + \frac{L_2}{I_2}\right) + M_3\,\frac{L_2}{I_2} = -\frac{1}{4}\,w_1\,\frac{L^3_1}{I_1} - \frac{1}{4}\,w_2\,\frac{L^3_2}{I_2} \quad \ldots(64b)$$

Then,

$$\frac{M_1}{K_1} + 2\,M_2\left(\frac{1}{K_1} + \frac{1}{K_2}\right) + \frac{M^3}{K_2} = -\frac{1}{4}\,w_1\,\frac{L^2_1}{K_1} - \frac{1}{4}\,w_2\,\frac{L^2_2}{K_2} \quad \ldots(64c)$$

Note.—The paper by W. L. Schwalbe, Esq., was published in August, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1936, by Messrs. Garrett B. Drummond, and A. W. Fischer; and December, 1936, by M. B. Gamet, Jun. Am. Soc. C. E.

[18] Chicago, Ill.

[18a] Received by the Secretary November 27, 1936.

with the addition of symbols for symbols and changing from common notation to that suggested by the author, the reader will arrive at Equation (1)—and the student will be confused.

Equation (8) is also of interest because it too adds to the student's problem. In school before the theorem of three moments is taught, the problem is solved by the calculus. Then, because this does not seem to be sufficient explanation, it is again derived by the moment-area method; and again by several variations, or very slight modifications, of moment areas. However, there is a general formula for finding moments, rotations, and movement of beams; this equation (derived from a knowledge of moment areas) is known as the slope deflection equation: $M_{AB} = M_{F\text{-}AB} - \frac{2\,E\,I}{L}\,(2\,\theta_A + \theta_B)$, or,

$$M_{AB} = + M_{F-AB} - \frac{2\,E\,I}{L}\,(2\,\theta_A + \theta_B - 3\,R) \quad \text{..........(65)}$$

Using standard notation (as in the three-moment theorem), the writer obtains the two equations:

$$M_{2\text{-}1} = - M_{F\text{-}2\text{-}1} - K_1\,(2\,\theta_2 + \theta_1) \quad \text{..............(66a)}$$

and,

$$M_{2\text{-}3} = M_{F\text{-}2\text{-}3} - K_2\,(2\,\theta_2 + \theta_3) \quad \text{................(66b)}$$

but $- M_{2\text{-}1} = M_2 = M_{2\text{-}3}$; so that:

$$+ M_{F\text{-}2\text{-}1} + K_1\,(2\,\theta_2 + \theta_1) = M_{F\text{-}2\text{-}3} - K_2\,(2\,\theta_2 + \theta_3) \quad \text{.......(67)}$$

Solving Equation (67) by elementary algebra, the writer derives an equation for a given θ-value:

$$\theta_2 = \frac{M_{F\text{-}2\text{-}1} - M_{F\text{-}2\text{-}3} - (K_1\,\theta_1 + K_2\,\theta_3)}{2\,(K_1 + K_2)} \quad \text{...........(68)}$$

which is as exact as any elastic beam theory used. If it does nothing else, this method at least omits many new and varied symbols that are confusing not only to the student but to the practicing engineer.

The writer omits the author's applications of slope deflection as presented in his paper, and calls attention to Equations (22), (23), (24), and (25)[18b]. By using the slope deflection equation,

$$M_{AB} = M_{F\text{-}AB} - K\,(2\,\theta_A + \theta_B) \quad \text{................(69)}$$

and correcting for the author's conception of signs from the simpler convention of slope deflection, the writer obtains from Fig. 7:

$$\theta_A = (\theta_A)_0 + \frac{M_A\,L}{3\,E\,I} + \frac{M_B\,L}{6\,E\,I} \quad \text{...............(70a)}$$

[18b] Correction for *Transactions*: Substitute — $(\theta_B)_0$ for $(-\theta_B)_0$ and insert a plus sign before the last moment in Equation (26).

$$\theta_B = -(\theta_B)_o - \frac{M_B L}{3 E I} - \frac{M_A L}{6 E I} \quad \text{.............. (70}b\text{)}$$

$$M_{AB} = M'_A + 2 E K (2 \theta_A + \theta_B) \quad \text{................ (70}c\text{)}$$

and,

$$M_{BA} = M'_B - 2 E K (2 \theta_B + \theta_A) \quad \text{................ (70}d\text{)}$$

A little time spent in following the procedure (or algebra) necessary to arrive at the formulas mentioned, should convince the student or engineer that no "take off" on moment areas or variation of the slope deflection method could be as simple and practical as the slope deflection method itself. Certainly, the writer has proved his point that, fundamentally, this paper is an expression of slope deflection confused by new symbols; the same slope deflection, that is given in the paper (in general, Equation (65)), was originally presented[19] by G. A. Maney, M. Am. Soc. C. E.

The author would create a boon for students if he would take slope deflection and use it with its accepted convention in his teaching practice. The writer has shown that the background for slope deflection, and probably its derivation, is already taught; it seems that the use of Equation (65) with algebra or arithmetic (as is common in engineering) brings a widening of the structural field and applied mechanics in the accurate commercial solution of problems. In aeronautics alone slope deflection should find many uses because of its accuracy and speed; it also lends itself easily to rapidly converging approximations; and with the coming of aeronautical materials into the every-day structural field, the method is indispensable to the Engineering Profession. The author has started on the solution of a very important and useful problem, however, which should end in a symposium on the slope deflection method with its wider use in to-day's engineering, for students and for reference.

JOHN E. GOLDBERG,[20] JUN. AM. SOC. C. E. (by letter).[20a]—Discussion of Professor Schwalbe's paper affords an opportunity to emphasize several significant facts in the development of modern American slope-deflection practice:

(1) As early as 1913, Allston Dana, M. Am. Soc. C. E., used the method of iteration to obtain a slope deflection analysis of secondary stresses in the Kenova Bridge.

(2) In 1915, G. A. Maney, M. Am. Soc. C. E., proposed[21] the present general form of the slope deflection equation and demonstrated its application to a wide variety of rigid frame problems. Previous to this time, the slope deflection method existed solely as a means of analyzing secondary

[19] *Bulletin No. 1, Eng. Studies*, Univ. of Minnesota, March, 1915.

[20] With Dept. of Buildings, Chicago, Ill.

[20a] Received by the Secretary, December 9, 1936.

[21] See "Secondary Stresses and Other Problems in Rigid Frames", by G. A. Maney, *Engineering Studies No. 1*, Univ. of Minnesota, 1915; see, also, "Wind Stresses in the Steel Frames of Office Buildings", by W. M. Wilson and G. A. Maney, *Bulletin No. 80*, Eng. Experiment Station, Univ. of Illinois, 1915. (See footnote at bottom of page 1, *Bulletin No. 80*, on credit for the theory.)

stresses in trusses. Professor Maney introduced the fixed-beam moment factor which makes possible the analysis of stresses in loaded frames and which forms the basis of the present simple and popular methods for the analysis of rigid or continuous frames. With the exception of the secondary stress problem, solution of the simultaneous equations was effected by algebraic methods.

(3) In 1931, the writer presented[22] a practicable method, based upon slope deflection and successive approximations, for the analysis of rigid building frames and similar structures under the action of gravity loads, largely developed by Professor Maney directly from the original slope deflection equations.

(4) In 1933, several years after its development, the writer presented[23] the method of wind-stress analysis of slope deflection and converging approximations which he developed directly from the basic theory.

[22] "Vertical Load Analysis of Rigid Building Frames", by John E. Goldberg, *Engineering News-Record*, November 12, 1931.

[23] "Wind Stresses by Slope Deflection and Converging Approximations", by John E. Goldberg, *Transactions*, Am. Soc. C. E. Vol. 99 (1934), p. 962.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ADMINISTRATIVE CONTROL OF UNDERGROUND WATER: PHYSICAL AND LEGAL ASPECTS

Discussion

BY MESSRS. G. E. P. SMITH, AND DAVID G. THOMPSON

G. E. P. SMITH,[16] M. AM. SOC. C. E. (by letter).[16a]—This very valuable paper is more comprehensive than its title indicates. It is a rather thorough treatment of the entire subject of ground-water law, but with special emphasis on the control of ground-water by the State. If the following discussion appears to be critical, it does not follow that the writer is unappreciative of the high value of the author's timely analysis of an intricate subject.

A great handicap in the development of ground-water law is that legislators and advocates in the Courts usually have little conception of the hydrology of ground-waters. Fortunately, this has led to an avoidance of the issues in many States, and questions which might have been settled wrongly are still open. Progress is being made in the understanding of ground-water regimen and hydrologic records are being accumulated, but it may be long before ground-water law becomes fully crystallized. Although uniformity in the seventeen Western States might be desirable, it can scarcely be hoped for; even in the use of terminology, the various State Courts are not in agreement at present. Hydrologists should have an active part in the development of the law, both statutory and judicial, and administrative control should be in the hands of engineers whose training includes hydrology.

The most notable pioneering advance in recent years is the legislation in New Mexico, enacted in 1931, placing underground streams and bodies of ground-water the boundaries of which "can be reasonably ascertained" under the effective control of the State Engineer. The excellent administrative control that has been established has solved the difficult problem of a seriously decreasing water supply and has even made a significant further

NOTE.—The paper by Harold Conkling, M. Am. Soc. C. E., was published in April, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1936, by Messrs. Joseph Jacobs, and W. D. Faucette and J. E. Willoughby; September, 1936, by R. E. Savage, Assoc. M. Am. Soc. C. E.; and December, 1936, by Messrs. H. J. F. Gourley and O. J. Baldwin.

[16] Prof. of Irrig. Eng., Univ. of Arizona, Tucson, Ariz.

[16a] Received by the Secretary November 21, 1936.

advance in the use of public funds, in part, in shutting off the loss of water from abandoned and leaky wells.

The author has described and contrasted the various systems of law relating to surface waters and ground-waters. The riparian doctrine, as modified and developed by the Courts in California, is so different from the original riparian theory introduced into this country from the English law that it ought to be given a new name for the sake of clarity. Among its disadvantages in an arid region should be mentioned that, in theory at least, it may lead to the spreading out of the water supply, in dry years, into a thin irrigation over too great an area, so that on no field is there a profitable crop. Furthermore, since all ditches will be wetted, the proportion of the water lost in the canal system will be high; and, if a real attempt were made by a State administrative body to prorate the waters from day to day through a drought period, the task would be found well-nigh impossible. It is understood that the irrigation interests in California have had to tolerate the riparian doctrine, but have not relished it.

In his comparisons of laws relating to ground-water the author has tended to minimize the disadvantages of the American and California systems, which are, first, the almost certain over-development of the supply to the injury of all the water users; second, the insecurity as to the future of a developed supply and the insecurity attending the 5-yr probationary period; and, third, the likelihood of ultimate scattered small patches of irrigated land instead of more compact areas. Under the appropriative doctrine a water user at least knows where he stands, assuming that the determination of the existing rights has been made by the State's administrative officer.

The writer questions the statement that the rule of law promulgated in the Katz *v.* Walkinshaw case was adopted in other States. The one exception was the Utah Oil Refining Company case, in which the Court adopted the correlative rule, but permitted export; and more recently Utah, by statute, has placed all ground-waters under the full appropriative rule.

In an Arizona case, involving a small seepage of water, the query was as to just what constitutes a spring. The Court added considerable dicta, and indicated a leaning toward the correlative rule, but such tendency should not be considered as adoption. When the issue arises directly in a case, the American or the California rule may possibly be followed. In a case in the State of Washington, where a hillside well had been drained by reason of a new railway cut, the chief issue was damages, and the references to the correlative doctrine were of little moment. Judges often wander afield in the preparation of decisions.

The author points out clearly that the doctrine of correlative rights is incompatible with the doctrine of priority of appropriation. By reason of that incompatibility the correlative rights theory should be rejected in arid States other than California, at least with regard to "commercial" supplies of water.

The author's analysis of the terminology of rules of law is timely. The term, correlative, has been used in decisions with several different meanings. He might well have emphasized that a number of States have a dual system

of law—the American system for "diffused percolating waters" and the appropriative system for underground streams flowing in definite channels. The division is not along the same lines as that between non-commercial and commercial supplies. New Mexico, Arizona, Nevada, Oregon, and Washington have the dual system. A recent case in point is that of Evans *v.* City of Seattle (47 Pac. (2d) 984).

Normally, the best pumping supplies are located in the Quaternary valley fill. If they are controlled under the appropriative system of law, it may be assumed that domestic supplies will be made available under the rule of preferential rights. If meager supplies are obtainable from Tertiary formations (at least in Arizona), it may be best to leave them under the rule of reasonable use, the American rule; rarely, if ever, will the effect of such use on the Quaternary supplies be of any considerable importance.

The writer agrees that the extension of the right of priority to the means of diversion, requiring that the water level at the well of the prior appropriator be not disturbed, is absurd. For the rule respecting means of diversion of surface water, the case of Salt Lake City *v.* Gardner (114 Pac. 147) is a leading one and was followed in the case of Pima Farms Co. *v.* Proctor, cited by the author. In the few cases where this rule has been upheld, it is not unlikely that sooner or later the Courts will reverse themselves, and will adopt the rule of reasonableness; even now in those jurisdictions the rule is ignored in practice.

The future success of administrative control over ground-waters will depend to a large extent on the right to reject applications when development has reached a safe limit. It will be simpler to prevent drilling of wells than to exercise constant control over surplus wells. A private communication from the State Engineer indicates that the attitude in Wyoming as to rejections is that "if an individual wishes to proceed * * * he is the one who must bear the burden of costs and should satisfy himself as to the chance of securing water for his appropriation in accordance with its date of priority." It is understood that, in California, despite frequent strong opposition, the Division of Water Rights refuses applications to appropriate when there is no unappropriated water, "getting away with it when they can."

A State administrative office has all the data on vested and initiated rights and on stream flow and storage, and is in a better position to determine when a water supply is fully appropriated than any other agency. Surely an "individual" in an isolated location with limited finances cannot be expected to go into the question thoroughly; nor can a promotional company do it. It does not promote the public welfare when great sums are invested in irrigation works and in clearing land, building homes, and planting orchards, subdividing land, and luring settlers, only to find later that the water supply is inadequate or undependable. The attitude in Wyoming seems to be to evade the responsibility; this is lamentable because engineers most favorable to State administration of water supplies consider Wyoming's record of administration as the ideal one.

The author states that there are no statutes on underground water in Arizona. On the contrary, the State Water Code includes water "in definite

underground channels, whether perennial or intermittent", and also springs, among the waters that belong to the public and are subject to appropriation. This statutory provision is far-reaching, and has been recognized in Court decisions. It remains now to establish just what constitutes a definite underground channel, and much thought is being given that problem. In the Southwest Cotton Company case (4 Pac. 2d, 369), the Court in its decision suggested a test which hydrologists agree is impractical. When another case comes before the Court, or by statute, both definition and test should be settled definitely.

The author states, also, that in the case of Pima Farms Co. *v.* Proctor (245 Pac. 369), the Court ruled that the ground-water beneath the inner valley of the Santa Cruz River is in a definite under-ground channel. No technical data or competent evidence on that point was presented to the Court. The litigants stipulated that it was a definite channel, and the Court stated, "it is assumed or conceded throughout that this body of water is a known independent subterranean stream." How the Court would "rule" on this point if the evidence were presented, cannot be known. The status of ground-water law in Arizona is one of uncertainty.

In no Western State has the final word been said; ground-water law is still in the making. There will be refinements, especially in border-line cases, and there will be some reversals. In the further development of the law and of administrative control of ground-waters, it should be kept in mind that the interests of the public are an important factor. On some points, property rights may be modified or may give way entirely. The public is interested primarily in having the limited water supplies fully utilized. Furthermore, in the future, it may be emphasized that, in cases of conflict, the most important uses must prevail over those less vital to the general welfare.

David G. Thompson,[17] Esq. (by letter).[17a]—In this paper Mr. Conkling discusses a problem of great importance in planning the efficient development of one of the country's valuable natural resources, namely, the underground water or, as it is commonly called, the ground-water. In some States almost all public and domestic supplies are derived from ground-water, and large industries and thousands of acres of irrigated land are dependent upon this source of water supply. There is abundant evidence that, in many areas, the supply of ground-water is not sufficient to meet the needs or demands of all who desire to use it.

The paper is written largely from the viewpoint of problems in the Western States where much water is used for irrigation. It is the aim of this discussion to raise for consideration certain viewpoints arising from a study of ground-water conditions in many parts of the United States, but most particularly in certain of the humid States. The writer has delved assiduously into the laws of many of the States and of numerous Court decisions relating to ground-waters, but he must emphasize the fact that his viewpoint is that

[17] Senior Geologist, Div. of Ground-Water, U. S. Geological Survey, Washington, D. C. Published with permission of the Director, U. S. Geological Survey.

[17a] Received by the Secretary December 12, 1936.

of one not versed in the principles of law. His approach is primarily from the viewpoint of a ground-water hydrologist whose duty has been to advise in the development of the country's ground-water resources.

Mr. Conkling has stated that, in his paper, he has not considered the phase of control of ground-water relating to waste. He is concerned primarily with the orderly development of the available supply to the greatest possible extent without exceeding the safe yield of the source. Meinzer[18] has pointed out that the problem of legal control of artesian water relates: (1) To the proper construction and operation of wells to prevent waste; and (2) to the equitable distribution of the water. It is the writer's belief that laws relating to control of the use of ground-water should not avoid consideration of the phase of prevention of waste. As a matter of fact, a definite requirement in the statutes of most of the States that have sought to establish a property right in either surface or ground-water, makes such a right dependent upon a "reasonable use" of the water. As a result of its quantitative studies of ground-waters in many parts of the country the Division of Ground Water of the U. S. Geological Survey now defines the safe yield of an aquifer somewhat as follows: "The safe yield is the rate at which water may be drawn perennially from the formation * * * without depletion of the supply or impairment of its quality to such an extent that withdrawal at this rate is no longer economically feasible." In other words, safe yield involves not merely the elements of quantity of water, time, and cost of pumping, but also the element of quality of the water; for example, the controlling factor in the maximum use of water from beds along a sea coast may be the rate at which water can be withdrawn without drawing in salt water. Elsewhere, the usefulness of fresh water-bearing beds may be impaired by improper well construction in oil fields, or in other areas where salt water is present, by disposal of waste in industrial areas, or by pollution when wells are used for drainage. Any law seeking to bring about a reasonable development of ground-water must recognize these dangers and give authority to control them just as much as it gives authority to limit quantity of withdrawal. However, this discussion is confined to the phase of equitable distribution or limitation of the use of ground-water.

Under "Extent of Underground Water Use" Mr. Conkling has stated that "in humid regions (that is, in regions of the United States east of the 98th Meridian) underground water is exploited to the point where far-reaching effects are produced only to secure supplies for metropolitan areas." However, in Table 1, the two States following California, in second and third rank (namely, Louisiana and Arkansas), and a large part of the irrigated area in Texas, are in the humid part of the country.

It is doubtless true that the problem of the control of ground-water has been most urgent in the arid Western States. There is evidence, however, that increasing attention must be given to the problem in other States. In a most interesting discussion of problems of legal control in England, H. J. F.

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[18] "Progress in the Control of Artesian Water Supplies", by O. E. Meinzer, *Engineering News-Record*, Vol. 113, pp. 167–169, 1934.

Gourley[19], M. Am. Soc. C. E., shows some of the legal difficulties encountered in an area that certainly is as humid as most of the United States. Within recent years the need for some kind of legal control has been recognized by State geologists and other officials in several Eastern and Central States, and legislation has been drafted in some States. In the last two or three years special impetus has been given to the consideration of projects for irrigation by ground-water in drought-affected areas. There have been numerous developments of ground-water for municipal supply and for private industrial use. Particularly has there been a great increase in the use of ground-water for air-conditioning, and this use is certain to increase. In all too few instances in the actual and proposed developments has adequate consideration been given to the safe yield of the source aquifers, and control of development becomes increasingly desirable. A few instances may be cited to supplement, for the Eastern States, Mr. Conkling's excellent summary of the present status of control in the Western States, and to show the further need for, and problems of, control.

Stated briefly, the writer's thesis is as follows: Much of the classification of ground-waters adopted in many Court decisions and by writers of legal textbooks is not consistent with scientific principles of ground-water hydrology. Efforts frequently have been made to distinguish between "percolating water" and water moving in "known and defined subterranean channels or watercourses", where geologic and hydrologic conditions do not warrant such distinctions. The rule adopted in some Courts to apportion the ground-water among land owners in accordance with the doctrine of correlative rights will not provide a universally satisfactory solution because in many localities the supply of ground-water available is inadequate to fulfill the reasonable needs of all land owners and, especially in industrial areas, there may be no relation between the actual reasonable needs of different individual land owners and the area of lands owned by them, respectively. If this doctrine were interpreted in such a way as to limit each user to no more than his *pro rata* share of the ground-water on a basis of area of land owned, in some localities such share would be so small that no ground-water development would be practicable; and this great economic resource would largely go unused. Therefore, if the ground-water resources are to be utilized most beneficially it is believed to be essential that a doctrine of appropriation and beneficial use sufficiently flexible to be adaptable to the conditions of each locality is the only practicable solution. It is realized that adoption of this doctrine in some States will encounter divergences of opinion and conflicts of interests among various groups which must be most carefully and sympathetically reconciled, but it is not believed that any insurmountable obstacles exist.

Attempts at, or Lack of, Control in Eastern States.—In the Grand Prairie region of Arkansas for about 20 yr from 100 000 to 160 000 acres of rice land has been irrigated with water from wells[20]. Since 1927, the water level in

[19] "English Law Does not Control Ground-Water Draft", by H. J. F. Gourley, *Water Works Engineering*, Vol. 89, pp. 1294–1297, September 30, 1936.

[20] "Ground-Water Supplies for Irrigation in the Grand Prairie Region, Arkansas", U. S. Dept. of the Interior (Geological Survey) Press Memorandum, January 26, 1931, 21 pp. (mimeographed); also, observations continued since this report was issued.

observation wells has dropped to lower and lower levels in successive years and the evidence seems indisputable that the safe yield of the beds has been exceeded. The area of land that is suitable for irrigation is much larger than that actually irrigated in any season, and conceivably at some future time, with conditions of price and other factors favorable to the rice farmer, there may be a great increase in the use of ground-water for irrigation in the Grand Prairie region.

Arkansas has no law providing for the control of ground-water, and so far as the writer knows, there have been no Court decisions upon the subject within the State. The available scientific facts disclose that limitation of the use of ground-water in the Grand Prairie region is desirable if not necessary to protect an important agricultural development. With conditions as they now exist in the region, undoubtedly it would be necessary to recognize the rights for water already developed, and if these rights exceed the safe yield of the aquifers, to require a proportionate decrease in use by present water users to bring the total consumption within the safe yield.

Within a radius of 25 miles of Houston, Tex., at least 80 000 000 gal of water per day is used for public supply and industrial purposes and irrigation. It is believed that the pumpage has not yet exceeded the safe yield of the aquifers, but there may be larger future development of ground-water, which (at least as far as mechanical problems are concerned) can be obtained from properly constructed wells. In this region, as in other areas in Texas, to protect the large investments already made, regulation of the use of ground-water is desirable.

There has been great lowering of artesian head in a large area around Chicago, Ill.[21]. Less severe drops have been reported from other parts of Illinois. It would seem that some control in this area would also be desirable.

Ground-water is used for irrigation in several areas of the Florida Peninsula and in certain areas some wells already yield salt water. There is reason to believe that any considerable increase in draft will cause further contamination. In an attempt to give some protection to the Gulf Coast area, the Florida Legislature in 1929 passed a law relating solely to Sanford, Manatee, and Charlotte Counties[22]. An effort has been made, but is as yet (1936) unsuccessful, to pass a law applying to the entire State. The present law defines waste from wells, sets up provisions for the prevention of waste, and violation of these provisions is a misdemeanor subject to the penalties prescribed by law. It also makes provision for inspection and supervision by the State Geologist of drilling operations in order to prevent leakage of water, known in some places to be salty, from one aquifer to another. The law further states that,

"Should the underground-water supply of any district become so depleted by heavy draft as to endanger the water supply of such district by saline contamination, the State Geologist, his assistants, or duly authorized representa-

[21] "The Artesian Waters of Northeastern Illinois", by Carl B. Anderson, *Bulletin 34*. Ill. State Geological Survey, pp. 93–95, Fig. 3, and Pl. IV, 1919.

[22] The text of the law is printed in "Ground-Water Resources of Sarasota County, Fla.," by V. T. Stringfield, Twenty-third and Twenty-fourth Annual Repts., Florida State Geological Survey, pp. 179–181, 1933. (This paper is also printed separately.)

tive, shall be empowered to regulate the draft upon all wells in such district so that the water resources can be protected and preserved."

This provision, however, relates only to the prevention of salt-water contamination. There is no provision of funds to enforce the law, nor for bond or other insurance that a well owner or driller will properly plug or otherwise control a leaking or flowing salt-water well. The prevention of leakage from one formation to another or of flow around the outside of wells has been difficult and costly, and laws should definitely require a bond by the driller or owner to assure proper protection of other well owners in areas where these difficulties are likely to be encountered.

In New Jersey there has been a partial control of ground-water by the State since 1907. This has been accomplished by laws which provide that new or additional developments of water, either surface or underground, for public supply, by municipalities or privately owned water supply companies, can not be made until the supervising commission (at present (1936), the State Water Policy Commission) has approved the plans for such development.

In the State of New York there is a control of public water supply systems exercised by the State Water Power and Control Commission, very similar to that in New Jersey. The power seems to be limited to an approval of plans and not to the granting of water rights. In 1933, the New York Legislature amended the law to provide that,

"* * * no person, firm, or corporation shall hereafter install or operate any new or additional wells in the counties of Kings, Queens, Nassau and Suffolk to withdraw water from underground sources for manufacturing or industrial purposes or for use in manufacture or industry where the capacity of such well or wells singly or in the aggregate is in excess of one hundred thousand gallons a day without first obtaining the approval of the Commission. * * * The provisions of this act shall not apply to the use of water for agricultural purposes.'

The four counties mentioned constitute all of Long Island. The first section of the law, after referring to reports by State and Federal agencies in regard to depletion of the underground waters of Long Island and the danger of encroachment of salt water, states that "this enactment is made in the exercise of the police power of the State and its purposes generally are to protect public health and public welfare in conserving the supply of water for domestic consumption."

Parenthetically, it may be noted, in contrast, that an Hawaiian Court expressed the opinion that the police powers of the Territory "cannot be deemed to justify, under the showing made in this case, the prohibition of the appellant's proposed well while at the same time permitting all existing wells to continue to be operated without diminution"; but it did recognize that the Territory could exercise the police power to the extent of regulating the exercise of "private rights to the end that the health and safety of the community may be preserved and that the right of co-owners may not be violated." (City Mill Co., Ltd. *v.* Honolulu Sewer and Water Commission, 30 Haw. 912.)

In both New Jersey and New York favorable action by the controlling commission apparently does not constitute a grant of any right to use the

water, but merely an approval of plans, etc. Nevertheless, the laws in these States have had the effect of controlling to some extent the development of ground-water supplies in certain areas where there is danger of the safe yield being exceeded, either by actual refusal of approval, or by modifying provisions as to location of wells or quantity to be withdrawn.

In both New Jersey and New York, except Long Island, the law does not apply to diversions of water by industries or others using the water for purposes other than public supply systems. This distinction has brought about a peculiar situation in that upon showing that a proposed development for municipal supply may injure an existing private supply the controlling commission may refuse, and (at least in New Jersey) has refused, to approve the public development; but, on the other hand, after a mnuicipal development has been completed, a private development may be made which may greatly reduce the capacity of the wells of the public system or perhaps even destroy its usefulness. A similar situation in England has been cited by Mr. Gourley[19].

Confusion in Classification of Ground-Waters.—There is considerable inconsistency or confusion in thinking concerning certain phases of the situation of control, or lack of control, of the use of ground-water as it now exists. These conditions are largely the cause of the present mixed, and to large extent unsatisfactory, status of legal control. This situation doubtless is due in part to the fact that the legal principles were established before there had been developed an adequate knowledge of the geologic and hydrologic conditions that govern the occurrence of ground-water; but even after these conditions have become better known, the hydrologists have, perhaps, not presented them clearly to the Courts and legislative bodies. It may be said that some progress is being made, but there is still room for more enlightment.

An outstanding confusion of ideas exists in the distinctions or classes into which underground waters have been divided from a legal viewpoint. In many cases the question arises as to whether the water under consideration is ground-water or part of a surface stream. From the viewpoint of many hydrologists this is a starting point of ambiguity and uncertainty. In his discussion of ground-water law, Mr. C. S. Kinney divides underground waters into three general classes, namely, "subterranean water courses or streams", "artesian waters", and "percolating waters."[23] The first class is subdivided into water flowing in "known and defined channels or water courses", or in "undefined and unknown" courses. Artesian waters apparently were originally placed in the class of "percolating waters", but as their characteristics became better known a separate class was made for them. Percolating waters have been subdivided into: Diffused percolations or percolating waters; percolating waters tributary to surface watercourses or other bodies of surface waters; percolating waters tributary to underground reservoirs or other bodies of underground waters; and, finally, seepage waters. Other subdivisions include "underflow dependent on surface streams", "percolating waters tribu-

[23] "A Treatise on the Law of Irrigation and Water Rights and the Arid Region Doctrine of Appropriation of Water Rights", by C. S. Kinney, Bender-Moss Co., San Francisco, Calif., Second Edition, p. 2095, 1912.

tary to springs", and "percolating waters supplying surface wells" (as distinguished from waters supplying artesian wells).

There is no doubt that, at the mention of the term, "known and defined subterranean channels or water courses", most geologists visualize streams flowing in channels with definitely identifiable banks, such as are found principally in solutional channels in limestone caverns (for example, the Echo River in the Mammoth Cave of Kentucky); or of so-called tunnels in lava flows, or in smaller openings with definite walls such as crevices, rather than minute interstices in sand and gravel, even is the water-bearing beds are beneath a definitely recognizable surface stream. On the other hand, the geologist would consider the water flowing in alluvium beneath a stream bed to be percolation just as much as water moving in an artesian aquifer, or ground-water tributary to surface streams or underground reservoirs is percolating water. (The Standard Dictionary defines "percolate" as to cause, as a liquid, to pass through fine interstices; filter; strain. It then defines "percolation" as the act of percolating or passing through small interstices, infiltration. Furthermore, The Standard Dictionary defines "channel" as: (1) The bed of a long body of water, especially the hollowed course of a stream; (2) the deep part of a river, harbor, straight, or estuary where the current or tide is strongest; etc. It defines "watercourse" as: (*a*) A stream of water; river; brook, especially, in law, a stream usually flowing (although not necessarily running all the time) in a definite channel, having a bed and banks; (*b*) the course or channel of a stream of water. The definition in Webster's Unabridged Dictionary is in essentially similar language.)

Further difficulties arise when one attempts to discover the distinction, for example, between underflow, percolation, and artesian water. In many localities, where a stream flows over a considerable thickness of permeable alluvium confined in a rock-walled canyon, most, if not all, Court decisions recognize underflow as part of the surface stream. The question becomes more complicated where the stream flows across a wide alluvial plain of essentially homogeneous material, so that the stream is continuous with the water-table beneath the plain for many miles on either side of it, the stream being at about the lowest part of the water-table. Perhaps there would be no question as to the portion of the ground-water body directly beneath the stream, provided the course of the stream is fairly straight; but what would be the determination where the stream, instead of flowing in a reasonably straight line, meanders in large ox-bow curves several miles in diameter across a wide plain as does the Mississippi River, and other rivers in wide alluvial filled valleys? In this case the movement of water in the lowest part—the thalweg[24] of the water-table—may be expected to be in a rather straight line toward the mouth of the stream and not parallel to and beneath the meanders of the stream. Thus, although certain drops of water may at one place be directly beneath the stream bed and, therefore, might be called underflow, upon reaching and passing down stream beyond a sharp sideward meander of the stream, they apparently cease to be "underflow" in the sense of water flowing

[24] "The Motions of Underground Waters", by C. S. Slichter, *Water Supply Paper 67*, U. S. Geological Survey, p. 32, 1902.

beneath a stream bed, and become "percolating water" which, in its journey to the sea, may reach a point several miles from the river before again passing under it.

In both the foregoing cases there is a question as to where boundaries shall be drawn to divide the underflow from the "percolating" water beneath the plains on either side of the river. In many localities this is hydrologically impossible because, at least in part, percolation laterally from the plains to the river maintains the underflow and at different seasonal periods or in different sections of the river system, this direction of movement may be reversed. In some localities it can be demonstrated both by studies of the geological history of the regions in question and by hydrological data, that the underflow is derived largely, if not wholly, from such lateral percolation, and not from the surface stream as is stated by Mr Kinney to be the legal presumption both under the common law doctrine and under the doctrine of appropriation[25]. In fact, the usual geological and hydrological conception of the relations between a permanent surface stream and the ground-water is that the stream comes into existence only after erosion has cut a channel deep enough to intersect the top of the zone of saturation or water-table. This view is in agreement with Mr. Conkling (see "Introduction") who uses the title of "underflow which may support a surface stream." The theory stated by Mr. Kinney that the stream supplies the underflow can be demonstrated completely only where the stream is influent; that is, where the water-table is some distance below the bottom of the stream and water percolates from it downward to the ground-water body. In this case it might be well argued that the person who diverted the surface water was taking water that would otherwise naturally supply the ground-water reservoir of percolating water.

Mr. Conkling has described certain conditions under the headings of "Definite Underground Channels" and "Percolation Through Basins." Presumably, these conditions comply with those described by Kinney as waters flowing in subterranean streams or watercourses and percolating waters, respectively. It will be noted that Mr. Conkling states ("Definite Underground Channels"): "If the underflow of a stream * * * is regarded as a definite underground stream, it may be said that such streams, usable for irrigation, often occur in the West" This suggests that there is some doubt in his mind as to what really constitutes a definite underground stream. Under the heading of "Percolation Through Basins", he describes the general hydrological features of the San Gabriel Basin. The writer does not know whether any Court decision has classified the movement of ground-water in the San Gabriel Basin as "percolation", but he believes that it is a proper classification according to the customary conception of that term as it is commonly used except perhaps for minor parts of the basin where certain geological conditions produce special conditions such as small artesian basins. This general classification is supported by maps of the water-table which show

[25] "A Treatise on the Law of Irrigation and Water Rights and the Arid Region Doctrine of Appropriation of Water Rights", by C. S. Kinney, Bender-Moss Co., San Francisco, Calif., Second Edition, pp. 2111–2115, 1912.

it to be continuous, with gradual slopes over a large area, and with no demarcation or indication of difference in water-table conditions beneath the wide alluvial wash channel of San Gabriel River and lands that extend for several miles on either side of the present channel[26].

San Gabriel Valley may be compared with San Fernando Valley. As far as the writer can determine from the statement of facts in the case of City of Los Angeles *v.* Hunter (156 Cal. 603), and from other information, the geologic and hydrologic conditions in the latter valley are essentially the same as in San Gabriel Valley in that the ground-water occurs in a vast body of alluvium surrounded by bed-rock hills, with a water-table continuous over practically the entire valley. Contours[27] of the water-table show that the ground-water for practically the entire valley is moving toward the locality where the Los Angeles River, which rises in the valley, passes through "narrows" with conditions essentially similar to the pass traversed by San Gabriel River between San Gabriel Valley and the Coastal Plain as described by Mr. Conkling; and yet, in the case of City of Los Angeles *v.* Hunter, the Court decided that water pumped from wells (some of which were 1 000 ft distant, but others 2 or 3 miles distant, from the Los Angeles River) was a part of the subterranean flow of that river and was not percolating water.

In another case relating to San Fernando Valley (City of Los Angeles *v.* Pomeroy (124 Cal. 597)) the difficulty in distinguishing between water in subterranean channels and percolating water is just as evident and the reasoning of the Court is confusing. In this case the decision of the Supreme Court quotes the instructions to the jury by the Superior Court. In Paragraph XVII of these instructions the Court stated certain physical conditions which, if they were considered to fit the case, should require the jury to decide the waters of certain tributary streams of the valley to be a part of Los Angeles River. In Paragraph XIX were other statements to show under what conditions the ground-water would not be in a definite watercourse, but would be percolating water. Both statements seem to fit the conditions in San Fernando Valley in so far as they go, and reading either of them alone one would conclude that the ground-water should properly be classified by that statement. In neither this case nor in Los Angeles *v.* Hunter (156 Cal. 603), does the writer find any discussion of the critical geological or hydrological factors that really would determine whether the subterranean channel or watercourse flowed in a definite, confined channel, with "beds, banks, or sides."[28] Such pertinent evidence should show either definite differences in the character of the water-bearing materials constituting the supposed subterranean channel of the river from the materials on either side that would be considered to constitute the banks; or it should show that the water-table or piezometric surface of the water in the materials constituting the banks and

[26] "Ground Waters and Irrigation Enterprises in the Foothill Belt, Southern California", by W. C. Mendenhall, *Water Supply Paper 219*, U. S. Geological Survey, Pl. VII and IX, 1908; also, "'South Coastal Basin Investigation—Geology and Ground-Water Storage Capacity of Valley Fill", by Rollin Eckis, *Bulletin 45*, California Div. of Water Resources, Plates D and E, 1934.

[27] *Loc cit.*, Rollin Eckis, Plate E.

[28] City of Los Angeles *v.* Pomeroy 124, Cal. 597, Paragraph XIX of charge to jury by Superior Court, quoted in the decision of the Supreme Court.

sides of the watercourse was discontinuous with the level of the surface water in the stream or in the water-table beneath it.

It would seem that, from the decision of Los Angeles *v.* Hunter and Los Angeles *v.* Pomeroy, by analogy, logically all the water in the vast underground reservoir of San Gabriel Valley, which eventually passes through the Whittier Narrows either as surface flow of San Gabriel River or underflow beneath it in a restricted channel, should be considered to be part of that stream. Mr. Conkling has pointed out ("Percolation Through Basins") how, with great increase in pumping from wells the quantity of "rising water" at the Narrows (that is presumably, the low-water flow of San Gabriel River) has greatly decreased; and this very fact suggests a relation between the body of "percolating water" and the river similar to that determined by the Courts for Los Angeles River in San Fernando Valley.

Since a large part of the firm flow of practically all streams of importance in the United States comes from the ground-water reservoir, from the foregoing reasoning it can also be argued that ground-water almost anywhere is a part of the nearest surface stream toward which the water-table slopes. The obvious consequence of this argument would be that riparian owners of surface waters might compel abandonment of many wells now diverting such percolating water. For most parts of the country this interpretation may seem absurd, but for certain regions, conceivably, it may have application, as, for example, along certain stretches of the Mohave River, in California, the Rio Grande in New Mexico and Texas, and the Platte River in Nebraska. On stretches along each of these rivers there is undoubtedly contribution of ground-water to the stream from lands some miles back from it.

In its length of more than 100 miles the Mohave River, in California, is alternately, here an effluent and there an influent stream, with stretches in which most of the time there is a surface stream and farther down stream no surface flow. Probably most of the flow comes from the San Bernardino Mountains, but undoubtedly some of the water moves ("percolates") toward the river beneath the bordering "mesas" or alluvial slopes that reach back to the mountains of the desert. In some, if not in all, of the stretches, both influent and effluent, the underflow might be considered to be part of the river. There is reliable evidence that, in a stretch between Hodge and Barstow, Calif., some of the underflow of the river—which is probably augmented by percolation from the surface flow when there is water in the river—leaves the valley of the Mohave River underground and moves northwestward beneath an area known as Hinkley Valley, past a surface divide and toward an undrained valley containing a "playa" known as Harper Dry Lake[29]. Apparently, at least in years of normal run-off, none of this ground-water that leaves the valley of the Mohave River, appears on the surface again, but is disposed of by evaporation from the soil and transpiration by plants. The question may now be asked: When it reaches Harper Valley, is this water still to be considered as part of the Mohave River although it is entirely

[29] "The Mohave Desert Region, California", by David G. Thompson. *Water Supply Paper 578*, U. S. Geological Survey, pp. 428–429, 1929; also "Mohave River Investigation", by Harold Conkling, M. Am. Soc. C. E., *Bulletin 47*, California Div. of Water Resources, pp. 27–28, and contours of water-table on Pl. 5-B, 1934.

outside the drainage basin of that river; although subsequently it does not reach any other stream; and, although it moves in a manner entirely similar to "percolating water" in numerous basins of the desert region? The answer might be different, depending upon whether a down-stream user along the Mohave River sought to enjoin a user in Harper Valley, or a Harper Valley user sought to enjoin an up-stream user along the Mohave River.

One more example may be cited to show the difficulties of attempting to set up definite distinctions between different types of occurrence. According to co-operative studies by the Nebraska State Geological Survey and the U. S. Geological Survey, between Chapman and Gothenburg, Nebr., about 30 000 acre-ft of water is pumped from wells on the flood-plain of the Platte River[30]. The writer does not know whether any Court decisions have defined this water in Nebraska, but under similar conditions in Colorado, it would certainly seem to be considered a component part of the Platte River The field studies have demonstrated beyond a doubt that a large quantity of water, estimated to be about 56 000 acre-ft per yr, percolates southward out of the Platte River Valley and beneath a high upland from a few to as many as 15 miles wide and re-appears in the tributaries of the Republican and Blue Rivers which flow at considerably lower altitudes[31]. The studies indicate that more water for irrigation can be taken from wells in the Platte River Valley as well as on the upland plains. If such development occurs to the extent that the flow of the Republican and Blue Rivers is seriously diminished, shall the ground-water beneath the upland (which normally would be considered to be "percolating water") and also the water used in the Platte River Valley, be considered to be part of the Republican and Blue Rivers? A case recently decided (Osterman *et al v.* Central Nebraska Public Power and Irrigation District, 268 N. W. 334), relates to the diversion of surface water from the Platte River into the drainage basins of the Blue and Republican Rivers. In this case the probability of underflow from the Platte River to the basins of the other rivers was recognized, but the ground-water conditions apparently were not specifically considered in the recent decision.

The foregoing examples have been cited primarily to point out inconsistencies in the reasoning by which certain principles of classification of ground-water have been set up in the course of a long period of litigation. Apparently, the complicated classification given by Kinney has been found necessary as case after case came up for decision in which the geological and hydrological conditions did not exactly fit the conditions of some previous case. He seems to have discovered the essential difficulty of the entire problem when he wrote, concerning the criteria for determining the classification of ground-waters[32]: "The difficulty has been not so much to determine the law

[30] "Ground-Water Resources of South-Central Nebraska with Special Reference to the Platte River Valley between Chapman and Gothenburg", by A. L. Lugn and L. K. Wenzel, U. S. Geological Survey. (Report released in manuscript form and waiting publication, mimeographed preliminary Interior Dept. Notice 106376, September 16, 1935.)

[31] For a cross-section showing this condition see "Water-Bearing Formations of Nebraska", by G. E. Condra and E. C. Reed, Nebraska Geological Survey *Paper No. 10*, p. 17, 1936.

[32] "A Treatise on the Law of Irrigation and Water Rights and the Arid Region, Doctrine of Appropriation of Water Rights", by C. S. Kinney, Second Edition, p. 2115, Bender-Moss Co., San Francisco, 1912.

as to rights which may be acquired in subterranean waters, where certain water has been determined to be of a certain character, as it has been to determine the character of the particular class to which the water belongs." This difficulty may be expected to become greater as still other cases, with peculiar but perfectly natural conditions, as in some of the areas suggested in the paper, come up for decision. The very existence of this difficulty raises a question as to whether it may be due to the fact that the classification that has developed in past cases, as outlined by Kinney, is not on a sound basis of geological and hydrological facts. All water beneath the surface of the ground, after all, is purely and simply ground-water, moving according to certain well recognized laws of physics. There seems to be no scientific reason why an elaborate and expanding classification of ground-waters should be necessary

One may wonder as to how the aforementioned confused state of affairs came about. It can be attributed to lack of knowledge of the geologic and hydrologic principles that govern the flow of water beneath the surface of the earth. It is not surprising, of course, that many facts now well known in regard to the occurrence of ground-water were not so widely understood when the early case relating to percolating water cited by Wiel[33] (Acton *v.* Blundell (12 Mees and W. 324)) was decided in 1843, although they were not wholly unknown even then. Some essential facts were known to a few scientific observers[34], but undoubtedly these ideas had not been very widely circulated. Unfortunately, even to-day not a few persons believe that subterranean water actually occurs in more or less distinct and open bodies with free surface similar to lakes and rivers that are seen above ground. The conception of "definite subterranean channels or watercourses" probably at least in part arose from this erroneous belief.

So far as percolating water is concerned the theory set up in early cases likewise rests on lack of knowledge of ground-water conditions. In Los Angeles *v.* Hunter (156 Cal. 603), the Court stated that percolating waters, in the common law sense of the term, were "vagrant, wandering drops moving by gravity in any and every direction along the line of least resistance." This judgment was in contrast to the Court's conception of water moving in a definite subterranean channel or watercourse of which it said,

"These waters percolate, it is true, but only in the sense that they form a vast mass of water confined in a basin filled with detritus, always slowly moving downward to the outlet, in the effort, in conformity with physical law, to attain a uniform level."

As a matter of fact, with the present knowledge of ground-water hydrology, it may be stated with considerable emphasis that practically all ground-water conforms with the requisite conditions set up in the last quotation because, until diverted by artificial means, the ground-water flow, in conformity with physical laws, at any given point is moving in a definite direction as a result of the interaction of geological and hydrological conditions, and, except for

[33] "Water Rights in the Western States", by S. C. Wiel, Second Edition, Vol. 2, p. 970, 1911.

[34] "History and Development of Ground-Water Hydrology", by O. E. Meinzer, *Journal*, Washington Acad. of Science, Vol. 24, pp. 14–16, 1934.

loss by transpiration and evaporation, nearly all ground-water is moving to maintain the flow of surface streams Furthermore, the direction and rate of movement can be determined within reasonable limitations of accuracy, although this may not everywhere be practical because of the cost involved.

If the foregoing arguments are tenable, namely, that from a logical continuation of the reasoning stated by the Court in such cases as Los Angeles *v.* Hunter and Los Angeles *v.* Pomeroy, it would seem to be a further logical conclusion that since nearly all ground-waters tend to maintain surface streams, they should be considered waters in subterranean channels, and thus practically all ground-waters should be subject to the laws that govern surface waters Under this reasoning, in States in which the doctrine of riparian rights now applies to surface water, this doctrine would logically be applied to ground-waters. In these States under a strict interpretation of the doctrine of riparian rights it would seem that the use of ground-water would be essentially prohibited unless all well owners were considered to have riparian rights equal to the riparian owners along the surface streams. At best, it would mean the application of the doctrine of correlative rights (see heading "Water Law in General"). On the other hand, in States in which the doctrine of appropriation is now applied to surface waters that doctrine would also be applied to all ground-waters, with consideration given to the question of the effect of their diversion on surface water rights already granted.

It may be stated that the application of the doctrine of appropriation to the control of ground-waters in certain areas in New Mexico is based on the scientific evidence of the movement of the water from its source to points of discharge in areas, which, although they may be of large extent, have boundaries that can be, and have been, definitely ascertained. To this extent there is recognized a similarity to "definite subterranean channels or water-courses." Thus, by recognizing definite boundaries and a definite direction of flow the doctrine of appropriation, which has been applied for many years to surface waters in New Mexico, has now been applied to ground-waters in areas in which, as pointed out by Mr. Conkling ("Underground Water Law in General"), the water occurs under conditions that formerly would have been considered to require a classification as "percolating water."

It is pertinent, perhaps, to state that the New Mexico law was first drawn up following an extensive study by the U. S. Geological Survey, of ground-water conditions in the Roswell Basin, to which the law was first applied[35]. This study showed that the safe yield of the artesian reservoir had been exceeded and that any increase in the use of ground-water would jeopardize the investments of prior users; and it recommended legislation to prohibit new developments of artesian water except as it can be shown that such developments will not injure present users of water. The information collected in the investigation furnished the necessary basis for the determination and declaration by the State Engineer that the Roswell Basin comes within

[35] "Report on Investigations of the Roswell Artesian Basin, Chaves and Eddy Counties, New Mexico", by A. G. Fiedler, Seventh Biennial Rept. of the State Engr. of New Mexico, pp. 21–60, 1926; also, "Geology and Ground-Water Resources of the Roswell Artesian Basin, New Mexico", by A. G. Fiedler and S. S. Nye, U. S. Geological Survey *Water Supply Paper 639*, 1933.

the purview of the new law in so far as the water is in an "artesian basin * * * having reasonably ascertainable boundaries." It is worthy of note that the law has indeed accomplished its objective in the Roswell Basin, and this is perhaps the outstanding instance of limitation of use of ground-water by law in the United States.

Doctrines of Correlative Rights and Appropriation.—The writer wishes to make himself clear that in what he has written thus far he is not suggesting what he believes should be a proper basis for the determination of water rights. He has merely endeavored to present what he believes must be logical conclusions if certain assumptions in legal decisions are accepted. At this point it is desirable to consider another phase of the problem, namely, the apportionment of water among land owners under the doctrine of correlative rights.

It is first necessary to consider just what is meant by the doctrine of correlative rights. Mr. Conkling states ("Underground Water Law in General") presumably referring to the doctrine of correlative rights, but not using the term specifically in the sentence quoted: "This rule holds that each land owner overlying a basin has a right to the underground water co-equal and correlative with that of all other land owners overlying the same basin." This interpretation is considered by some to require, in part, that if there is not sufficient water for unlimited use by all owners of land overlying water-bearing strata, there shall be a proportionate distribution among all. This view is stated, apparently as dictum of the Court—that is, mere discussion upon some collateral question of law not necessary to the decision of the case before the Court—and not as a part of the actual decision, in several cases, notably Katz *v.* Walkinshaw (141 Cal. 116 (especially p. 136)); and City Mill Co., Limited *v.* Honolulu Sewer and Water Commission *et al* (30 Haw. 912 (especially p. 922)).

The writer is aware that this view is not shared by others. Some appear to consider the doctrine of "correlative rights" as applied to ground-waters to be synonomous with the doctrine of "reasonable use." (See such expressions as the following in which the two phrases are used together, quoted from the annotation appended, Clinchfield Coal Corporation *v.* Compton (55 A. L. R. 1376): "a reasonable exercise of such right; or as said by the Court, the rights are correlative"; "* * * in Katz *v.* Walkinshaw, 1902 (141 Cal. 116, 64 L. R. A. 236, 99 Am. St. Rep. 35, 74 Poc. 766, 80 Poc. 633), the first case in which the rule of reasonable use applied to correlative rights is applied"; "the American rule, or rule of reasonable use, or as expressed by some Courts and writers, the rule of 'correlative rights'"; and, finally, in Horne *v.* Utah Oil Ref. Co. (59 Utah, 279, 31 A. L. R. 883, 202 Poc. 815), "the doctrine of correlative rights or reasonable use." On the other hand, in at least one case the Court recognized a distinction between, and defined, these two doctrines, separately. (See heading "Underground Water Law in General"; also, City Mill Co., Limited *v.* Honolulu Sewer and Water Commission *et al* (30 Haw. 912 (especially p. 922)). In this case a definite part of the doctrine of reasonable use was stated to be that the water shall be used on "the owner's land" (in cases in other jurisdictions limited to lands of the owner overlying the

water-bearing strata). As a matter of fact, in many, if not all, cases where the doctrines of reasonable use or correlative rights (whatever the application of this term may be) have been considered, water was taken for sale by public supply systems or for other use on lands distant from those beneath which the water was found; and this has generally been held to be an unreasonable use (Clinchfield Coal Corp. *v.* Compton (55 A. L. R.), annotation p. 1404). The writer has failed to find a single case in which there was actually decided by the Court the question of division of water among owners of land overlying the water-bearing strata when the supply was insufficient for all desired use, without the collateral question of use on distant land. Therefore, it is probable that the meaning of the term, "correlative rights", as applied to such a condition of insufficient supply, is still an open question except as Courts subsequently may be influenced by the dicta in the aforementioned cases.

Next, there is to be considered the doctrine of correlative rights from the viewpoint of ground-water hydrology. Of the total quantity of ground-water that flows beneath the average parcel of land, only part of it originates on that land and much, if not most of it, originates on lands farther up the ground-water gradient. In the case of an artesian aquifer covered by an impervious bed beneath the entire parcel no water would be contributed from rainfall on that parcel, and it would all move in from lands beyond its boundaries, perhaps from an outcrop many miles away. In fact, an artesian aquifer, perhaps, is one of the best examples of a condition that, by stretching considerably the geological concept, might fit the description of a subterranean watercourse with definitely bounded walls.

The supply of ground-water available in any area is dependent upon the rainfall either in the immediate area, or in some other area, perhaps fairly remote, from which it moves to the area in question either in a surface stream, which loses water to the underground reservoir, or through some underground aquifer. In many localities, particularly in the arid and semi-arid States, ground-water is available in large quantity only where geologic conditions are such as to concentrate part of the precipitation or surface run-off from large areas, or areas of heavy precipitation, into ground-water reservoirs of comparatively small size, or where large reservoirs have been filled by infiltration from small areas continuing through long geologic periods. Under certain conditions the available supply is also limited by the permeability of the water aquifer; that is, the rate at which it can transmit water. As a consequence of these factors, there is a definite limit to the safe yield of water that can be taken perennially from wells in any locality. This limit has been reached and passed in several parts of the United States, although the supply may not yet be exhausted. In many other parts of the country this limit is far from being reached and, at present, gives no concern. In some areas it probably never will be reached. However, it seems necessary that the possibility of it being reached must be recognized in any effort to provide adequate control of ground-waters. This is particularly so where the possible use of ground-water might greatly exceed the safe yield of the water-bearing formations.

In Antelope Valley, California, for example, 200 000 acres or more is suitable for irrigation, but the annual contribution to the ground-water supply is of a magnitude of only about 50 000 acre-ft[36]. If this quantity were distributed over 200 00 acres it would be only 0.25 acre-ft per acre, which is not enough to do material good in irrigation. Obviously, if the water were divided among all lands proportionately under the doctrine of correlative rights, as suggested by the Court in Katz *v.* Walkinshaw, much of the land would be nearly worthless. Prior beneficial users would be insecure. Only by a reasonable application of the doctrine of appropriation can the maximum economic use of water be obtained in Antelope Valley.

In the Borough of Brooklyn, New York, a number of industrial concerns pump large quantities of water from wells on their property, and as a result there has been a serious drop in the water-table. However, if all industries of the Borough undertook to pump from wells, claiming a share of water under the doctrine of correlative rights as interpreted in Katz *v.* Walkinshaw, either salt water would quickly be drawn in, spoiling the water for many purposes, or, if the water was rationed in such a way as not to exceed the total safe yield, each owner's share would be so small that if no other supply were available many of the industrial concerns could not operate. Although the principle of correlative rights may be said to hold in the humid States, in many localities where much ground-water is used the actual situation is that there is no equitable distribution of water in proportion to ownership of land, but the water is available primarily only to those best able to pay for the deepest wells or the largest pumps.

Another important fact, which is not generally appreciated, is that, as stated by Mr. Conkling under "Administration: Underground Water", it is impossible to take water from any well either by natural flow from an artesian well in which the static head is above the surface, or by pumping from wells in which it is below the surface, without causing a drop in head, or static level, beneath the territory surrounding the well. Theoretically, this drop in head should extend ultimately to the outermost borders of the ground-water body under consideration[37]. The loss of head resulting from the withdrawal of water from several wells if within the cones of influence of each other, may be significant in amount over a large area, perhaps many square miles. Some loss of head can not be avoided even if the quantity of water withdrawn is only a small part of the total safe yield of the aquifer; and if a considerable part of the safe yield is to be obtained in some regions there must be a considerable loss of head. It should be distinctly understood, however, that loss of head does not necessarily mean that the permanency of a well owner's supply is endangered. These facts are of interest in connection with the situation in Idaho, where, as pointed out by Mr. Conkling ("Summary, Discussion, and Conclusions (Item 15")) a strict interpretation of the Court's

[36] "The Mohave Desert Region, California", by D. G. Thompson, U. S. Geological Survey *Water Supply Paper* 578, p. 322. 1929.

[37] In this connection see "The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge in a Well Using Ground Water Storage", by C. V. Theis, *Transactions*, Am. Geophys. Union. Pt. 2, pp. 519–524, 1935.

reasoning in Silkey *v.* Trego, (5 P. (2d), 1049), would prohibit every one from using ground-water except the first appropriator.

Examples of Lack of Appreciation of Fundamental Hydrologic Principles.— The failure to appreciate these facts has resulted in some unfortunate situations. Several years ago the writer had an opportunity to read the transcript of evidence in a case in a Western State. It was a striking fact that in this testimony wells were said to have gone "dry" when the water had merely ceased to flow over the top of the casings at or near ground level. Apparently, it was not evident to counsel or expert witnesses for the defendant that the wells were, in fact, not dry, and that it would only be necessary to install pumps in the wells to obtain as much water as, or more than, was originally obtained by natural flow. The failure to understand this simple hydrological principle is almost unbelievable, but it is borne out by other evidence that in the same region until a few years ago both well drillers and water users did not understand other essential principles to be considered in the construction of wells and developments of ground-water.

In 1935, the Colorado Legislature passed a law (Chapter 80) which provided that "it shall be unlawful for any person, persons, or corporations to pump water from an artesian well for any purpose or purposes whatever except for domestic or manufacturing purposes." An artesian well was defined as a well "the waters of which, if properly cased, will flow continuously over the natural surface of the ground adjacent to such well at any season of the year." The Act was worded so that apparently it was intended to apply principally to a particular area in which there are numerous flowing wells used for irrigation. As written, the law obviously prohibits pumping, but does not prohibit the drilling of a well that may perchance flow. The framers of the law appear to have been unaware of, or to have overlooked, the fact that the damage they sought to prevent by prohibiting pumping can be brought about by drilling flowing wells of large capacity in parts of the artesian basin where the head above the ground surface is high; for, as stated previously, a cone of depression, or an area of influence and interference, is created when a well with artesian head above the ground surface is permitted to flow just as much as when a well with head below the surface is pumped.

According to the definition of artesian well in this law, a well that may be near the border of the area of flow and which does not flow at any season of the year is not an artesian well and not affected by the law. Its owner can put in a pump and operate it to his heart's content, creating a considerable drop in head in wells in the near-by "artesian" part of the basin. On the other hand, a man whose well is situated just inside the "artesian" area, so that it flows in some seasons of the year and not at others, or one whose well is still farther inside the "artesian" area, so that the well flows continually but in some seasons the flow is very small (perhaps only a few gallons per minute) can not install pumps to obtain water from his wells. Since the lowest head, and hence the period of low flow or non-flow, is generally in the height of the irrigation season, the owners of these wells are prohibited from pumping their wells to irrigate crops. It thus appears that if this law were applied strictly as worded it would in fact prohibit the

use of ground-water by certain established users in a zone between the truly non-artesian area where water may be pumped from wells and the part of the artesian area where adequate supplies may be obtained by natural flow.

In connection with the situations just cited, it may be noted that in some cases it has been ruled that one riparian owner, in taking water from a surface stream, has so changed conditions at the intake or other control works of another riparian owner in such a way as to make such works useless, the latter owner does not have cause for redress, but must remodel his works at his own expense. By analogy it may be argued that when one well owner deepens his well or pumps it heavily in such a way as to lower the water level in another owner's well, or even dry it up, the latter must deepen his well at his own expense, provided the former is not taking more than his share of the water. This has, indeed, been the conclusion in cases of record relating to correlative rights. The result, as pointed out by Mr. Gourley[19], is a race to construct the deepest well, in which the weakest party financially, who may be the earliest user, "gets left."

Ground-water laws in some States have been written very poorly, in part, because of lack of understanding on the part of the writers as to principles of ground-water occurrence, construction of wells, etc. The poor wording of laws applies, for example, to sections defining artesian wells. In some laws, this covers only wells that flow. The quotation from the Colorado law, previously cited, is an example. A definition in another law includes not only wells "through which water is raised or carried to or above the surface of the ground by natural pressure or gravity", but also wells "through which water is or may be raised or carried to or above the surface of the ground by artificial means." This definition would include practically any well from which water is taken, whether by natural flow, by pump, or even by bucket; and the term, "artesian", which here is used very loosely, is superfluous. The really significant hydrologic principle involved in a scientific definition of artesian water and artesian wells is that the water is under artesian pressure beneath an impervious stratum so that it will rise above the level at which it is struck, regardless of whether the water flows at the surface or how many impervious layers may be between the aquifer and the level to which the water will rise[22]. A State legislature, in general, has the right to define terms as it wishes. Such action, however, does not help in developing scientific accuracy that is sought by technical workers and it can not change the natural laws and principles involved or, by mere edict, rectify the difficulties that the law is designed to meet. One is reminded of the law introduced in a State legislature a few years ago which, in order to simplify geometry for students and engineers, said that hereafter and forevermore the value of π, the factor used to designate the relation of the circumference of a circle to its diameter, shall be 3.00.

Conclusion.—The writer has discussed what he believes to be inconsistencies and misconceptions that have grown up both in written laws and in Court decisions relating to the control of underground waters. The need of control is being shown more and more, not only in the arid and semi-arid

[22] "Outline of Ground-Water Hydrology", by O. E. Meinzer, U. S. Geological Survey. *Water Supply Paper 494* (definitions of zone of saturation and water-table, pp. 21–22, and definition of artesian water, p. 39), 1923.

States but also in the humid States. A more definite control must come some time. The longer it is deferred the more complicated will the problem become. It is desirable therefore, that action be no longer delayed.

What shall the method of procedure be? It is the writer's opinion (and this is shared by his colleagues in the Division of Ground Water of the Geological Survey who have given serious thought to the problem), that all lines of argument lead to the conclusion that the only feasible and reasonable method of control of the use of ground-water for any purpose can be under the doctrine of appropriation, with the necessary control exercised by some State official. The recognition of the doctrine of appropriation as applied to ground-waters "whether flowing in definite channels, or standing in lakes or basins, or percolating through the soil, in areas of which the boundaries may be ascertained" is the first of several recommendations in regard to the control of ground-water, submitted by an advisory committee of the National Land Use Planning Committee[39]. As indicated by Mr. Conkling's statement ("Underground Water Law in General: Utah") in regard to the 1935 law of Utah, in a draft of a proposed "uniform underground water law for Western States", a committee of the Association of Western State Engineers has also applied the principle of appropriation for beneficial use[40]. (The draft of this law in many respects is patterned after the New Mexico law).

The Committee recognizes that the principle is not now applicable in States wherein the doctrines of riparian rights or correlative rights are recognized. The two organizations mentioned have concerned themselves primarily with conditions in the arid and semi-arid States, but it is the writer's belief that similar action can be recommended both for the humid States and those arid or semi-arid States in which the doctrines of riparian rights and correlative rights are now recognized; and definite steps should be taken to make possible the application of the doctrine of appropriation for beneficial use in these States.

In some States it may be necessary to apply the doctrine of appropriation with modification, for example to exempt, or to give special preference to the comparatively small use of ground-water for rural domestic use and watering stock. It may be desirable to provide control only in areas where there is large use of water, by permitting the formation of water districts; but control should be initiated at the earliest possible time in any area where it seems likely that at any time in the future the supply will be deficient. It may be desirable to provide some means for water districts to acquire, for the common good, established rights in some parts of a ground-water basin, which, if they were utilized, would prevent a full and efficient use of the available supply. Thus, to satisfy early rights to a small quantity of water on high lands might mean the prevention of pumping which would lower the water level beneath such lands.

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[39] "Suggested principles of State legislation relating to the use of underground water. *Publication No. 3* (6 pp.) (mimeographed), prepared by Technical Advisory Committee on Reclamation, Drainage, and Irrigation Policies, W. W. McLaughlin, *Chairman*. The National Land Use Committee, organized in February, 1932, is not to be confused with the present National Resources Committee.

[40] "*Proceedings*, Seventh Annual Conference, Assoc. of Western State Engrs. (1934), p. 45.

How may the principle of appropriation be introduced into States where it is not now recognized? This is a problem for the law-makers and the Courts to decide, of course, but it seems not inappropriate that the ground-water hydrologist give thought to the problem. Perhaps, in some States, it can be done by passing a law stating that all waters of the State, whether above or under the ground, are the property of the public, and, subject to all existing rights to the use thereof, hereafter shall be subject to appropriation for beneficial use under laws of the State existing or to be enacted. This method has been adopted in New Mexico, Utah, and Oregon. In some States it may be necessary to place such a declaration in the State Constitution by amendment. It may be difficult to arouse sufficient public interest to bring about the necessary legal enactment. However, it is not inconceivable that the Local Sections of the Society, the American Water Works Association, and similar organizations, could bring about the desired results, especially if campaigns of education were pushed during times of drought or other emergencies which emphasized the need of administrative control.

Even after the enabling declarations are obtained the problems of the control of ground-water will necessarily continue to be complicated. There will be the matter of determining rights established before the new principle became effective. All this will require much study of legal, hydrologic, and engineering problems, and funds for such study must be made available. Appropriation of funds raised by taxation perhaps will not be readily approved by the general public. However, inasmuch as the waters would be considered to be owned by the public, the developer using them by permission, it would not be unreasonable to require a fee in proportion to the quantity of water used. Since 1917 by law the New Jersey State Water Policy Commission and its predecessors has collected a fee of $1 per 1 000 000 gal for all water used by public supply systems in excess of certain initial allowances. This fee is very small, and it could be increased to several times as much without becoming an undue burden to the water user. The New Jersey Commission has used the móney thus collected not only to carry on its work relating to current applications for approval of plans to divert water, but it has also undertaken studies as to the requirements of different parts of the State 10, 25, and 50 yr or more in the future, in order that it may be guided to a most efficient use of the water resources of the State. Some kind of fee has been collected for use of certain waters in other States, for example, in Ohio and Pennsylvania.

One other fact should be borne in mind, namely, that the movement of ground-waters is not limited by State boundaries, and sooner or later (in some States already) interstate problems must be considered. When these interstate problems are considered it further becomes evident that the general problems should be considered not from the standpoint of any one section of the country, but on a country-wide basis By this statement the writer does not refer to the possibility of any Federal control of waters, which is doubtless both undesirable and unlikely of attainment, but merely the desirability of having the laws of adjacent States sufficiently similar to simplify the solution of interstate problems relating to ground-waters. This conclusion

is readily reached when it is realized that certain of the water-bearing beds that underlie such of the Great Plains States as Nebraska and Kansas, extend into Iowa and Missouri; other formations of the latter States extend into Wisconsin and Illinois; and so on from one State to another with related problems.

Some progress has been made in the problem of legal control of ground-water, but the situation is still far from satisfactory. Conditions are ripe for another forward step. Probably no single organization has in its membership as many persons who are in close touch with the variety of problems of the control of ground-waters as the Society. Some of its members are involved primarily in the development of ground-water for public supply, industrial use, or irrigation; others in problems of the contamination of waters or of drainage; and others in positions of State administration. Accordingly, the Society is in a position to take the first step in initiating discussions on this important problem. However, the discussions should be on a very broad basis and should not be restricted to problems of one section of the country. They should include not merely the consideration of control of ground-water from the standpoint of the problem of water supply for irrigation or for public use, but also from the standpoint of contamination of ground-water supplies by oil-field developments or salt-water encroachment, the effect of ground-water developments on navigation of rivers, of drainage projects and the construction of dams on ground-water developments and *vice versa.* The discussions should be participated in by a variety of specialists, representatives of those who use the water—the public supply systems, industrial concerns, irrigators—to get their viewpoint; the well drillers and engineers to determine construction problems to be considered; the ground-water hydrologists to show the controlling natural principles; and, the State officials who generally are best aware of the difficulties of administration now encountered and who will have the duty of administering the laws subsequently to be enacted. Experienced men in these various lines can be obtained from a number of companion associations such as the American Water Works Association, the Section of Hydrology of the American Geophysical Union, the American Public Health Association, the Society of Economic Geologists, the Geological Society of America, the Association of State Geologists, the American Association of Petroleum Geologists, the American Petroleum Institute, the Association of Western State Engineers, and the American Association of Water Well Drillers. The Legal Profession, also, must be consulted; but is it not pertinent to suggest that in this problem—concerning whch all who have practical contact with it agree that conditions are most unsatisfactory—when once the majority of the technologists agree, the position of the lawyer is to show not what the desired result shall be, but how it can be brought about? The writer has seen a statement (but can not now find definite reference to it), attributed to the late Chief Justice Marshall in which he indicated his belief that the Courts should be permitted to call more freely upon men of technical training entirely independent of the experts produced as witnesses in cases. In the case of Los Angeles *v.* Hunter (156 Cal. 603), Justice Henshaw stated: "For experts learned in geology to give their theories

as to the manner in which Nature has created or developed a given physical condition is not an invasion of the domain of the law." From these statements it seems entirely fitting that the technologists take the initiative in endeavoring to solve the problem of the control of ground-water which is necessary to the most efficient use of this valuable resource.

Acknowledgments.—The writer wishes to acknowledge his indebtedness to his colleagues, Messrs. O. E. Meinzer, A. G. Fiedler, Walter N. White, and J. F. Deeds, of the U. S. Geological Survey, and to H. D. Padgett, of the U. S. Resettlement Administration, whose opinions have served to clarify the writer's viewpoint. Attention should also be called to a recent paper by Mr. S. C. Wiel[41], which deserves consideration in connection with the problems considered herein.

[41] "Fifty Years of Water Law," by S. C. Wiel, *Harvard Law Review*, Vol. L, pp. 252–304, 1936.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SIMPLIFIED METHOD OF DETERMINING TRUE BEARINGS OF A LINE

Discussion

BY MESSRS. F. L. MCREE, F. J. DUARTE, AND LEONARD C. JORDAN

F. L. McREE,[17] Esq. (by letter).[17a]—Some mention should be made of the accuracy of the values obtained from the table prepared by Mr. Inch. In working with Table 1 the writer was surprised to discover how little error is introduced by the use of an average parallax and refraction correction. The principal error seems to be due to straight-line interpolation being used between tabular values. This error, fortunately, is partly compensating for values of Z (measured from the north) greater than 90 degrees. An examination of the following examples will show that when $A \sin \delta$ is negative and is added to B, although the individual values of $A \sin \delta$ and B, as computed from Table 1, differ from the exact values, the result does not differ materially from the exact value for $\cos Z$. However, when Z is less than 90° (when $A \sin \delta$ is positive), this compensation does not exist, and larger errors may appear. Unfortunately, the range of Table 1 does not allow investigations for smaller values of Z. The writer would like to raise this question with the author: Do larger errors occur for interpolated values from the tables when Z (measured from the north) becomes small?

To illustrate the writer's viewpoint with respect to the accuracy of the results obtained from the tables the following examples are given.

Example 4.—Latitude = 35° 32.0′ (North); observed h = 35° 36.1′; declination = N 23° 24.0′; and, temperature = 50° C.

By the exact formula:

$$\begin{aligned} \text{Observed } h &= 35^\circ\ 36.1' \\ \text{Parallax and refraction correction} &= 0^\circ\ 1.1' \\ \hline h &= 35^\circ\ 35.0' \end{aligned}$$

NOTE.—The paper by Philip L. Inch, Assoc. M. Am. Soc. C. E., was published in September, 1936, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: November, 1936, by Messrs. Earl F. Church, Paul E. Wylie, James B. Goodwin, C. H. Swick, Philip Kissam, and George D. Whitmore; and December, 1936, by Messrs. O. H. Chilton, Chalmers C. Schrontz, Frank M. Johnson, Walter H. Starkweather, and C. I. Day.

[17] Associate Prof. of Civ. Eng., Texas Technological Coll., Lubbock, Tex.

[17a] Received by the Secretary November 9, 1936.

$$\cos Z = \frac{\sin 23^\circ\ 24.0'}{\cos 35^\circ\ 35.0' \quad \cos 37^\circ\ 32.0'} - \tan 35^\circ\ 35.0' \tan 37^\circ\ 32.0'$$

$$\cos Z = \frac{0.39715}{0.81327 \times 0.79300} - 0.71549 \times 0.76825$$

$$\cos Z = 0.61581 - 0.54968 = 0.06613$$

$$Z = 86^\circ\ 12.5'$$

By the author's tables:

Factor A			Factor B
$h = 35^\circ;\ \phi = 37^\circ$	1.52811		0.52723
36.1×32.0	$= 0.01155$	36.1×33.1	$= 0.01195$
32.0×34.3	$= 0.01098$	32.0×32.4	$= 0.01037$
	1.55064		0.54955
$\sin \delta$	$= 0.39715$		
$A \sin \delta$	$= 0.61584$		

$$\cos Z = 0.61586 - 0.54955 = 0.06629$$
$$Z = 86^\circ\ 11.9'$$

Example 5.—Latitude $= 40^\circ\ 30.0'$ (North); observed $h = 21^\circ\ 32.4'$; declination $=$ S $22^\circ\ 30.0'$; and, temperature $= 0^\circ$ C.

By the exact formula:

Observed h	$= 21^\circ\ 32.4'$
Parallax and refraction correction	$= 0^\circ\ 2.4'$
h	$= 21^\circ\ 30.0'$

$$\cos Z = \frac{\sin\,(-\,22^\circ\ 30.0')}{\cos 21^\circ\ 30.0' \quad \cos 40^\circ\ 30.0'} - \tan 21^\circ\ 30.0' \tan 40^\circ\ 30.0'$$

$$\cos Z = \frac{-\,0.38268}{0.93042 \times 0.76041} - 0.39391 \times 0.85408$$

$$\cos Z = -\,0.54089 - 0.33643 = -\,0.87732$$

$$Z = 151^\circ\ 19.2' \text{ (measured from the North)}$$

By the author's tables:

Factor A			Factor B
$h = 21^\circ;\ \phi = 40^\circ$	1.39790		0.32143
32.4×16.1	$= 0.00522$	32.4×28.3	$= 0.00917$
30.0×35.0	$= 0.01050$	30.0×19.3	$= 0.00579$
A	$= 1.41362$		$B = 0.33639$

$$\sin \delta = -0.38268$$

$$A \sin \delta = -0.54096$$

$$\cos Z = -0.54096 - 0.33639 = -0.87735$$

$$Z = 151° \ 19.4'$$

Example 4 gives a variation from the exact value as obtained from the formula of 0.6′. If Table 1 had been based on exact values of h, and if corrections had been made for parallax and refraction, the values of Z for this particular example would still be 86° 11.9′.

In both examples, it is noted that, from Table 1, that $A \sin \delta$ is larger and B smaller than the like expression in the exact formula. When $A \sin \delta$ is negative and is added to B the discrepancies compensate partly so that $\cos Z$ is not materially in error.

Examples 4 and 5 show that the proposed tables cannot be used where extreme accuracy is desired, in which case a more precise instrument than the transit should be used. It has been the writer's experience that the ordinary observer does well to obtain results accurate to the nearest minute with a transit. There are many instances in which azimuths or bearings to the nearest minute are sufficiently accurate and Table 1 seems to be well within this limit of accuracy.

The writer would like to see the complete table published. Due to his own labors in that direction he realizes the enormous amount of labor involved in its preparation, and that the author deserves considerable credit for his work.

F. J. Duarte,[18] Esq. (by letter).[18a]—A more complete table than Table 1 could be computed if the corrections for mean refraction and parallax were omitted. It would then be sufficient to compute another subsidiary table, for every degree of altitude of the mean refraction less the altitude parallax. With this auxiliary table the observed altitude angle would be corrected before entering Table 1.

The differences for 1′, both for altitude and latitude, have been computed on Table 1, dividing the first differences by 60 (chord interpolation). It would perhaps be more convenient to compute the derivatives (tangent interpolation), as follows:

$$f'(x) = \text{unit variation} = \frac{\Delta_1 - \frac{1}{2}\Delta_2 + \frac{1}{3}\Delta_3}{60}$$

The method described is easy to apply and precise enough for the orientation of a topographic map. The novelty of the method consists in Table 1 designed to facilitate the solution of the formula. This work, however, could be accomplished more rapidly by the use of a table of natural circular functions and a calculating machine, than with Table 1.

[18] Director del Observatorio de Caracas, Caracas, Venezuela.

[18a] Received by the Secretary November 20, 1936.

LEONARD C. JORDAN,[19] M. AM. SOC. C. E. (by letter).[19a]—For his effort to simplify and reduce the labor of field computations, Mr. Inch is to be commended. A transitman cannot always wait until evening to compute his bearings by candlelight in the office tent, but, sometimes, must take a solar observation, compute the results (perhaps on a windswept mountain side), and proceed to run lines in accordance with predetermined bearings. Any method that aids in making quick and accurate field computations will be welcomed by those men who find it suitable to their needs.

Attention should be called to the particular necessity for frequent correction of bearings on surveys that extend over considerable east and west distances. On such surveys, bearings very soon become incorrect, the errors increasing with the latitude of the location.

A base line, run in a straight line, 90° from any meridian, and from any latitude, will cross the equator when it is one-fourth of the distance around the world. Obviously, for a line north of the equator, the true bearing will be slightly southward, within the course of a few miles.

In sighting the sun, the writer has found it most convenient to focus the instrument on a distant object and, with his back to the sun, direct the telescope by means of its own shadow until it is nearly in the correct position. Then, the standard method of focusing the sun's image upon a page of field notebook held at the eye-piece (with the bisecting cross-hairs plainly visible) gives accurate results. Five independent observations, computed individually, generally are found to be in agreement. The most convenient transits for these observations are those which have additional hairs at 45° and at equal distances from the intersection of cross-hairs. These diagonal hairs are placed so that they cut thin segments from the sun's image. The slightest error in direction will cause one segment to be conspicuously longer than the opposite one. Looking through the instrument, no matter how shielded, should be avoided since the observer is almost certain to look past the telescope and directly into the sun. His work during the succeeding few minutes would be unreliable.

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[19] Cons. and Designing Engr., New Rochelle, N. Y.

[19a] Received by the Secretary November 27, 1936.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ANALYSIS OF CONTINUOUS FRAMES BY BALANCING ANGLE CHANGES

Discussion

By Ralph E. Byrne, Jr., Jun. Am. Soc. C. E.

Ralph E. Byrne, Jr.,[18] Jun Am. Soc. C. E. (by letter).[18a]—Another method for the analysis of continuous structures is presented in this paper. It is a close parallel to the method of moment distribution, but inasmuch as the quantities dealt with are rotations and displacements rather than moments, the procedure must necessarily differ. Some of the most important differences between the two methods occur in the definitions of some of the terms involved. In the moment-distribution method, stiffness is defined as the moment required to produce unit rotation at one end of a member, the other end being fixed; the carry-over factor is defined as the ratio of the moment at the fixed end to the moment at the end being rotated. In the method of balancing angle changes, stiffness is defined as the moment required to produce unit rotation at one end of a member, the other end being hinged; and the carry-over factor is defined as the ratio of the angle change at the hinged end to the angle change at the end being rotated. Likewise, the method by which applied loads are brought into the analysis is different for the two methods In the former method, fixed-end moments are used, these being the moments which would occur at the ends of the loaded member if the ends were fully restrained. In the latter method, the quantities which are analogous to the fixed-end moments are the angles of rotation of the two ends of the member under the applied loads, the ends being hinged. These quantities are easily calculated for use in either method as long as the members are prismatic; the calculation of the quantities becomes a major task when haunching occurs.

Various tables are available giving the values of stiffness, carry-over factors, and fixed-end moments for various types of haunched beams and various loadings, for use in the method of moment distribution. The writer knows of no

Note.—The paper by L. E. Grinter, Assoc. M. Am. Soc. C. E., was published in September, 1936, *Proceedings*. Discussion on this paper has been published in *Proceedings*, as follows: December, 1936, by Messrs. John E. Goldberg, G. A. Maney, Paul Andersen, and William F. Luce.

[18] Structural Designer, Pacific Elec. Ry., Los Angeles, Calif.

[18a] Received by the Secretary November 27, 1936.

similar tables giving directly the analogous quantities for use in the method of balancing angle changes. However, simple relationships exist between the various quantities which greatly simplify the calculation of the quantities required for the latter method, those required for the former method being known.

The following notation will be used in establishing these relationships: S and K will denote stiffness, and r and C will denote the carry-over factor, as defined, respectively, for the method of moment distribution and the method of balancing angle changes; F will be used to denote the fixed-end moment, and ϕ, the angle change at the end of the simply supported member, due to applied loads; and the subscripts, A and B, applied to these quantities will distinguish them for the two ends of the member. In this connection, it should be noted that r_A and C_A denote the carry-over factor to the end, A.

Referring to the member shown in Fig. 16: First, consider the member fully restrained at End B and a moment applied to End A to produce unit rotation of End A (Fig. 16(a)); this moment is defined as S_A. Next, release

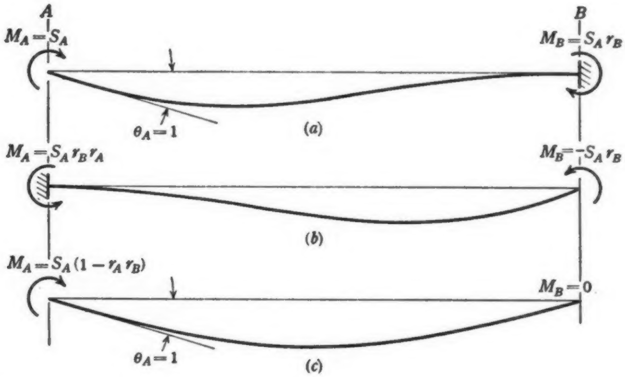

FIG. 16.

End B, the moment at B now becoming zero (Fig. 16(c)). This is accomplished by applying a moment of $-S_A r_B$ to the end, B (Fig. 16(b)). Adding the results of Fig. 16(a) and Fig. 16(b) the results of Fig. 16 (c) are obtained. The angle of rotation of End A is unity, and the moment at End B is zero; hence, by definition, the moment at End A is the stiffness at End A as defined for the method of balancing angle changes. Expressing this relationship as an equation,

$$K_A = S_A\ (1 - r_A\ r_B) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Consider the member shown in Fig 17. In Fig. 17(a), a moment, M, is applied at End A, End B being simply supported. The moment diagram is shown in Fig. 17(b). Assuming the origin of co-ordinates to be at End B,

the following equations may be written directly, using the two area-moment propositions:

$$\theta_A = \frac{1}{L}\int_0^L \frac{m_x\, x\, dx}{E\, I} = \frac{M}{L^2}\int_0^L \frac{x^2\, dx}{E\, I} \quad \text{............}(16a)$$

and,

$$\theta_B = \frac{1}{L}\int_0^L \frac{m_x\,(L - x)\, dx}{E\, I} = \frac{M}{L}\int_0^L \frac{x\, dx}{E\, I} - \frac{M}{L^2}\int_0^L \frac{x^2\, dx}{E\, I} \quad \text{.....}(16b)$$

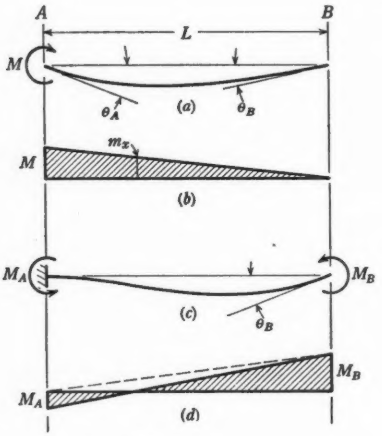

FIG. 17.

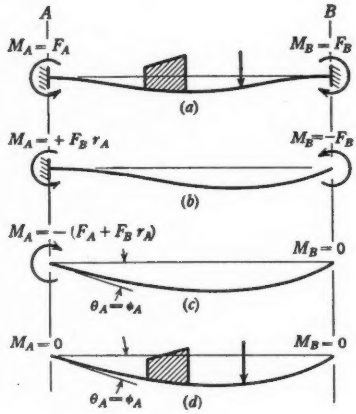

FIG. 18.

The carry-over factor, C_B, as defined for the method of balancing angle changes, may be expressed, therefore, as follows:

$$C_B = \frac{\theta_B}{\theta_A} = \frac{L\displaystyle\int_0^L \frac{x\, dx}{E\, I} - \int_0^L \frac{x^2\, dx}{E\, I}}{\displaystyle\int_0^L \frac{x^2\, dx}{E\, I}} \quad \text{..............}(17)$$

Next, consider the same member, as shown in Fig. 17(c), fixed at End A and a moment, M_B, applied to End B. The moment diagram is shown in Fig. 17(d). Again, assuming the origin of co-ordinates at End B, the following equations may be written directly, using the two area-moment propositions:

$$\frac{M_B}{L}\int_0^L \frac{(L - x)\, dx}{E\, I} - \frac{M_A}{L}\int_0^L \frac{x\, dx}{E\, I} = \theta_B \quad \text{...........}(18a)$$

and,

$$\frac{M_B}{L}\int_0^L \frac{(L - x)\, x\, dx}{E\, I} - \frac{M_A}{L}\int_0^L \frac{x^2\, dx}{E\, I} = 0 \quad \text{..........}(18b)$$

Equations (18) may be solved for M_A and M_B, obtaining,

$$M_A = \theta_B \frac{L \int_0^L \frac{x\,dx}{E\,I} - \int_0^L \frac{x^2\,dx}{E\,I}}{\int_0^L \frac{dx}{E\,I} \int_0^L \frac{x^2\,dx}{E\,I} - \left[\int_0^L \frac{x\,dx}{E\,I}\right]^2} \qquad \text{(19a)}$$

and,

$$M_B = \theta_B \frac{\int_0^L \frac{x^2\,dx}{E\,I}}{\int_0^L \frac{dx}{E\,I} \int_0^L \frac{x^2\,dx}{E\,I} - \left[\int_0^L \frac{x\,dx}{E\,I}\right]^2} \qquad \text{(19b)}$$

The carry-over factor, r_A, to End A as defined for the method of moment distribution, may be expressed as follows:

$$r_A = \frac{M_A}{M_B} = \frac{L \int_0^L \frac{x\,dx}{E\,I} - \int_0^L \frac{x^2\,dx}{E\,I}}{\int_0^L \frac{x^2\,dx}{E\,I}} \qquad \text{(20)}$$

Comparing Equations (17) and (20), it is seen that,

$$C_B = r_A \qquad \text{(21)}$$

Similarly, it could be shown that $C_A = r_B$.

Consider a member under any system of transverse loading fixed at both ends, as shown in Fig. 18(*a*). The fixed-end moments are F_A and F_B. Release End B, by adding to Joint B a moment equal to $- F_B$, resulting in a moment at End A equal to $+ F_B\, r_A$ (Fig. 18(*b*)). By adding, the condition is obtained in which the beam is fully restrained at End A, and hinged at End B; for this condition the fixed-end moment at End A is seen to be $F_A + r_A F_B$. Next, release End A by applying to that end (Fig. 18(*c*)) a moment equal to $- (F_A + r_A F_B)$. It was shown (see Equation (15), that a moment equal to $S_A\,(1 - r_A\, r_B)$ applied at End A produced unit rotation at A when End B was hinged. Therefore, the angle change produced at End A when a moment of $F_A + r_A F_B$ is applied to End A must equal the ratio, $\frac{F_A + r_A\,F_B}{S_A\,(1 - r_A\,r_B)}$. This angle change has been defined as ϕ_A for the method of balancing angle changes; hence,

$$\phi_A = \frac{F_A + r_A\,F_B}{S_A\,(1 - r_A\,r_B)} = \frac{F_A + r_A\,F_B}{K_A} \qquad \text{(22a)}$$

and, similarly,

$$\phi_B = \frac{F_B + r_B\,F_A}{S_B\,(1 - r_A\,r_B)} = \frac{F_B + r_B\,F_A}{K_B} \qquad \text{(22b)}$$

Using Equations (15), (21), and (22), and any tables giving values of S, r, and F, for use with the method of moment distribution, the corresponding quantities, K, C, and ϕ, may be computed. The extra step entailed by these computations may be considered an advantage of the method of moment distribution over the method of balancing angle changes; however, for any case in which tables are not available, it will be somewhat easier to calculate K, C, and ϕ, than the corresponding S, r, and F.

The method of analysis introduced in this paper has both advantages and disadvantages when compared with other available methods. The author has enumerated some of them; others might be mentioned. Inasmuch as the choice of method must depend largely on the designer's preference, it should be to the advantage of each individual designer to list and weigh the various advantages and disadvantages of available methods for himself. In this way, he will be better enabled to judge the most advantageous method to use in any particular case.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE MODERN EXPRESS HIGHWAY

Discussion

By Messrs. F. L. McRee, Theron M. Ripley, W. W. Crosby, Richard S. Kirby, Harold M. Lewis, George Conrad Diehl, and William E. Rudolph

F. L. McRee,[15] Esq. (by letter).[15a]—Considerable food for reflection and thought is presented in the paper by Mr. Noble. There is no doubt that steps should be taken to reduce the appalling death rate on highways in the United States. However, it is a moot question whether or not increased safety in design will bring about the desired result. Since a large percentage of the automobile accidents are caused by a small percentage of the drivers, can engineers produce a design that will be safe for these few inherently reckless drivers?

The trend is, and will continue to be, toward faster motor-vehicle speeds, unless legal steps are taken to prohibit the increase. Motor-vehicle manufacturers have increased the speeds of their vehicles from year to year because the people have demanded it. As the author points out, there are few existing highways that can accommodate these speeds. John W. Wheeler, M. Am. Soc. C. E., has made[16] the statement that some highways are antiquated before they are built—a very broad statement, but which, nevertheless, may be true.

The author's idea of a dual highway appears to be good, but will the people submit to the cost of the suggested wide right of way? It seems to the writer that the width could be reduced considerably and still serve the purpose. It is doubtful whether truck traffic will need two lanes in each direction for many years to come. Furthermore, why should the people have their money tied up in an extensive investment to provide for truck lines of the future?

Note.—The paper by Charles M. Noble, Assoc. M. Am. Soc. C. E., was published in September, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1936, by Messrs. Fred Lavis, Joseph Barnett, G. E. Hawthorn, John F. Fairchild, Leslie R. Schureman, and C. H. Purcell; and December, 1936, by Messrs. Elmer R. Haile, Jr., H. W. Giffin, and T. T. Wiley.

[15] Associate Prof. of Civ. Eng., Texas Technological Coll., Lubbock, Tex.

[15a] Received by the Secretary November 9, 1936.

[16] *Civil Engineering*, June, 1935.

Restricting rural entries to 10-mile intervals makes it very difficult for the rural resident who may have to travel 10 miles to visit his neighbor who lives directly across the highway. Rural entries are undoubtedly a source of danger, but it seems unreasonable to restrict these entries to such long intervals. The people living in rural communities help pay for the highways, and, in return, expect to make use of them. The only basis on which rural entries could be restricted would be to provide parallel slow-speed roads which would increase costs and still impose a hardship on rural residents.

It is very necessary that curves be properly superelevated. The flat or insufficiently superelevated curve becomes a source of great danger with present high motor-vehicle speeds. The frictional resistance between tires and pavement surfaces should be given serious consideration in determining the superelevation. Investigations conducted at the Iowa Engineering Experiment Station[17] indicate that a working coefficient of 0.30 can be used safely, even for wet pavements. This will permit much higher speeds than the theoretical for which the curve is superelevated.

Equation (1) can be modified to take account of the frictional resistance. From mechanics, it can be shown that,

$$\tan\,(\theta + \phi) = 0.067\,\frac{V^2}{r} \qquad (25)$$

or, in terms of the degree of curve, D,

$$E = \tan\,(\theta + \phi) = \frac{DV^2}{85\,800} \qquad (26)$$

in which θ = the angle of superelevation; $\phi = \tan^{-1} f$; and f = the coefficient of friction. If the coefficient is used as 0.3, ϕ becomes 16° 42′. As an example, suppose that the minimum superelevation is required for a 3° curve and a speed of 100 miles per hr: $\tan\,(\theta + \phi) = \dfrac{3 \times (100)^2}{85\,800} = 0.35$; and $\theta + \phi = 19° \; 17'$. For $f = 0.30$, $\phi = 16° \; 42'$; and, therefore, $\theta = 2° \; 35'$; $\tan 2° \; 35' = 0.045$; and, by Equation (26), $E = 0.045$ ft per ft.

A solution of the author's problem of finding the sharpest curve for a maximum superelevation of 1 in. per ft by the foregoing would be: $\theta + \phi = 4° \; 46' + 16° \; 42' = 21° \; 28'$; $D = \dfrac{85\,800 \times 0.393}{(100)^2} = 3° \; 22'$; or, $r = 1\,700$ ft.

In any consideration of superelevation of curves, minimum speeds should be considered. When ice forms on a pavement the coefficient of friction sometimes becomes very small and vehicles tend to slip toward the inside of the curve. This is particularly true for slow-moving, heavily loaded trucks. These considerations should limit the maximum superelevation to be used.

The minimum speed on superelevated curves is given by Equation (26) solving for V.

[17] *Bulletin 120,* Iowa Eng. Experiment Station, Iowa State Coll., Ames, Iowa.

Assuming a coefficient of friction equal to 0.10, the minimum speed for the example noted would be zero since the friction angle is 5° 43′; that is, $\theta - \phi$ is negative, showing that, theoretically, a negative speed could occur without skidding.

Fig. 7 shows the minimum superelevation for a speed of 100 miles per hr and a coefficient of friction of 0.30, and also the maximum for a speed of 10 miles per hr and a coefficient of friction of 0.10, 10 miles per hr being assumed as the minimum speed likely to occur. It is not reasonable to assume that the curves would be built flat up to 2° 30′, therefore, the suggested superelevation is indicated. Diagrams similar to Fig. 7 can be drawn for any coefficient of friction that is assumed to be reasonable.[16] Table 4 shows values of the maximum speed for $f = 0.30$; maximum and minimum speeds for $f = 0.10$; and the theoretical speed for which the curve is superelevated ($f = 0$), all based on the suggested values of superelevation.

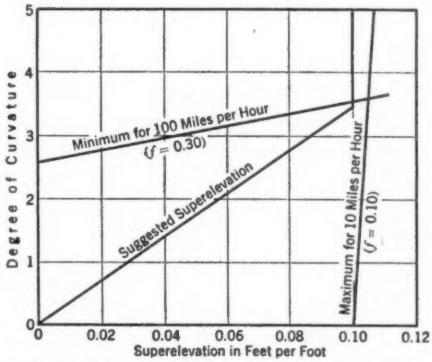

FIG. 7.—MINIMUM SUPERELAVATION FOR A SPEED OF 100 MILES PER HOUR; MAXIMUM SUPERELEVATION FOR A SPEED OF 10 MILES PER HOUR; AND THE SUGGESTED SUPERELEVATION.

TABLE 4.—MAXIMUM AND MINIMUM SPEEDS, AND THE THEORETICAL SPEED FOR WHICH THE CURVE IS SUPERELEVATED

Curvature, D, in degrees	Superelevation, E, in feet	VELOCITIES, V, IN MILES PER HOUR FOR:				Curvature, D, in degrees	Superelevation, E, in feet	VELOCITIES, V, IN MILES PER HOUR FOR:			
		$f = 0$	$f \leqq 0.30$	$f \leqq 0.10$	$f \geqq 0.10$			$f = 0$	$f \leqq 0.30$	$f \leqq 0.10$	$f \geqq 0.10$
(1)	(2)	(3)	(4)	(5)	(6)	(1)	(2)	(3)	(4)	(5)	(6)
1	0.028	49	169	105	0	4	0.10	46	95	66	0
2	0.057	49	125	82	0	5	0.10	41	84	59	0
3	0.086	49	106	72	0	6	.....	..	...	...	.

The writer cannot agree with Mr. Noble that spiral curves are unnecessary, or that a motor vehicle has sufficient room within its lane to form its own spiral. For a motor vehicle to enter or leave a curve smoothly the curve should be spiraled and the spiral should be of sufficient length to accommodate the maximum speed for which the highway is designed.

There is considerable disagreement among highway engineers as to the proper length of spiral. Past practice has been to use spirals that are too short. The American Railway Engineering Association has found by experiment that the maximum rate at which the tilt of superelevation can be attained without discomfort to passengers is about 2 in. per sec. There is no valid reason why the same rule will not apply to highway curves. Using 58 in. as the average width of automobile tread and E as the superelevation, in inches per

inch of width, the total tilt would be 58 E in. The minimum time in which this tilt should be attained is $\frac{58\,E}{2} = 29\ E$ sec which gives the minimum length of spiral in terms of the velocity, V_s, in feet per second, as $L_s = 29\ E\ V_s$. Changing the velocity, V, to miles per hour the expression becomes,

$$L = 42.5\ E\ V \ldots\ldots\ldots\ldots\ldots\ldots\ldots(27)$$

in which E = the superelevation, in feet per foot.

Another method proposed for determining the minimum length of spiral is based on the maximum rate at which a motor vehicle may change its radial acceleration without discomfort and danger. Experiments conducted at the Iowa Engineering Experiment Station[16] indicate that this value is from 2 to 3 ft per sec per sec per sec, with 2 ft as the recommended value. The engineers of the State Highway Department of Oregon use a value of 3 ft per sec per sec per sec for their spirals in planning their highways for a speed of 100 miles per hr. The radial acceleration varies from zero at the beginning of the spiral ($r = \infty$) to $\frac{V^2}{r}$ at the end. The time in which this acceleration is attained is $\frac{L}{V}$ sec. The rate of change of acceleration, then, will be:

$$d = \frac{V^2_s}{r} \div \frac{L}{V_s} = \frac{V^3_s}{L_s\,r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(28)$$

Using a value of 3 ft per sec per sec per sec for this rate of change, changing V_s to miles per hour, and replacing r by $\frac{5\,730}{D}$:

$$L_s = \frac{D\,V^3}{5\,450} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

The writer favors using the expression that gives the greatest length of spiral.

Some engineers follow the practice of spiraling the center line of the pavement and offsetting to obtain the line for the pavement forms. It is much easier, and more accurate, to run separate spirals for each edge of the pavement. With widened pavements separate spirals should always be run on account of the longer length of the inside spiral. A good method is to compute the minimum length of spiral from one of the foregoing expressions, use this length to the nearest 10 ft for the outside edge of the pavement (unless the run-off of the superelevation requires a longer spiral), and then compute the length for the inside edge. This inside length is fixed by the length of the outside spiral and the widening.

Referring to the preceding statement that a motor vehicle does not have room to form its own spiral within its lane, it can be shown that the shift or offset, p, for a spiral is given by the following formula:

$$p = \frac{D^3\,V^8}{408 \times 10^{10}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(30)$$

The derivation is based on the A.R.E.A. spiral expressions and Equation (29). Using a 3° curve and a speed of 100 miles per hr, the offset is $p = 6.6$ ft. If a value of 2 ft per sec^3 for the maximum rate of change of radial acceleration had been used in developing Equation (29), the spiral offset for this degree of curve and velocity would be 15 ft. Obviously, a motor vehicle cannot form its own spiral within a 10 or 12-ft lane when the offset is so large.

It appears that there is a tendency among highway engineers to avoid the spiraling of highway pavements because of the supposedly complicated computations involved. The A.R.E.A. spiral can be adapted easily to highway pavements, and the writer cannot agree that the computations involved are too complicated. Even if they were complicated the engineer should be willing to do the necessary work when increased safety is the goal.

THERON M. RIPLEY,[18] M. AM. SOC. C. E. (by letter).[18a]—The "express highway" is given as the principal antidote for accidents and fatalities and in this development the highway engineer is implied to be the contributing cause of the present accident record due to improperly designed travel lanes. The experience of seventeen years in the design, construction, and maintenance of highways and the study of traffic compels the writer to take issue with the foregoing assumption. He has seen the 14-ft, the 16-ft, and the 18-ft, two-way pavement, each in turn, abandoned for the 20-ft lane with all the attendant improvements in grade, alignment, and crossing elimination. The self-propelled vehicle has changed from an oddity and a toy to the commonplace and a necessity, and the dirt road to a paved street; yes, for many hundred miles, to a boulevard.

The discussion of the highway traffic situation, apparently, has arrived at that stage which can be represented by the man suddenly awakened by an unusual sound: when he is so awakened his first impression is one of bewilderment and he attributes the sound to one of great magnitude and danger. This is a natural reflex from the first law of Nature—self-preservation. In May, 1934, under the title, "The Balance Wheel", the president of a large corporation attempted to keep the feet of his organization on the ground by writing a few truths which are as applicable to the present highway situation as they then were to the petroleum industry. He said in part:

"This is a time when mental indigestion is epidemic. The seats of learning have become sounding boards for intellectual jazz; and the Doctor Cooks of economics are having a field day.

"The statement that two and two are four lacks intrigue or interest, and arguments to the effect that, in this modern time, two and two make five are the sex appeal of current economics. It is a day of intellectual emancipation, and it should not be turned into a period of mental anarchy.

"These observations apply only to current discussions on religion, morals, politics, business, art, and science. Otherwise, things are normal.

"The balance wheel has its uses even though it is not spectacular."

Under "Introduction" the first subject mentioned by the author is "the enormous loss of life and property on American streets and highways" and, included in this paragraph, is a table (Table 1) showing the number of per-

[18] Cons. Engr., Buffalo, N. Y. Mr. Ripley died on November 30, 1936.
[18a] Received by the Secretary November 16, 1936.

sons injured, the number killed, and the total number of accidents in 1935. For the convenience of the reader the original table[19] is reproduced as Table 5 for the record.

After stating that "the reckless and unsafe driver must be considered", the author finishes his "Introduction" with the statement that "the injuries and the loss of life and property on American highways are issues squarely facing the highway engineer. It is his duty to design the highways so that the traveling public is safeguarded." A study of the accident record, perchance, may show where the engineer has failed.

TABLE 5.—CONDITIONS RESULTING IN PERSONS KILLED AND INJURED IN 1935

Item No.	Description	ACCIDENTS		PERSONS KILLED		PERSONS INJURED	
		Number	Percentage of total	Number	Percentage of total	Number	Percentage of total
	(*a*) EFFECT OF THE DIRECTION CARS WERE TRAVELING						
	Vehicles:						
1	Going straight	653 090	79.0	31 480	87.2	703 690	78.6
2	Turning right	20 670	2.5	610	1.7	23 280	2.6
3	Turning left	56 220	6.8	1 120	3.1	63 560	7.1
4	Backing	12 400	1.5	330	0.9	13 430	1.5
5	Skidding	29 760	3.6	1 260	3.5	31 330	3.5
6	Parked, or standing still	23 970	2.9	610	1.7	25 070	2.8
7	Slowing down or stopping	26 450	3.2	400	1.1	30 440	3.4
8	Miscellaneous	4 130	0.5	290	0.8	4 480	0.5
9	Totals	826 690	100.0	36 100	100.0	895 280	100.0
	(*b*) EFFECT OF HASTY, NEEDLESS, AND CARELESS ACTS OF DRIVERS						
	Driver:						
10	Exceeding speed limit	121 460	22.8	7 240	30.7	161 550	22.9
11	On wrong side of road	85 770	16.1	3 940	16.7	111 460	15.8
12	Did not have the right of way	135 840	25.5	3 580	15.2	191 880	27.2
13	Cutting in	17 580	3.3	420	1.8	23 980	3.4
14	Passing standing car	2 130	0.4	70	0.4	2 820	0.4
15	Failed to signal or signaled improperly	27 700	5.2	260	1.1	35 980	5.1
16	Passed another car on a curve or hill	8 520	1.6	400	1.7	11 290	1.6
17	Passed another car on the wrong side	2 130	0.4	50	0.2	2 820	0.4
18	Absent; car ran away	3 200	0.6	280	1.2	4 230	0.6
19	Drove off the roadway	55 940	10.5	3 390	14.4	64 190	9.1
20	Drove recklessly	51 670	9.7	3 020	12.8	67 020	9.5
21	Miscellaneous	20 780	3.9	920	3.9	28 220	4.0
22	Totals	532 720	100.0	23 570	100.0	705 440	100.0
	(*c*) EFFECT OF THE ACTION OF PEDESTRIANS						
	Pedestrian Crossing at Intersection:						
23	With signal	10 990	3.8	210	1.3	10 780	3.9
24	Against signal	36 200	12.4	1 070	6.7	35 130	12.7
25	No signal	37 280	12.7	1 870	11.7	35 410	12.8
26	Diagonally	5 350	1.8	370	2.3	4 980	1.8
	Pedestrian:						
27	Crossing between intersections	78 100	26.7	4 550	28.4	73 550	26.6
28	Waiting for, or stepping on or off, street car	3 210	1.1	80	0.5	3 130	1.1
29	Standing on safety aisle	1 210	0.4	100	0.6	1 110	0.4
30	Getting on or off another vehicle	3 210	1.1	260	1.6	2 950	1.11
31	Playing in street	45 850	15.7	1 600	10.0	44 250	16.0
32	At work in roadway	6 220	2.1	450	2.8	5 770	2.1
33	Riding or "hitch-hiking" on vehicle	3 920	1.4	320	2.0	3 600	1.3
34	Coming from behind parked car	34 340	11.7	1 140	7.1	33 200	12.0
35	Walking on rural highway	14 650	5.0	3 030	18.9	11 620	4.2
36	Not on the roadway	5 920	2.0	340	2.1	5 580	2.0
37	Miscellaneous	6 220	2.1	640	4.0	5 580	2.0
38	Totals	292 670	100.0	16 030	100.0	276 640	100.0

[19] See "Live and Let Live", Travelers Insurance Co., Hartford, Conn., p. 4.

TABLE 5.—(*Continued*)

Item No.	Description	Accidents		Persons Killed		Persons Injured	
		Number	Percentage of total	Number	Percentage of total	Number	Percentage of total
	(*d*) Effect of Prevailing Weather Conditions						
39	Clear weather	699 710	84.6	28 600	85.6	671 110	84.6
40	Fog	15 940	1.9	870	2.6	15 070	1.9
41	Rain	92 160	11.2	3 310	9.9	88 850	11.2
42	Snow	18 880	2.3	630	1.9	18 250	2.3
43	Totals	826 690	100.0	33 410	100.0	793 280	100.0
	(*e*) Effect of Road Surface Condition						
	Road Surface:						
44	Dry	619 930	75.0	25 860	77.4	594 070	74.9
45	Wet	136 830	16.6	5 240	15.7	131 590	16.6
46	Snowy	23 970	2.9	770	2.3	23 200	2.9
47	Icy	45 960	5.5	1 540	4.6	44 420	5.6
48	Totals	826 690	100.0	33 410	100.0	793 280	100.0
	(*f*) Effect of Light Conditions						
49	Daylight	479 650	58.0	14 000	41.9	465 650	58.7
50	Dusk	30 100	3.6	1 540	4.6	28 560	3.6
51	Dark	316 940	38.4	17 870	53.5	299 070	37.7
52	Totals	826 690	100.0	33 410	100.0	793 280	100.0

As Table 5(*a*) shows that 79.0% of all accidents, 87.2% of all fatalities, and 78.6% of injuries happen "going straight", it seems reasonable to assume that doing away with all curvature is not the answer to the prevention of trouble. If the engineer is to blame he must look further into the causes.

Table 5(*b*) is a breakdown of the items in Table 1 of the paper. Of Table 5(*b*) the compilers[19] wrote:

"Although total deaths last year [1935] exceeded fatalities of the year before by around 300, mistakes of drivers figured in nearly 1 500 more deaths than in 1934. That means the time for dilly-dallying about what to do to restrain careless drivers has long since passed. Drivers who will not act safely must be forced to change their conduct. Else they must be kept off the road."

The greatest number of fatalities, under one heading, is seen to be "Pedestrians"; these are segregated in Table 5(*c*). It is believed to be a fair assumption that all the items in Table 5, from Items Nos. 23 to 38, inclusive, can be assumed as wholly within urban communities. If this assumption is used for argument, then it is found that 12 020 fatalities occur in cities and villages and 3 030 fatalities in rural communities, with 980 fatalities to be distributed. If the 980 fatalities are divided between the two known localities on the basis of the assumed known amounts, the 16 030 total will be divided into 12 805 urban and 3 225 rural. It is self-evident that rights of way of 300 ft, or more, in width cannot be secured in urban centers; therefore, in order to save 80% of the disappearing pedestrians the engineer must look to some other savior than the width of the highway.

Considering the remaining 20% of walkers: It is possible that the construction of sidewalks in special localities might be of greater value than widening the right of way or separating the vehicle movements. Pedestrians will not walk over rough ground, through wet grass, or in the mud in greater numbers simply because a change has been made in the pavement or highway limits.

Assuming the statements in the preceding paragraphs to be true as to facts and probable action, the fatalities chargeable to the highways can be reduced by 16 030, to 20 070; but Table 5(*b*) shows 23 570 fatalities due to "Hasty, Needless, and Careless Action of Drivers" and, therefore, some of these quantities overlap. It may be only a coincidence that this overlapping is almost equal to the number "Walking on Rural Highways" (Item No. 35, Table 5(*c*)).

Considering now the matter of pavement condition, attention is called to Table 5(*d*) and Table 5(*e*), which show that 84.6% of all accidents happen on clear days; that 75.0% happen on dry surfaces; that 85.6% of fatalities happen on the former and 77.4% happen under the latter.

When the matter of design is considered in relation to fog, snow, and ice, and their effect upon traffic, it is seen that these "Weather Conditions" account for 14.4% of the fatalities and these "Road Conditions" for 6.9 per cent. The handling of ice and snow conditions upon the highway is, primarily, a function of maintenance. When the designing engineer has provided the necessary room for the proper functioning of the snow-removal equipment and personnel he will have done his duty. Ice and packed snow on the pavement must be looked after by the maintenance organization.

Artificial lighting of improved highways has been discussed for a number of years, but the question of cost and the inadequacy of the lighting equipment available have been deterrents in the adoption of a general policy. An idea of the influence of light upon fatalities and injuries can be secured from Table 5(*f*), which shows that "Dusk" and "Dark" claim 58.1% of the fatalities and 41.3% of the injured. Lighting, properly done, is not merely the placing of electric lamps along the roadside; the writer has seen more than one installation of that character which was more of a hazard than a help. The new lamps in the experimental installation near Schenectady, N. Y., will help to solve this problem.

The foregoing remarks, born of Mr. Noble's paper, may seem to exonerate the designing engineer, but such is not the intention. The engineer has his work to do. The writer desires to impress upon the reader that the principal reason for automotive mortality is the operator of the machine and, in a lesser degree, the pedestrian or his guardian; and that the reduction of accidents is to be accomplished, primarily, by education and law enforcement.

To design for a speed of 100 miles per hr, as suggested by the author, would require a major change in the designing trend. Dr. F. C. Stanley, Chief Engineer for the Raybestos Company, of Bridgeport, Conn., has published a number of papers and graphs relative to the stopping distances required by automotive vehicles under ideal conditions. His studies have

resulted in the adoption of an empirical formula which shows that 444 ft is the distance to stop a car traveling at 100 miles per hr—this with a dry pavement and a car in perfect condition, and the operator with a reaction time of 0.75 sec. A speed of 45 miles per hr, which is now recognized as legal in some States, would require, under the same conditions, 90 ft in which to stop.

As the whistling snorer refuses to admit that he keeps the family awake, so, the driver of an automobile refuses to admit his responsibility for the highway slaughter.

Recently the technical and lay press has been writing about "friction" as being a major cause of highway accidents. Vehicle friction, or highway friction, or traffic friction may be good or bad slogans. The writer has never seen or heard brain-friction mentioned and yet it may have a legitimate place in the discussion.

As an example of public reaction, to curb highway traffic damage, the State of New York voted $300 000 000 for the elimination of railroad-highway grade crossings. The record then showed that 4.5% of fatalities and 0.5% of all accidents happened at such crossings. Much of this money has been spent and many crossings eliminated but the slaughter continues, the motorist is as discourteous as ever, and traffic rules and laws are broken as of yore.

What the speed of highway traffic will be ten or twenty years hence no one knows and few are competent to make an intelligent guess. What is known is that a nap at the wheel is a free pass to eternity; that alcohol in the stomach and gas in the tank is a combination that can make shambles of the public highway; and that halfwits, nitwits, and morons are at the throttle of machines of 60 hp and more, driving from somewhere to nowhere to do nothing when they get there and, in their joyously mad ride, they are a fatal menace to all other traffic on the highway.

Looking back to the water-bound macadam days of New York State and the time when a motor vehicle was a "red devil" which scared all the horses, and most of the drivers; and tracing the changes through the years to this day, it seems that both the civil and the mechanical engineer have done a most commendable job. Neither has reached perfection, but one has only to see what has been done adjacent to the larger cities in New York and other States, particularly in New Jersey, to realize that the highway designer is awake to the situation. To find the sleepers one must look in other directions.

The most potent cause of accidents, as well as the most difficult to control, is the driver. Fool-proof highways can not be built; therefore, a sincere effort must be made to keep the fools away from the steering wheel.

W. W. Crosby,[20] M. Am. Soc. C. E. (by letter).[20a]—This frank and forceful paper is a subject for congratulation; the cogent statements contained in it are most opportune. The author points out some fundamental defects of present highway designs. These latter are the direct reflections of engineering

[20] Cons. Engr., Coronado, Calif.

[20a] Received by the Secretary November 23, 1936.

ideals, imagination, and effort, and he rightly implies that "safety" should be a feature of the results. This latter point the writer has been trying to emphasize for some time[21].

Too often, however, the engineer is restricted by higher authority (such, for instance, as his Highway Commission or Department of Public Works) from developing advanced or new ideas, or from attaining effective results along progressive lines. Even Governors have prevented highway authorities from attempting novelties; and legislatures have usually restricted the authority and activities of highway departments. An illustration of the latter may be found in many States where laws limit the width of public rights of way for roads to 100 ft, or similarly inadequate widths, and compel the highway officials to produce results within that insufficient strip.

Safety can be enhanced by better road design, but room for that must be provided for engineers to use. The provision of this "elbow room"[22], or right of way, generally depends more on the electorate, the legislatures, and the authorities than on the engineers. It is the duty of the engineer of course, to force his higher authorities toward—and even hard against—their limits when his technical knowledge and experience convince him that this should be done; but the extension or removal of those limits is in most cases the responsibility of others rather than that of the engineer.

The author's statement as to "broad highway economics" (see "Introduction") is mild. Until recently the preponderance of discussion on the subject has been along regrettably narrow lines. The writer welcomes the author's assertion because he has tried on various occasions to combat the narrower views expressed[21].

Under "Fundamental Concepts", the author's brief statement that the highway design should be such that "the motorist is led unconsciously to choose the safe act rather than that which is unsafe" deserves emphasis. The writer has previously expressed himself on this subject[23] and wishes here to applaud the author's statement. Similarly, Mr. Noble's endorsement of the "Dual Highway" has been supported by the writer[24] who, however, offers herein some discussion or emphasis of the details given[25].

If the central "110-ft reservation" (Fig. 1) is intended to be used later for the through ways (and each "pavement-one-way" for "service roadways"), the width (110 ft) may be insufficient. Otherwise, it is probably three to four times as much as is desirable, although the total width of right of way should be preserved.

The "Acceleration Lane" (Fig. 2) may be a questionable provision in many cases. The writer has learned from experience that dangers exist for a driver swerving from a closely parallel lane into a swift traffic current in the same direction. There is inevitably a "blind spot" in the left rear of a

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[21] *Roads and Streets*, March, 1926, p. 154, and March, 1936, p. 48; also, "Notes on Highway Location", by W. W. Crosby, Chapter V *et seq.*, Gillette Pub. Co.; and the *Transactions* of the Society.

[22] *Roads and Streets*, October, 1934, p. 362.

[23] *Loc cit.*, March, 1926, and March, 1936.

[24] *Loc. cit.*, November, 1936.

[25] *Transactions*, Am. Soc. C. E., Vol. 100 (1935) pp. 1052 *et seq.*, and p. 1085.

driver. It would be much better to bring the traffic from the side in on a track much more nearly normal to the through traffic so that the approach of the latter may be included in the usual front and side vision of the entering driver, even at the sacrifice of speed momentarily by the latter. The tangent of the curve on the turn to the "trunk highway" should go off the first half of that curve as drawn, making the angle with the trunk highway in no case less than 30°, and the "acceleration lane" should be abandoned as such.

Under "Details of Design", the writer agrees with Mr. Noble as to the need for longer vertical curves; but he believes that, for a maximum grade limit, the length of the grade should enter the consideration, together with the actual minimum rate of rise. With the improvements in cars, brakes, and tires, is not an increase of acceptable maxima for rates of grades now justifiable in many cases?

The writer wishes to add whatever emphasis he may be able to contribute to the author's statement following Equation (4), that "the passing menace is one of the most serious problems facing the highway designer."

Although in the past the writer has expressed himself quite fully as to "Signs"[26], he wishes now to applaud the author's remarks as to the need for signs "in advance" (see heading, "Signs"). He also agrees with Mr. Noble as to "obstructions, such as poles", etc., and as to "guard-rails". The writer would point out that, where guard-rails are unavoidable, their main objective should be to assist the driver to regain control of his vehicle on the highway.

In the "lighting" of streets the considerations are different in some respects from those on open roads. In both cases the glare of opposing headlights is a large factor in creating dangers. The dual highway removes this factor, and greatly simplifies the problem, if, in many cases, it does not eliminate it. "Roadside stand lighting" would probably be unobjectionable if the stands were back where they belong and where they would be on proper widths of highway right of way.

The author's remarks on "Terminals and Interchange Facilities" are extremely important. Their essence seems to be that these provisions (and many others in the design) should be made so that the driver will go automatically where he should, without delay or hesitation that may create danger for himself or others. "What looks right may be wrong, but what looks wrong cannot be right".[27]

The writer heartily endorses the first two paragraphs of Mr. Noble's "Conclusion." A few States might be credited with actually expressing some of the principles mentioned. The State of Washington is reported to have adopted the "dual highway" for all its East and West State routes. New Jersey seems to be establishing the ideal, and possibly there are other States that should be mentioned. Italy set the example which Germany is developing. Great Britain has provided legal authority for it.[28]

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[26] "Notes on Highway Location", by W. W. Crosby, Gillette Pub. Co., Chapter VI and p. 72.

[27] *Loc. cit.*, pp. 78, 130–132.

[28] *Roads and Streets*, October, 1936, p. 33.

In the third paragraph of "Conclusion" the author seems to go a bit too far when he states that the highway engineer "controls the destiny of billions of dollars and tens of thousands of lives." The engineer is fated to be loaded with the responsibility, of course, but actually the final authority or "control" is beyond him many times. Prompt general recognition of this fact should result in an earlier realization of desirable ideals.

The author's reference (see heading, "Conclusion") to the obsolescence of highways in the United States deserves emphasis and perhaps elaboration. The writer, therefore, ventures to append the following at this opportunity. In these days, "three things are certain: Death, Taxes, and Obsolescence." Obsolescence is a greater and more formidable menace on roads than on any comparable line of approach. It seems that the highway authorities to whom the public has the right to look for relief are actually doing the most to bring about the enormous penalty of antiquated, inefficient, and obsolescent public roads.

Generally, it may be stated that public road authorities are spending hundreds of millions of dollars annually "slicking up" a type of highway, which, when "perfected" will not only be inadequate for its purposes, but which also will *per se* hasten its own condemnation as being antiquated, effete, inefficient, out of date, and as requiring a substitute.

With a few notable exceptions, State and National highway authorities seem to cling to the ideal of the single-roadway type, and seem to be making no provision for its transformation into a better one even if they actually are conscious of its imminent obsolescence.

The opportunity for escape from the impasse ahead is so often neglected that one is almost forced to believe that they are not even looking ahead. Merely securing the extra width of right of way for the double-roadway type (while it is undeveloped and, in many cases, of no considerable value), would postpone the obsolescence indefinitely, beside securing concomitant benefits of immeasurable value. Instead, the practice seems to persist (even across waste lands) in taking only a 100-ft right of way, or even less, when 300 ft or 500 ft might just as well have been had. Later widening of the right of way is then almost always impracticable.

It is probably too early to forecast accurately all the effects on highway design that may come from the future of "trailers." Their use is estimated as 100 000 in 1936 and this number is increasing rapidly. It is certain that they will hasten the arrival of the "Pairway"[26], and, hence, the need for width in which to build and to operate it.

According to a recent article[29], expenditures of \$15 000 000 000 since the turn of the century, in the United States, have yielded a highway system unfit for modern motor traffic. The statement may be a bit extreme, but there is no doubt that the enormous expenditures made and annually being continued are rapidly bringing this country to that position when the highway authorities will have to retrace their steps, or seek a new course, at an immense cost of money and lives.

[29] *Fortune Magazine*, August, 1936.

Highway engineers should urge the authorities that rank above them to adopt a new ideal for the public road. They should visualize divided roadways instead of the single broad pavement as something to work toward more or less gradually. They should urge the provision, as promptly as may be, of ample rights of way for that purpose; and, as engineers, they should do their utmost to have the safety built into, and the absolescence built out of, the highway.

RICHARD S. KIRBY,[30] M. AM. SOC. C. E. (by letter).[30a]—It is high time that automobile manufacturers and highway builders interchanged views. The former, in urging the public to invest in cars capable of speeds as great as, and even more than, 100 miles per hr are lacking in a sense of social responsibility and are blinking at certain obvious physical and mental limitations of the average human machine. Perhaps the latter are following where they should lead. It is obvious that there is a need for better designed highways, on which driving will be safer and less nerve-wracking; but why urge that public money be spent so that some Zelik Forlansky, who should be piloting a road roller, or some Percey Asteroid Vandervell, III, who should be attending his classes at college, can tear along over the country at rates 50% greater than that of the speediest limited express trains? What, in particular, is gained for society as a whole? The millions would far better be spent on careful selection of drivers, special training for those who obviously need it, elimination of the unfit, and swift, impartial, and certain punishment for flagrant offenders.

All of which is in no sense a condemnation of Mr. Noble's excellent presentation of a pressing technical problem. Suppose, however, that, ten or twenty years hence, engineers have developed highways which in their perfection of design are even beyond Mr. Noble's fondest dreams. If there still persists the present *laissez faire* attitude toward those persons who are allowed to drive cars on these superways the taxpayers' good money will have been spent to little purpose, for the killings and the maiming will be going on in ever-increasing volume. The country does indeed need safer highways for rational use; but persons who think they must travel at 2 miles per min will never be satisfied with any highways that engineers may build, because such individuals require three-dimensional space instead, and the good Lord has provided plenty of such space already.

HAROLD M. LEWIS,[31] M. AM. SOC. C. E. (by letter).[31a]—The development of express highways has taken place within the last ten years. Only within the last few years has it been recognized that the dual highway, recommended by Mr. Noble, which provides separate roadways for each direction of traffic, should be made a feature of all major express routes. The safety of a highway would thus be tremendously increased and operation at higher speeds made possible.

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[30] Prof., Eng. Drawing, School of Eng., Yale Univ., New Haven, Conn.

[30a] Received by the Secretary November 21, 1936.

[31] Engr. and Planning Consultant, Regional Plan Assoc., Inc., New York, N. Y.

[31a] Received by the Secretary November 25, 1936.

The first comprehensive system of express highways for a metropolitan area was that presented in the Regional Plan of New York and Its Environs in 1929. This included about 253 miles of routes in the New York-New Jersey Metropolitan Area, only 3½ miles of which had then been developed as express routes. This system was supplemented by a separate system of parkways, most of which would also provide express movement, but all of which would be limited to passenger vehicles. The express highway system was designed for both passenger and commercial vehicles.

Express highways were needed to relieve the interference from cross-traffic on intersecting routes and to provide short cuts between population centers and by-passes around intensively built-up districts. In a few cases they have been developed as viaducts, or on embankments, where there could be no interference from the uses of abutting property, but where they have been built along existing grades, the permanency of free movement has been overlooked. An example is U. S. Route No. 1, between Newark and Trenton, N. J., which was reconstructed over an entirely new right of way and hailed as the first comprehensive express highway. For the first few months after it was opened to use, it was adequate for through traffic which traveled over it, but it soon became lined with gasoline stations, refreshment stands, and other commercial concessions so that parts of it are now little better than the old Lincoln Highway which it supplanted. It is recognized that a new and better route will be required to relieve it.

Mr. Noble has described a type of route which will be a great advance in highway design. He mentions that "a safety factor is to prevent frontage of any type on the highway and to exclude farm and local road entry. It appears reasonable to restrict access to approximately 10-mile intervals." The writer would like to emphasize the importance of this safety factor and believes that it is an essential feature if permanent efficiency is to be maintained.

A highway built with State or Federal funds without local assessment can logically be designed primarily for non-local uses. Why not make it entirely for through traffic and exclude the right of any abutting property to local access, except at those points designated by the agency building the highway and incorporated into its design?

The "freeway" or "limited way" is the term that has been used to designate such a highway, on which general traffic can have some of the advantages which passenger cars now enjoy on the modern parkway. The word, "freeway", was coined several years ago by Edward M. Bassett, Attorney, of New York City. Other countries have made notable advances in constructing modern highways of this type, notably the toll roads built by private corporations in Italy under Government authorization and with Government assistance, and the "Reichautobahnen" under construction in Germany as part of a national highway system.

Thus far, freeway construction has made little progress in the United States due to the necessity of new legislation authorizing public highway agencies to lay out and build highways to which abutting owners will not have the right of access.

Definition.—Probably the briefest definition of a freeway is as follows[32]: "A highway to which there is no vehicular access from abutting properties." In the report from which this definition is quoted a freeway was defined as one type of express highway.

A more complete definition of a freeway is obtained by summing up the various qualities which such a route should possess, as follows. (1) For fast-moving vehicles, long haul as against local traffic; (2) for mixed traffic, including passenger cars, buses, and motor trucks, but excluding horse-drawn vehicles; (3) free from crossings at grade, and left-hand turns, unless in isolated cases in rural areas where the construction necessary to eliminate them would not be justified; (4) providing no right of access to abutting property, except at points provided by the agency designing and constructing the freeway; (5) abutting service facilities, such as gasoline and parking areas, to be provided only by, or under the control of, the public agency building the highway at sites incorporated in the design of the freeway; (6) infrequent connections to other highways; and (7) low grades and easy curvature.

The freeway system should form a part of a more comprehensive system of express routes containing two other types of express facilities. One of these facilities would be a system of parkways, restricted to passenger vehicles and providing attractive, landscaped routes with grade separations at important intersections, expanded at places to provide recreational parks. The other would be express routes composed of highways also serving abutting property, but where freedom of movement is facilitated by occasional grade separations, side roadways for local traffic, or other special treatment.

Design of Freeway.—The general design of a freeway is well indicated by the foregoing Items (1) to (7) which list the qualities it should possess. Although the need of freeways is more acute in the closely developed parts of metropolitan areas, it is more feasible to establish them in suburban and rural areas. Accordingly, the widths of right of way to be provided will vary with the character of the territory. In congested areas of high land values, compromises with existing conditions will have to be made.

In general, the design should call for three lanes of traffic in each direction, and dual roadways separated by a central unpaved area. Two lanes in each direction will often be sufficient for initial construction and, in some cases, for final development. Border strips should be provided along each side.

The Regional Plan Association of New York City recommends that outside lanes and lanes next to the central unpaved strips be 13 ft in width. Lanes not bordering on unpaved shoulders or central strips could be 11 ft wide. A minimum width of 26 ft, including curbing, is recommended for the central strip. Within this width a screen planting of shrubs and trees will eliminate the glare of head-lights at night from drivers going in an opposite direction. A minimum width of 160 ft would thus provide an adequate right of way for a freeway with an ultimate capacity of six lanes, three in each direction.

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[32] Rept. of the Committee of the City Planning Division, Am. Soc. C. E., on Street Thoroughfares Manual, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1049.

In rural areas a width of 400 to 500 ft is advisable, but this might very well vary in accordance with the topography and the ownership of the land through which the freeway passes. In this case a minimum border strip of 50 ft is desirable which might well be supplemented by the prohibition of billboards and signs within 200 ft or more of the right of way. Through rural areas where land is under cultivation a narrower right of way, supplemented by an easement over adjacent land, will reduce initial and maintenance costs and give the motorist a more interesting and informative view of the country side. As much of the motor travel in the United States is of a recreational nature, many of the parkway features, such as attractively landscaped overlooks at scenic points, picnic areas, and bridle paths, might be incorporated advantageously in freeway design in rural areas.

A series of cross-sections, prepared by the Regional Plan Association and indicating the progressive development of a freeway in suburban areas contiguous to large population centers, is shown in Fig. 8. This calls for a

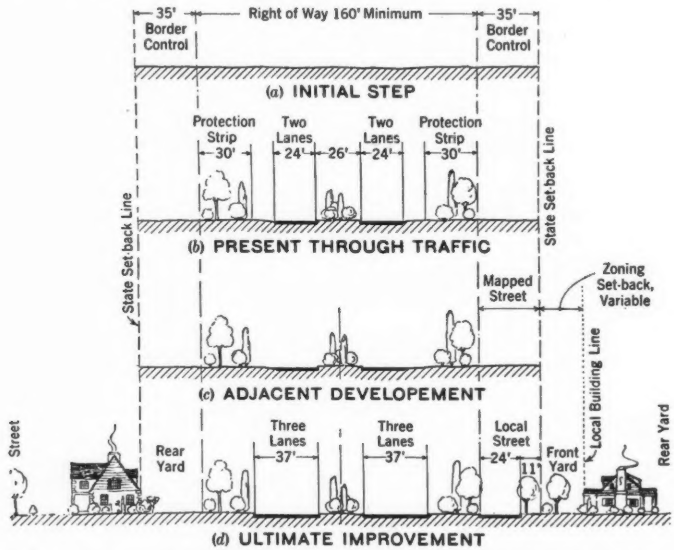

FIG. 8.—ADAPTATION OF ROAD CAPACITY TO TRAFFIC VOLUME AND LAND USE

minimum right of way of 160 ft and a 35-ft set-back line to be established by the State (see Fig. 8(*a*)), assuming that the freeway would be constructed by the State Highway Department. Early in the development (Fig. 8 (*b*)), an economical pavement is installed and trees are planted in their final location. As transition from an open area to a more intense use occurs (Fig. 8(*c*)), local circulation can be provided in "control borders." Local building lines should be established at this time. Ultimately, when future traffic justifies it, the roadway can be expanded to three lanes in each direction without disturbing the planting. A more effective separation is possible between traffic and buildings when abutting property is developed as shown on the right side of Fig. 8(*d*).

In urban areas marginal border strips might be cut down to 4 ft and the central strip to 8 ft. With these extreme measures, which should be applied only where local conditions make them necessary, a six-lane freeway could be constructed on a 90-ft right of way, or a four-lane freeway on a 64-ft width. In certain sections the elevated highway or vehicular tunnel must be resorted to for short distances.

Proposed Legislation.—Although there is general agreement as to the need of freeways and legislation to enable governmental departments to build them,

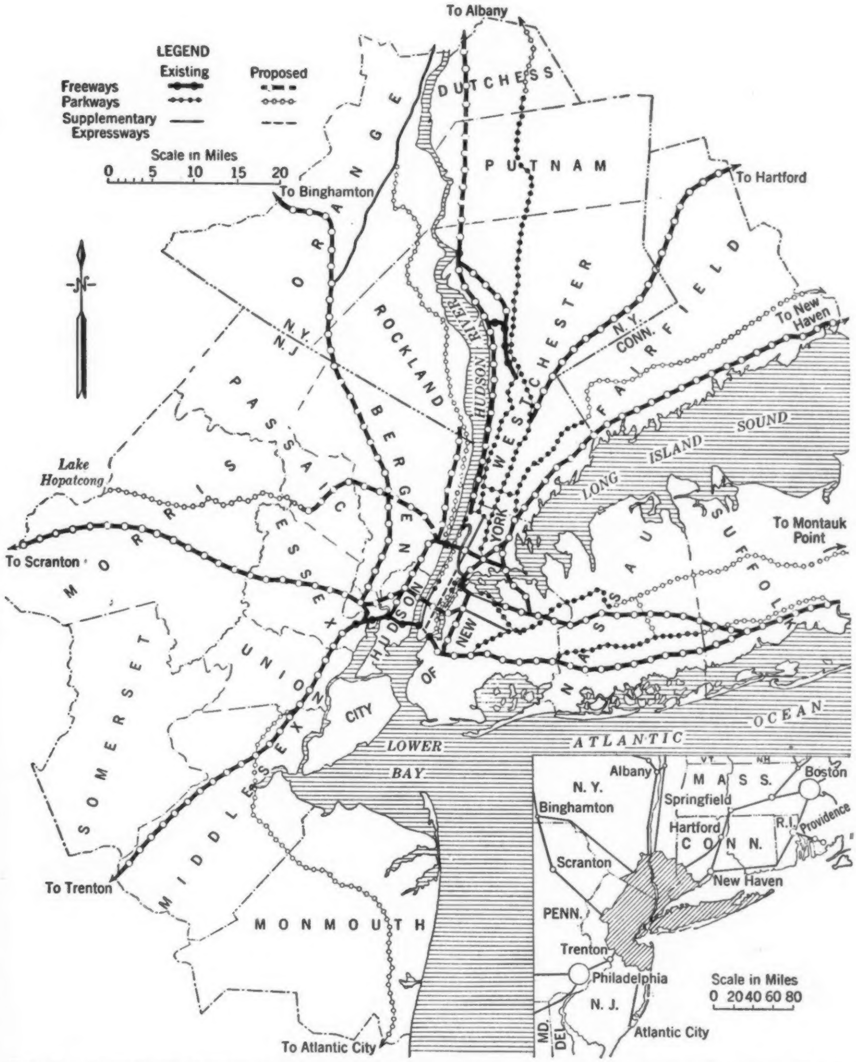

FIG. 9.—STUDY OF RADIAL FREEWAYS FOR NEW YORK AND ITS ENVIRONS, SHOWING THEIR PLACE IN AN ULTIMATE EXPRESSWAY SYSTEM

there has been considerable difference of opinion regarding the details of such legislation. The only freeways now in existence in the United States are some short sections of general traffic roads in the Westchester County Parkway System, in New York State, and the Norris Freeway constructed by the Tennessee Valley Authority. The Westchester routes were built by the expedient of placing them within parts of the County Park System, turning the roadways over to the jurisdiction of the Division of Highways of the State Department of Public Works. These roadways were constructed with State funds supplemented by Federal aid. The Norris Freeway was constructed by the Tennessee Valley Authority, a Federal agency, without any special State enabling legislation.

Freeway legislation has been introduced in the States of California, Connecticut, and Massachusetts. It has been under consideration by the State Planning Boards in Maryland and New Jersey, and the Division of State Planning in New York. It seems likely that some of these States will soon provide themselves with such legislation.

Application to New York Region.—Fig. 9 shows a study prepared by the Regional Plan Association for the application of the freeway principle to a complete system of expressways for the New York-New Jersey-Connecticut Metropolitan Area. It shows a much more extensive system of general traffic express routes than was contemplated in the express highway system proposed in the Graphic Regional Plan in 1929, already cited.

In 1933, a report[33] on the preceding four years of progress in the regional development of New York and its environs showed progress on 179 miles of express highways, or 72% of the new mileage proposed in the original plan, which had looked forward to the year 1965. This tremendous growth in the adoption of the express-highway principle justifies an expansion of the original proposals. Fig. 9 might be described as an ultimate plan of expressways for the Region looking even further ahead than the published Regional Plan. Its advantage would be that any routes constructed in conformity with it would be permanently useful for the expeditious movement of vehicles along attractive rights of way and the past sad experience of the clogging of State highways with local traffic to and from roadside development will be eliminated along new routes.

George Conrad Diehl,[34] M. Am. Soc. C. E. (by letter).[34a]—Traffic accidents in New York State during 1935 were about 10% of the total for the United States, which is also approximately New York State's percentage of population and of automobile ownership. In New York State less than 10% of the accidents occurred on State highways. The great preponderance occurred on city streets.

The mileage of modern express highways would probably never equal 30% of the total mileage of State highways. These express highways would be located principally near large centers of population. It would be safe to

[33] "From Plan to Reality", Regional Plan Assoc., 1933.

[34] Cons. Engr., New York, N. Y.

[34a] Received by the Secretary December 1, 1936.

assume that the number of accidents not occurring on State highways by reason of the use of express highways would be less than 3% of the total in the State of New York and far less in more sparsely populated States.

In New York State, the prevention of 1 800 accidents with 60 fatalities is well worth studying, but the foregoing brief statement is intended to indicate that the express highway is not a general panacea for motor-vehicle accidents.

Less than 4% of reported accidents were due to mechanical defects of the vehicle; 10% of drivers were violating speed laws; and 75% were traveling in a straight line (not on curves, etc.). Doubling or tripling allowable speeds would certainly increase accidents due to speed and those due to mechanical defects (tires, brakes, steering apparatus, etc.). It is rather doubtful whether express highways designed for 100 miles per hr. would materially decrease motor-vehicle accidents. They might even increase them. It seems that too much emphasis is placed upon speed.

The German highways designed for 115 miles per hr are primarily military roads and not "peace or commercial" highways. The German roads, like those in Italy, can scarcely be compared with the modern express highways, as they are little used and are not intended to be self-sustaining; nor is highway safety a prime object in their design.

The enormous number of vehicles that are safely carried by the Holland Tunnel, the George Washington Bridge, the Triborough Bridge, the Long Island, and the Westchester, Parkways (all in New York) with maximum speeds of 40 miles per hr, would indicate that a maximum hourly speed of 50 miles and not 100 miles should be assumed for the express highway. Any higher rate would be unsafe if roads were in general use. Although a special examination might prove the mental and physical stamina necessary to use such high speeds safely, it is unlikely that public funds could be obtained if only those especially qualified could use these modern highways.

Assuming that the vehicles could be made mechanically safe, the individual operator is certainly limited as to speed, no matter how well the engineer designed the vehicle and highway. Some limit must be fixed in the interest of public safety, and the rate should be less in the case of long, high-speed buses, of trucks, and of cars with trailers, and of vehicles of unusual length.

Often the fact is overlooked that increasing speed beyond certain limits decreases the capacity of the highways. This decrease is due to the extra spacing required for safe operation. Vehicles traveling at 50 miles per hr, according to Fig. 4, can be operated safely at intervals of 150 ft, but at a speed of 100 miles per hr, the spacing should be 600 ft, four times the spacing at twice the speed, reducing the capacity one-half. Only one-half the number of vehicles can be carried safely. The only remaining advantage is speed. Much of this advantage is lost when speed is reduced to minutes saved. This saving will be comparatively small, as sections of such modern express highways, generally considered, cannot be justified economically for extensive mileages.

The various forms of transportation—air, rail, and highway—should be co-ordinated so that all three can be operated successfully. If greater average

speeds than 50 miles per hr are desired, rail service could safely be developed from 50 to 100 miles per hr and air service much in excess of 100 miles per hr.

Expensive public improvements must be justified technically, economically, and socially. The express highway is a necessary transportation utility, but it involves unusually large expenditure. Fortunately, the mileage of such highways is very small when compared with the total.

Toll highways have not been popular in the past. In very few cases are there sound arguments in favor of the toll method of financing. If the toll method is not adopted, then the public, as well as appropriating bodies, must be convinced that the cost of express highways is justified and that there will remain in the available highway funds sufficient sums to care for all lesser classes of highways. Doubtless, the public will also insist that the work on all classes be carried on proportionately and contemporaneously.

Modern express highways must be justified by the traffic and probably a safe method of financing would require the users of such highways to pay from present registration or gasoline taxes a considerable part of the total cost. The organized motorist objects to any increase in motor-vehicle taxation, through tolls, or otherwise.

Every express highway which can be financed soundly should be built, but by the same reasoning it would be gross extravagance to construct such highways where less expensive roads would suffice.

If proposed speeds are slowed down the beautiful parkways with picturesque landscaping near the largest cities can often serve as modern express passenger highways, but their mileage should be limited, and recourse should be had to those rules of economy which properly apply to parks and parkways, as well as to business highways.

It would seem that definite general rules could be adopted for the design and construction of express highways for a period of, say, thirty years. This would permit a sufficient lapse of time to care for amortization of bonds or borrowings. Technical, economical, and social justification could be computed on the basis of thirty years, or on such other period of time as might be more equitable and desirable Proponents of each form of transportation could then develop and solve their own problems without unfair overlapping or competition.

Support for the large expenditure for express highways from legislators and those who plan the budget, whose home localities are not greatly affected by these great arteries, will be dependent upon a general plan which will provide for the equitably apportioned construction of all classes of highways within the sums available.

For at least twenty years the writer has urged the importance of "highway classification" and of "traffic distribution." All the many hundreds of thousands of miles of public highways and streets never can, or should, be designed for equal capacities, loadings, or speeds.

The individuals composing budgeting or appropriating bodies are prone to believe that the highways in their home localities are deserving of the highest and costliest types.

Formerly, the writer had suggested that the highways outside of cities be divided into four classes. Names which would be more or less descriptive to legislators were used: (1) High-cost roads; (2) low-cost roads; (3) little used roads; and (4) remote roads. It was endeavored to work out an economic justification by assuming that the actual payments of registration fees and gasoline taxes should equal one-half the road costs, including construction, maintenance, and amortization.

On this theory: (*a*) High-cost roads were assumed to carry as many as 600 000 vehicles annually; (*b*) low-cost roads were assumed to carry from 200 000 to 600 000 vehicles annually; (*c*) little used roads were assumed to carry 30 000 to 200 000 vehicles annually; and (*d*) remote roads were assumed to carry less than 30 000 vehicles annually.

The weight and length of vehicles to be permitted on high-cost roads were to be the full legal limit. On low-cost roads the maximum allowable capacity was $3\frac{1}{2}$-ton trucks and less in the case of school buses. Long trailer truck trains or long, high-speed buses were to be excluded (Incidentally, this restriction only excludes 1% of the total number of vehicles permitted on high-cost roads.) Little used roads are limited the same as low-cost roads, except that trucks must not exceed $2\frac{1}{2}$ tons. Remote roads are limited to 2-ton trucks and no trailers or buses (except smaller vehicles such as school buses) are permitted on them.

The maximum speeds varied from 35 to 60 miles per hr, with special restrictions on sharp curves of the lower types. Stage construction was planned so that one type could be changed to the next higher type with a minimum of unnecessary expenditure. A more complete description of these types cannot be made within the limits of a discussion of this paper.

The first class, namely, "high-cost roads", should be again subdivided, or at least a new class should be added; that is, the modern express highway. This latter highway could not be justified probably at less than 2 000 000 vehicles annually, and if clover-leaf intersections were adopted generally, a larger number of vehicles would be required. Very few miles of highways carry 10 000 vehicles daily and then only near large centers of population. Traffic congestion is local.

The mileage of high-cost roads is approximately only 10% of the total. The lower, less expensive types comprised 90% of the total. Formerly, traffic censuses were inadequate in number and scope to calculate the allowable mileage and justifiable costs of the several classes of highways. During the past few years, through Federal participation, there has been a great addition to traffic data and the more populous States have sufficient data to develop a fairly accurate general plan for road classifications and economic planning.

The writer's experience in the enforcement of road rules and restrictions convinces him that the traffic of the various classes could be practically, and readily, controlled. Closely allied to the subject of road classifications is the matter of traffic distribution on city streets as well as on suburban and rural roads On the most crowded Saturdays, Sundays, and holidays, there are,

in the vicinity of many large cities, mile after mile of comparatively empty roads used by few vehicles, but open to many. Pleasure seekers use the busy roads because they are not directed to idle roads. Even when one's destination is fixed, little used parallel roads are available for the passenger car and lighter trucks. The average tourist could readily be routed to arrive at or to leave on, busy city streets at other than rush hours with peak loads.

Many city streets permit of easy and undisturbed travel when greatly accepted routes are slow and difficult. For instance, it is possible to drive from mid-town New York City to the Westchester County line by several routes at the same hour of day, one route taking 35 min, another, 1 hr and 15 min, and others at intermediate periods. On the busiest road not only is $\frac{1}{2}$ hr wasted, but the traffic is still further cluttered. A continuing study and control can govern traffic by properly routing drivers who go daily between fixed destinations.

Traffic distribution is an important item in planning city terminals and within certain of the most congested areas, involves the question of compulsory mass transportation as contrasted with the use of these areas by the individual passenger car.

Obviously, the average taxpayer cannot analyze the traffic problem or study its intricate details, and yet he will re-act unfavorably to the enormous cost per mile of the express highway unless provision is made for road classification and traffic distribution. In these days of socially planned legislation one cannot disregard the lonesome vehicles on "forgotten highways" leading into hundreds of smaller cities and serving many thousands of villages and townships. Space does not permit a further discussion of traffic distribution, but much more should be said. It must be apparent that much travel could be taken from the express highway and the busiest high-cost roads.

The statement often made that most highways designed twelve or fifteen years ago are obsolete is true only of some of the crowded main highways near large centers. Literally, thousands of miles of the cheaper types are well designed for the purpose intended.

Such statements give an erroneous impression to laymen and to members of budgeting and appropriating bodies. As a matter of fact, it might better be said that the fine work of highway engineers during the past ten or twelve years has resulted in highways fully adequate for the next fifteen or twenty years. These excellent results should be emphasized so that highway engineers will be highly regarded by the general public, and their recommendations estimated at their true worth.

WILLIAM E. RUDOLPH,[35] M. AM. SOC. C. E. (by letter).[35a]—Although there is much of interest and pleasure to be gained from reading this paper, the writer considers that its basis is altogether erroneous. Mr. Noble introduces his subject with a compilation of accidents in the United States during 1935 (Table 1). Having been away from his native land for nearly three years,

[35] Chf. Engr., Mauricio Hochschild & Co., Ltd., Potosi, Bolivia.

[35a] Received by the Secretary November 9, 1936.

the writer was not a little surprised to learn that accidents accounted for injury or death to nearly a million persons annually in the United States. Then, upon turning the page, he was still further surprised to find that the author was proposing high-speed highways to accommodate faster cars, in order to "safeguard the traveling public."

Is Mr. Noble "looking in the right direction"? Year after year the manufacturer has been producing cars, each in its turn safer than the previous model, without a doubt, and also faster. Year after year the engineer has been building safer and faster highways, until Mr. Noble now suggests designing for 100 miles per hr.; and yet, with safer cars and safer highways, engineers are confronted with Mr. Noble's tabulation of a single year's terrible toll of accidents. Are they working in a vicious circle?

Safer cars, equipped with safer tires which will blow out gently if they must blow out, will probably present no difficult problem to the super-enlightened automobile manufacturer of the present day. Safer highways can be designed and constructed by the engineer, as outlined in Mr. Noble's excellent presentation of his subject; but the safer driver, the person who is not subject to ordinary human weaknesses, such as fatigue, mental lapse, or a strong cocktail—what is being done about him? Would it not seem in order to correct such factors concurrently with the physical factors, even if it became necessary to establish special schools for instructing the driver regarding the laws of motion, and the proper management of the immense amount of power at his command when he starts the motor of his car? Later, perhaps, with drivers who would depend upon good judgment rather than upon mere good luck for avoiding accidents, there might be a real need for higher-speed highways and the inevitable higher-speed cars; but they should not be projected upon the basis of a tabulation showing an annual total of nearly a million accidents.

It might be that engineers in the United States are taking their cue in this matter from high-speed highways recently constructed in Europe. This is ill-advised—for Europe requires such highways because of the inevitable war she is facing, whereas conditions in the United States are quite different (or at least it is to be hoped they are).

In Bolivia, Nature has imposed great obstacles for the road builder: Highways must wind in spectacular fashion in and out of rugged canyons, over steep mountain passes; at places there is scarcely width to pass another car. Many roads run through river bottoms, where there is no passage during the rainy reason; others become extremely difficult to travel when the clay surface becomes wet, and, hard roads are almost unknown. Despite vertical cliffs and other hazards, however, the accident factor in Bolivia is indeed low. Moreover its citizens work and produce and are happy, although they do not live so fast. Road conditions are constantly being improved, but, fortunately, they have not reached the point, thus far, which permits high speeds that might occasion risks quite out of proportion to the useful ends to be gained by such speeds.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SELECTION OF MATERIALS FOR ROLLED-FILL EARTH DAMS

Discussion

BY MESSRS. WILLIAM C. HILL, A. FLORIS, AND FRED D. PYLE

WILLIAM C. HILL,[18] JUN. AM. SOC. C. E. (by letter).[18a]—Engineers concerned with highway construction and soils will agree with the author that "probably the most severe test for the stability of earth material is as a wearing surface for an earth road." During the last few years a wearing course of sand and gravel stabilized with a binder soil has been developed for low-cost, dustless, road surfaces and bases for bituminous mats.

In Minnesota, the gravel to be stabilized in this manner is mixed with binder soils ranging from a sandy loam to a clay with plasticity indexes from 18 to 45. Care is taken that the grading of this mixture falls within the limits of a narrow band developed through the efforts of the soils engineers of the Highway Department from observations on a large number of existing gravel roads.

These roads were examined to determine their surface performance under varying traffic and climatic conditions. Samples were taken and analyzed in the laboratory to determine the gradation of those mixtures which were yielding most satisfactory performance. On the basis of these results, together with subsequent observation on some of the earlier projects stabilized by the Maintenance Department, the present specification was developed. This band differs from that of the United States Bureau of Public Roads given by the author, as shown in Table 8. (In a later report[19] than that cited by the author[9] the Bureau has discontinued the use of the No. 270 sieve in favor of the No. 200, for practical purposes).

NOTE.—The paper by Charles H. Lee, M. Am. Soc. C. E., was published September, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1936, by Messrs. T. T. Knappen, and Paul Baumann.

[18] Dist. Soils Engr., Dist. 4, State Dept. of Highways, Brainerd, Minn.

[18a] Received by the Secretary November 16, 1936.

[19] *Public Roads*, May, 1936, p. 50.

[9] *Loc. cit.*, February, 1935, p. 277.

TABLE 8.—PERCENTAGE PASSING BY WEIGHT

Sieve	U. S. Bureau of Public Roads grading	Minnesota grading
1-in.	100	....
¾-in.	85 –100	100
⅝-in.	....	95 –100
⅜-in.	....	70 – 95
No. 4	55 – 85	....
No. 10	40 – 65	35 – 65
No. 40	25 – 50	20 – 40
No. 100	....	15 – 30
No. 270	10 – 25	10 – 20

Fig. 13 shows the relation of the Minnesota band to the theoretical Talbot curve obtained by use of Equation (1) with the exponent, n, equal to 0.33. This curve indicates the theoretical grading to give maximum densities for

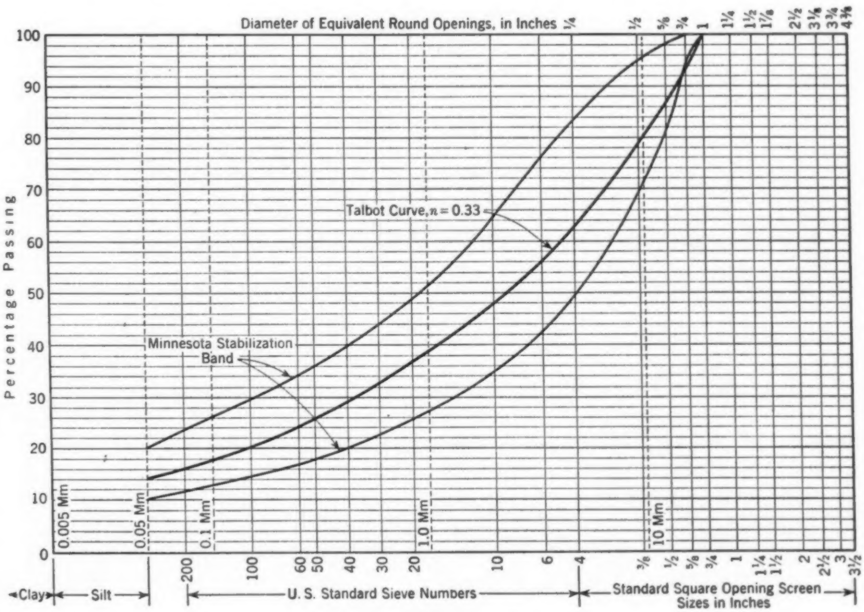

FIG. 13.—MECHANICAL ANALYSIS

aggregates having a maximum size of 1 in. to 2 in. It will be noted that the Talbot curve falls along the middle of the band. Where a close control of the mixing of the gravel with the binder soil has kept the grading near the middle of the band, a hard dense road surface has resulted.

The author points out that there is an optimum moisture content for maximum density. C. A. Hogentogler, Assoc. M. Am. Soc. C. E., has shown[20] that temperature influences this optimum point. In a series of experiments Mr. Hogentogler used three fine-grained soils and demonstrated that the

[20] *Public Roads*, May, 1936, pp. 48–50.

higher the temperature the greater would be the density with the lower moisture content and equal compaction. He follows this with a demonstration proving that the stability of soils varies inversely with the temperature and is a reversible action.

A. FLORIS,[21] ESQ. (by letter).[21a]—In this scholarly paper the author discusses earth testing as an aid to the selection of materials for rolled-fill earth dams and establishes five principal requirements regarding suitability of material for this type of dams; Stability, water-tightness, workability, insolubility, and reasonable cost.

The first of these requirements is essentially one of statics and, according to the author, can be secured in a well-graded material through mechanical compaction. In other words, the author, in line with other authorities on the design and construction of earth dams, believes that the stability of a rolled-fill earth dam can be best assured, if a properly graded material is used. By analogy one might eventually conclude, that the safety of concrete dams can be taken for granted if the material is properly prepared. This is not correct, of course, because in addition to the quality of the concrete the forces acting on the dam must be investigated. This holds equally true for the earth dams treated by the author. Under the action of water pressure and dead weight, concrete dams can be analyzed quite satisfactorily. No such analysis is generally available for earth dams, and, unfortunately, the scope of the paper does not embrace this phase of the subject.

Too much emphasis has been laid upon the material and very little attention, if any, has been paid to the statical condition caused by the forces acting upon earth-fill dams. As far as the writer is aware the first attempt to remedy this unfavorable condition was made by S. Ehrenberg[22], whose analysis is based upon new concepts of soil mechanics.

FRED D. PYLE,[23] M. AM. SOC. C. E. (by letter).[23a]—In discussing the factors to be considered in examining materials for the construction of rolled-fill dams, Mr. Lee has performed a notable service to engineers. Personal observations made many years ago of priming of canal banks constructed by teams, indicated that where material is reasonably well graded no trouble is encountered, but where it is of uniform size (whether clay, sand, gravel, or rock) there is a leakage with resultant sloughing on the outside of the bank where the materials are sufficiently fine to prevent free drainage. Except for the top 2 or 3 ft, satisfactory highway fills may be constructed of ungraded materials, sand, gravel, or rock, provided there is sufficient compaction, drainage, and lateral support to prevent excessive settlement.

[21] Dipl.-Ing., Los Angeles, Calif.

[21a] Received by the Secretary November 18, 1936.

[22] "Grundlagen der Berechnung von Staudaemmen," von S. Ehrenberg, *Wasserkraft und Wasserwirtschaft*, 1929 No. 23.

[23] Hydr. Engr., Div. of Development and Conservation, Water Dept., City of San Diego, San Diego, Calif.

[23a] Received by the Secretary November 18, 1936.

It would appear that the following factors, in addition to those discussed by Mr. Lee, might well be considered:

(*a*) Consolidation by weight, which may be determined by testing and which will become evident in the completed dam by the rate and amount of settlement;

(*b*) Allowable percentages of colloidal and clay materials; and

(*c*) Capacity of materials, when placed in rolled-fill dams, to adjust themselves to settlement without cracking or formation of planes of lessened resistance to percolation of water.

Referring to Factor (*a*), consolidation tests should include, in addition to tests of normal material, tests of such material with the introduction of water after a consolidation head equal to one-third the height of the dam is reached. The weight load of the fill tends to increase density with corresponding reductions in the voids, and permeability. More information should be secured by engineers on the rate and amount of settlement of all dams constructed of earth and rock, both during and after construction.

An excess of fine materials (see Factor (*b*)) often means a large capacity for water which may result in excessive cracking and checking on loss of moisture content and excessive settlement.

Concise, brief definitions of certain words or terms used by the author and by others would be of assistance to those interested in studying dams constructed of earth, whether by rolled or hydraulic methods.

In order that comparisons may be made quickly with a minimum of effort it is believed advisable that gradation graphs be in millimeters, with inches and sieve sizes shown at the top and the classifications—clay, silt, sand, gravel, etc.—shown across the middle of the sheet. It is interesting to note that material having diameters between 0.04 in. and 0.2 in. is classified as gravel on gradation graphs.

As it is practically impossible, in the construction of rolled-fill dams, to mix materials from several sources to secure a modified material having or approaching the ideal characteristics, it becomes necessary to determine the suitability of local materials by inspection and by applying the laboratory tests, suggested by Mr. Lee.

Under the heading, "Impermeability", the author discusses permeability coefficients without mention of the degree of pressure or loading on the materials when tested. As the pressure has an effect on the voids, there must be some effect on the permeability coefficient.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STRUCTURAL APPLICATION OF STEEL AND LIGHT-WEIGHT ALLOYS A SYMPOSIUM

Discussion

BY MESSRS. J. CHARLES RATHBUN, FRED L. PLUMMER, C. F. GOODRICH, G. K. HERZOG, JOHN H. MEURSINGE, P. G. LANG, JR., AND W. L. WARNER

J. CHARLES RATHBUN,[116] M. AM. SOC. C. E., AND D. M. MACALPINE,[117] JUN. AM. SOC. C. E. (by letter).[117a]—The field of photo-elasticity has been covered by Mr. Brahtz in a most creditable manner. It does not, however, mention the multi-material use of the photo-elastic method. The idea has been suggested before, but the writers know of no attempt being made to use it. This type of model differs in several important respects from those described in the paper. At the College of the City of New York the photo-elasticity equipment is composed of a standard polariscope, with Nicol prisms and a carbon light source. Although this apparatus is equipped with a specially designed hydraulic loading device, it was considered more satisfactory to apply the loads as weights attached to the lower end of the models.

The advantage of this arrangement can be visualized best in the solution of the problem of determining the partition of a load among the rivets in a butt-joint connection. The model for this case consisted of three plates riveted together, the load being applied to the center plate and the reaction being furnished by the two outer plates.

An attempt was made to coat the inner surface of one of the outer plates with a reflecting material, with the intention of obtaining the patterns after the polarized light had passed through the same plate twice. The results were not satisfactory and the idea was abandoned. A mirror was then inserted between the plates to take the place of the reflecting coating. This appears

NOTE.—This Symposium was presented at the meeting of the Structural Division at Pittsburgh, Pa., October 14–15, 1936, and published in October, 1936, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: December, 1936, by Messrs. E. Mirabelli, R. W. Vose, Raymond H. Hobrock, William F. Clapp, J. C. Hunsaker, Horace C. Knerr, and F. T. Sisco.

[116] Assoc. Prof., Civ. Eng., Coll. of the City of New York, New York, N. Y.

[117] Instructor, Coll. of the City of New York, School of Technology, New York, N. Y.

[117a] Received by the Secretary, October 26, 1936.

to be unsatisfactory for several reasons, among them being the effect of the mirror on the stress distribution and also the effect of the distortion of the mirror when stress was applied to the model. Tin foil, aluminum foil, and silvered glass were all tried.

Models were then made which consisted of a central plate of marblette and two outer plates of celluloid, the idea being that, since marblette is much more sensitive than celluloid, the error due to the effect of the less sensitive material could be neglected in the first approximation to the solution. This error may possibly be evaluated and allowed for, if it is considered necessary, by using two sets of models alike in every respect except that different materials are used for the outer plates. There would then be two solutions of the same problem, each containing a set of errors. With the properties of the two materials, and the ratio between the errors known, the absolute value of the error might be computed or estimated, and the correction made.

The first model tested consisted of two strips of celluloid, 1 in. wide and 0.1 in. thick, riveted on either side to a strip of marblette 1 in. wide and 0.25 in. thick. Five rivets, 0.25 in. in diameter and spaced at 0.75-in. centers, were used, the rivets being in line. The value of two 0.1-in. plates of celluloid, when multiplied by the value of E (350 000 lb per sq in.) for celluloid, is approximately the same as that of one 0.25-in. plate when multiplied by the value of E (250 000 lb per sq in.) for marblette, so that the model represented three plates of approximately equal thickness riveted together. Furthermore, since the rivets were made of marblette, the plates had an effective thickness equal to the diameters of the rivets.

The problem of using rivets having a snug fit, but at the same time not introducing any initial stress in the sensitive marblette, was solved by using rivets having a loose fit and coating them with liquid marblette; that is, cementing the rivet into place with a very thin coat of marblette.

This design of model was used because both experimental and mathematical checks on the results were available. Although common practice assumes the partition of load as uniform between the rivets, it is easily shown that the outer rivets carry much more load than the inner ones. The partition obtained by the photo-elastic analysis of the multi-material model checked those obtained analytically within less than 2% in all cases and within less than 1% in most cases.

This experience has encouraged the writer to run a series of tests on rivet groups (not in a line) of five and six rivets when the load applied is eccentric to the grouping and also when the eccentricity is zero. That the results differ materially from those obtained using the assumption made in practice when designing the rivet groups, is quite obvious. It is hoped that a series of these investigations will yield very useful data. This same multi-materials method can be applied to the partition of the load among the rivets in the standard connection between beams and columns in building construction.

For the purpose of experimental study, models were made in which the three plates were of the same material. The resulting confused pattern could not be separated into the separate patterns of the inner and outer plates. This same tendency ensued when high loads were used with the multi-materials

models. If the loads used were great enough to produce isochromatics and isoclinics in the celluloid, some confusion was found in interpreting the results. When an isoclinic from the celluloid crossed one from the marblette they destroyed each other, producing a light spot and not a dark spot. This confusion was avoided by using stresses so low that the effect on the celluloid was not visible whereas the isoclinics of the marblette were quite definite. A further advantage encountered in the use of low loads on marblette models is the reduction of creep to a negligible amount. The investigations found little evidence of creep under these loading conditions. This multi-material use of the photo-elastic method is worthy of further study.

FRED L. PLUMMER,[118] ASSOC. M. AM. SOC. C. E. (by letter).[118a]—The design of any part of a machine or structure, the size of which is determined by the stresses or deformations which may be imposed upon that part, represents a structural problem which deserves the attention of a competent structural engineer. Too many structural engineers think of fixed structures—buildings and bridges—as the only proper field for their activities. The design of the side frame or the truck of a railway car, the structural frame of an airplane or of a bus, and the hull and deck structures of boats are also structural problems. They sometimes present very difficult design problems which challenge the talents of the best engineers.

Only a few years ago, many structures of this type were built by processes which placed definite limitations on the choice of sizes and shapes of the various parts. As a consequence, much of the analysis and design was reduced to an arbitrary "rule-of-thumb" or "cut-and-try" process. The introduction of high-strength and light-weight metals, together with the use of new fabrication processes, has caused such methods of design to become quite inadequate and has created a wide and comparatively new field of activity for the structural engineer. New problems are involved; however, structural engineers are better trained than any others to analyze and design these mobile structures properly. It will be unfortunate if through neglect the manufacturers of such structures and machines are forced to re-train men who are now trained primarily to design mechanisms and not structures.

The design of any structure involves first an attempt to predict what forces will act upon the completed structure and the conditions under which those forces will be active. It is then necessary to determine as accurately as possible (and the degree of precision is seldom even close to perfect) the stresses and distortions that will probably be created in the structure by the applied forces, whether those forces be applied loads or destructive agencies of entirely different character. Finally, the material must be selected, and the proper quantity and shape of that material must be determined, which will provide most efficiently a safe and useful service life for the projected structure. Included in this final step is the selection and design of proper connections so that the parts of the structure may be assembled and act as a

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[118] Associate Prof., Structural Eng., Case School of Applied Science, Cleveland, Ohio.

[118a] Received by the Secretary, November 4, 1936.

unit. This problem, which is perhaps the most difficult of the group, frequently receives too little study.

High-strength steels and the light-weight alloys are usually selected for use in a given structure because of superior corrosion resistance properties or because the weight of the structure can be decreased by the use of such metals. This Symposium is devoted to a consideration of light-weight structures. Since these metals cost more than the materials now in common use, their selection can only be justified if the resulting structure is safer, if a longer service life can be expected, or if the extra cost can be balanced by lower maintenance and operation costs, or by greater income-producing possibilities.

The papers presented by Messrs. Ragsdale, Hartmann, and Winston have outlined the economical use of such metals in buildings, bridges, and erection equipment; excavating and materials-handling equipment, including cranes, trucks, etc.; in transportation equipment, including trains, buses, airplanes, and airships; on boats; and for a number of other miscellaneous uses.

The structural designer must re-study the fundamentals and not assume blindly that the principles followed in the analysis and design of structures built of materials now in common use must necessarily hold if similar structures are built of these newer metals.

In presenting these papers, the various authors of this Symposium have called attention to such factors as: (1) The non-linear distribution of unit stresses; (2) the effect of surface conditions on the strength of metal parts; (3) the effect of pulsating stresses "which may result in fatigue failures"; (4) the possible results of a small margin between yield point and ultimate strength, making less probable the relief of stress concentrations by the yielding of some of the over-stressed parts; (5) the difficulties of obtaining satisfactory joints of high-strength materials; (6) the problems created by the heat of welding if that method of fabrication is used; (7) the stress distribution between rivets in joints involving large plates and a great number of rivets; (8) the thin sections which frequently result when high-strength materials are used, give rise to a number of problems (corrosion may have a much greater relative effect; and failure by buckling becomes a greater possibility); and, (9) low moduli of elasticity and thin sections both contributed to greater distortions, more flexibility, and possible serious vibrations.

Not many years ago, few structural engineering organizations included on their staffs men capable of analyzing a highly redundant structure. It was common practice, in so far as possible, to design and construct all structures as simple non-continuous elements. Now, every large design staff includes a number of men trained in the analysis and design of statically indeterminate structures. Engineers are trained to meet and solve new and difficult situations, and they would be false indeed to the traditions of their profession if they were to allow the foregoing difficulties to prevent the use of these new metals. On the other hand, they must not use them without making a most careful study of the probable performance of the resulting structure as well as a thorough investigation of the economic factors involved.

The papers by Messrs. Ragsdale, Hartmann, and Winston contain a wealth of accurate design data. The authors have discussed in detail the proper treatment of many of the difficulties which accompany the use of these metals. Very briefly, they have indicated some of the economic factors that determine whether such metals should or should not be used. Each author might well amplify this part of his paper.

The increased use of these newer metals seems inevitable. Every structural engineer should study for himself the comparative design of at least one structure so that he can more fully appreciate the possibilities that lie in the development of these metals.

C. F. Goodrich,[119] M. Am. Soc. C. E. (by letter).[119a]—All structural engineers who are faced with the problem of designing any large steel structure should read carefully the paper by Mr. Moisseiff because, although it contains no new technical formulas or data, it presents facts which, if studied and followed, will lead to better and more economical designs. There are two points, especially, which might well be more strongly emphasized:

(1) The very rapid development of the high-strength steels in the last decade and the contrastingly slow previous development; and,

(2) The note of caution against the promiscuous use of the various high-strength steels for structures, or for parts of structures, where ordinary standard carbon steels would have been more economical.

Prior to 1927 there were only two high-strength, low-alloy, structural steels in use: Nickel steel and silicon steel. Since the introduction of the so-called manganese steel used in the Bayonne (N. J.) Arch, high-strength steels of various chemical content and heat treatment have appeared on the market. It took ten or fifteen years for silicon steel to come into such general use that it could be obtained from any rolling-mill or from stock, or could be fabricated in any shop and at a cost only \$10 or \$15 per ton greater than that of medium structural steel. The high-strength steels that have been produced since 1927 are far from standard at present. There is a tendency among many engineers to seize upon one or another of these newer steels, which exhibit desirable physical qualities, and apply them to structures, without much knowledge of their cost, simply because their use results in a lighter structure.

Mr. Moisseiff cautions against the use of high-strength steels in a small percentage of the members of a structure. The designer must realize that quantity is a large factor in the production cost of steel. Three or four members of high-strength steel in a structure, built generally of medium carbon steel, will cost far more per pound than this same steel in a structure built almost entirely of it.

The condition, pointed out by Mr. Moisseiff, which has existed in the last two decades (wherein the engineer is continually demanding a higher grade

[119] Chf. Engr., Am. Bridge Co., Pittsburgh, Pa.

[119a] Received by the Secretary November 7, 1936.

of steel from the producer) is a very healthy one. The laboratory, the mills, and the shops need this stimulus if the art of steel-making is to advance, but this growth must not be of the mushroom kind. The engineer must determine how his demands affect the producer: Whether they can be met at reasonable cost, or whether they will defeat the purpose for which they were made—economy and ultimate low cost to his client. Interruptions to continuous production, at the mills, of one grade of steel in order to introduce a special grade, cost money and the engineer's client pays for that extra cost. Unfortunately, the client, being a layman, does not always know that, but must depend upon his engineers for proper economy. Even the engineer sometimes makes the mistake of thinking that his case may be a "special one", that the mills will be glad to get his order and so will absorb the extra cost. There is no need to point out that this is false economy, but it happens constantly.

Very short deliveries are now being specified. The standard structural steels lend themselves to these short deliveries far better than the special steels, because they are standard product.

The economical problems placed before the designer and the producer by the introduction of these many new high-strength steels are not simple. Their solution will take time and study even if the demand for stronger and better steel is ever pressing.

G. K. HERZOG,[120] ESQ. (by letter).[120a]—The tremendous increase in the use or the stainless, high-alloy steels for structural purposes should make the paper by Mr. Morris of particular interest to civil engineers who are, perhaps, as yet not quite as familiar with their advantages as are mechanical and chemical engineers. For the latter they are the solution of many a heretofore insoluble problem. As Mr. Morris states, the method of manufacture and the production of these steels are in a state of flux and rapid progress is being made. The same is true of the modifications of the compositions of the various types of steel to increase their strength or corrosion resistance, or to improve their fabricating properties. He has briefly touched upon some of these modifications in composition, but has perhaps not emphasized their importance quite enough.

The addition of 2 to 4% molybdenum to the 18–8 (18% chromium and 8% nickel) type of steel not only increases its resistance to corrosion, but also has a marked favorable effect on its creep strength at elevated temperatures. The increased resistance to corrosion is such that in a number of applications for which ordinary or plain 18–8 steels cannot be used, the grade containing molybdenum is entirely satisfactory; that is, the addition of molybdenum so enhances the corrosion-resisting properties of this type of steel that it opens up entirely new fields of application.

The same is true of the use of columbium. The ordinary grade of Alloy 18–8 cannot be used satisfactorily in certain temperature ranges because of the structural changes which this steel undergoes when exposed for any length

[120] With Electro Metallurgical Co., New York, N. Y.

[120a] Received by the Secretary November 21, 1936.

of time to temperatures within this range. This may result in premature failure of parts operating at these temperatures. The addition of ten times as much columbium as the carbon content stabilizes the steel and entirely prevents these structural changes. This improvement is brought about without in any way impairing any of the desirable physical or chemical properties of the steel.

In discussing the plain chromium, or ferritic type of stainless steel, Mr. Morris mentions briefly the addition of small quantities of nickel, molybdenum, and silicon to improve the physical properties and the corrosion resistance of this type. A very interesting and important development in these steels is the addition of nitrogen to improve their physical properties and workability. In the form of castings or ingots these steels, without nitrogen, possess a coarse grain structure which makes them relatively brittle and difficult to forge, roll, or otherwise hot work. The addition of a quantity of nitrogen equivalent to about 1 part per 120 parts of chromium greatly refines the grain, improves ductility and strength, and makes it much easier to hot work the steels.

Another important new development in the plain chromium steels is the addition of columbium to prevent so-called "air-hardening." When some of these steels are heated above a critical temperature (as, for example, in welding), and are then allowed to cool normally in air, they become brittle. This disadvantage is entirely eliminated by the use of the proper quantity of columbium. Furthermore, these improved properties are obtained without any sacrifice of resistance to corrosion at ordinary or elevated temperatures.

These improvements in physical and chemical properties are perhaps of greatest interest to the chemical and mechanical engineer who must design equipment to operate at higher temperatures and under more corrosive conditions than are in general encountered by civil engineers. However, the writer believes the latter will be interested in these developments.

JOHN H. MEURSINGE,[121] ASSOC. M. AM. SOC. C. E. (by letter).[121a]—This Symposium happens to be of unusual interest for the Civil Engineering Profession, because it calls attention to the fact that structural engineering has entered into the third phase of its existence, the period called "neotechnics" by the late Patrick Geddes.

As many civil engineers will not be familiar with the symptoms which accompany this event, it will be necessary to dwell for a moment on the philosophy of that great Scotchman. His ideas have recently been worked over and broadened by Lewis Mumford[122] whose work has furnished most of the information which will be used for this discussion.

Mumford has shown[122] how everything on earth grows or develops according to the same pattern. This phenomenon holds true for Man and beast, for machines, architecture, political science, etc.—and also for structural engineering. The pattern distinguishes three periods that fade out in each other.

[121] Huntington Park, Calif.

[121a] Received by the Secretary November 21, 1936.

[122] "Technics and Civilization", by Lewis Mumford.

For human beings one may refer to babyhood, childhood, and manhood or womanhood. For machinery, utilities, etc., Mumford calls these phases: Eotechnics, paleotechnics and neotechnics. In the eotechnic stage, the object is still close to Nature; it is poorly developed, but nevertheless it is at most times still attractive to the eye. In the paleotechnic stage, the object increases its speed (if it has any) and it grows quickly, quite often out of proportion; it becomes ugly. In the neotechnic stage, it becomes more efficient, it improves its appearance, and it adapts itself to its surroundings.

Each phase utilizes its own power and materials. According to Mumford: "The eotechnic phase is a water and wood complex, the paleotechnic phase is a coal and iron complex, and the neotechnic phase is an electricity and alloy complex."

If the structural engineer wishes to use the alloys to their fullest advantage he must understand the characteristics of the "neotechnic" period. As a matter of fact he has already obtained results that suit the demands of the neotechnique. Where in the paleotechnic area the cost of a refined design was often greater than that of the materials that could be saved, in the "neotechnic" area, due to the more expensive alloys, the refined design has become a profitable item. This change has been emphasized by most of the authors of the Symposium. Mr. Karpov states: "More refined designs will involve additional engineering work." In the "Synopsis" of his paper, Mr. Brahtz states: "The more expensive alloys make it imperative to the designer, that he avail himself of every possible means of refined stress analysis"; and Mr. Winston states: "* * * careful attention to small details has been an important factor in the success of these applications." Mumford also has stressed the importance of the same fact:

> "The lightness of aluminum is a challenge to the more careful and more accurate designer in such machines [and utilities] as still use iron and steel. The gross over sizing of standard dimensions with an excessive factor of safety based upon a judicious allowance for ignorance, is intolerable in the finer designs of airplanes; and the calculations of the airplane engineer must in the end react back upon the design of bridges, cranes, steel buildings; in fact, such a reaction is already in evidence. Instead of bigness and heaviness being a happy distinction, these qualities are now recognized as handicaps; lightness and compactness are the emergent qualities of the 'neotechnic' area."

The technical philosopher and the structural engineer have reached the same conclusion. The importance of this fact should not be under-estimated, for it proves that they can work together. The structural engineer can profit by this in regard to his future.

What can he expect? To sum up the characteristics of the neotechnic area: It does not call for bigger structures, but for more efficient, better looking, edifices which adapt themselves to their surroundings. Whereas the paleotechnic complex (which, for many structures, is still in existence), called for a centralization of the population, the neotechnic complex tends toward decentralization. Subsequently, modern demands will be for structures the sizes of which are within the present scope. Therefore, it is interesting to note that Messrs. Moisseiff and Hartmann call attention to the limitations of

the alloys. Mr. Winston calls for shorter members, at the same time mentioning the high fatigue endurance of this alloy.

This last characteristic means higher efficiency, a typical requirement of the neotechnic phase. This higher efficiency goes also with all the noncorrosive alloys. To quote Mr. Aston: "* * * the tangible depreciation of the cost of abandoning structures is exceeded in a monetary sense by the more intangible effects of designing structures heavier than the requirements of working stresses * * *." Stronger and lighter bridges, earthquake-proof buildings, and better homes will call for the application of the new alloys. The structural engineer and the architect should find the world waiting for the application of these new materials.

The demands for the architect's services should increase as shape (appearance) plays an important rôle in the neotechnic complex. In certain areas where high wind pressures are the rule, stream-lined buildings may have a future. Shape will not be less important for the earthquake-proof structures. A material equally distributed throughout the entire building will keep the earthquake stresses low, not to mention the advantages already obtained by the use of the lighter material.

The average structural engineer should have no difficulties in adapting himself to those first two requirements of the neotechnic phase. The third requirement, adaptation to its surroundings, may cause more difficulties, because this means a better place in present-day society or, better stated, the neotechnic society. Although many people cannot see it, the fact remains that pecuniary differences will disappear. To quote Mumford:

"There is no qualitative difference between a poor man's electric bulb of a given candle power and a rich man's, to indicate their differing pecuniary status in society, although there was an enormous difference between the rush or stinking tallow of the peasant and the wax candles or sperm oil used by the upper classes before the coming of gas or electricity."

The structural engineer should understand that the improvements of the neotechnic phase tend toward a classless society. Those who disagree should not waste their efforts on the design and the application of the new materials, for only a mind freed from all the traditions, prejudices, and superstitions of the present era can expect to be successful in this kind of work. Hence, fiction writers, whose imagination is not kept within certain limits by tradition, cost estimates, or conservative employers, have often been able to encroach into the structural engineer's territory.

In the Nineteenth Century the French writer, Jules Verne, visualized a modern submarine long before any engineer had given thought to it. To-day, history is ready to repeat itself. Writers such as H. G. Wells and others indicate that the structural engineer is lagging behind in imagination and application. Is he going to take up the challenge?

If he wants to, he should prepare himself for a struggle. The structural engineer will have to adapt himself to the new circumstances he has brought forth by his own efforts. Many of them soon will find themselves in "no man's land." On one side neotechnics will call for an elaborate design at a high engineering cost, and on the other side will be the employer who is

still firmly submerged in the paleotechnic ideas of the profit system. The last one, no doubt, will insist on keeping the engineering cost down to the lowest possible level. The structural engineer, by the use of a great diplomacy, will have to make the best of it, so that he may continue his services to mankind. To succeed he will have to study his technical heritage.

P. G. LANG, JR.,[123] M. AM. SOC. C. E. (by letter).[123a]—A scholarly and illuminating treatment of this subject, Mr. Moisseiff's paper merits careful study by all whose duties concern the design, construction, or maintenance of steel structures. The statements made therein concerning mass and automotive transportation, long-span bridges, and highway bridges for motor traffic are most appropriate, and are complete in a degree which leaves no margin for elaboration or criticism. The paper epitomizes in concise and logical form the present status and prevailing trends of the structural steel industry, especially in its relation to bridges.

In forecasting tendencies in railroad bridge design, a factor of major importance must necessarily be the anticipated developments in railroad equipment, and the present indications are, and have been for some time, that the average weight of such equipment and the consequent severity of its effect on railroad structures is likely to undergo little if any increase in the near future. The demand for increased speed on railroads, however, is continuous and unabated, and it is impossible to surmise any limit to developments in this field. Increasing speeds are inevitably accompanied by increased vibration in the bridges carrying such traffic, and this, in turn, calls attention to questions of impact and fatigue. Apropos of this phase of the matter, it is worthy of note that Mr. Moisseiff establishes a clear distinction between bridge and building work, on the basis of the primary difference that impact and stress reversal, characteristic of the former, are absent in the latter.

Specific research objectives, designed to benefit producer and consumer, alike, may well include enhancement of the corrosive-resistant properties of structural metal. This is of obvious importance from a maintenance standpoint. In an effort to improve the corrosive-resistant quality of steel, some specifications were amended to provide for a copper content of 0.2 per cent.

Considering the fact that present-day, ordinary, carbon steel has been in use for forty years or more, and that its reliability has been well tested in practice, when alloy steels are used some consideration should be given to temperature effects, particularly the effects produced by comparatively low temperatures. Structures are frequently built in locations where temperatures range well below zero Fahrenheit, 40° to 50° below being not uncommon in the United States.

Since steel is a mass production commodity, economy of manufacture, in a major degree, is predicated upon uniformity of requirement on the part of the consumers. Any deviation from established practice is expensive and disruptive, and it is probable that in the steel industry the conflict between

[123] Engr. of Bridges, B. & O. R. R., Baltimore, Md.

[123a] Received by the Secretary November 24, 1936.

scientific and economic justification of new processes is most sharply defined. Each producing unit represents a vast investment, and changes in procedure involve alterations in equipment and accessories which are admitted only when absolutely unavoidable. Thus, the development of appliances and processes tends to stagnate. From the standpoint of the consumer, it is necessary that the material be purchasable at a moderate price, and, if it possesses the qualities that fit it for structural use, there is little incentive to assume the additional cost necessary to procure a steel of improved quality, unless this improvement is so great as to leave no room for doubt of economic justification. It is only natural that any specification which provides for material differing from the standard product should meet with some opposition.

Steel of the quality in common use for structural purposes at this time was specified and manufactured as early as 1889, nearly fifty years ago. During this long period, although there have been occasional demands for quantities of high-strength steel for large and important bridges, the demand which composes the major volume of the steel business, and determines its policies and practices, has been for material to be used in ordinary bridges, of moderate size. The actual need as to material for structural work, at this time, is for an improved quality of steel to be used in structures of this latter character, and the inability to produce material of more desirable characteristics seems more apparent than real.

Until 1926, structural steel for use on the Baltimore and Ohio Railroad was purchased to a minimum specification requirement of 30 kips per sq in. A review of test reports covering the actual properties of about 14 000 tons of steel used in bridges on that railroad, and bought to the specification named, indicated that more than 99% of the tonnage represented possessed an elastic limit of 35 kips per sq in., or more, and the specification was changed accordingly to establish that value as the minimum. This change was followed by prompt and vigorous protest, which subsided as soon as it was demonstrated that for a long time steel had actually been furnished which met or exceeded the new specification requirement, and that, obviously, no increase of cost was involved.

Within the recent past, welding has achieved recognition—and, for work of certain classes, or work situated in certain localities it has received preference—as an articulative process for metal structures. The stability of structures with such connections is necessarily conditioned upon the strength and homogeneity of the jointure welds, and, in devising procedure and materials to insure this condition, the character of steel work becomes a factor of much greater importance than in the case of a structure with riveted or bolted connections.

For many years no attempt has been made to impose a specification requirement limiting the carbon content of structural steel, and, at present, only a small proportion of the structural steel produced and used is subjected to a formal carbon limitation. With the introduction and extensive adoption of structural welding, this phase of the problem has become accentuated, and

research indicates that for steel structures thus connected it is highly desirable that carbon be limited to 0.25% on check analysis. The difficulties of production largely vanish when the pertinent facts are ascertained and considered. Although little attempt has been made to impose a definite limitation on carbon for many years, tests of ordinary steel have included ladle and check analyses of carbon content, and a review of test reports covering a very large tonnage of steel work actually furnished for structures of various kinds, principally railroad bridges, clearly shows that only approximately 1% would have been rejected on the basis of carbon content had the 0.25% maximum been stipulated. Hence, no economic grounds exist for objection to such a requirement.

To-day's apparent tendency in the development of specifications for steel construction is toward precision and exactitude, taking cognizance in the design of every ascertainable element, and reducing to the minimum the factor of safety or, as it is sometimes denominated, the "factor of ignorance." Continued progress in this direction is dependent upon the accumulation of exact knowledge concerning the characteristics of structural steel. With respect to the economic phase in its relation to design, as stated by Mr. Moisseiff, it is necessary to balance carefully, the advantages of reduced cost arising from the adoption of stronger steel and, consequently, lighter sections against possible objectionable flexibility in the structure.

In the case of certain alloy steels the cost increment is unquestionably a factor of considerable importance, requiring economic justification. The prices of nickel and silicon steel exceed that of ordinary carbon steel by about 3 cents and 1 cent per lb, respectively. Furthermore, the use in any structure of a steel differing materially from that readily procurable on the market, or the use of steels of different characteristics in the same structure, should be admitted with caution, in view of complications which are likely to arise incident to repairs or partial replacements in the future.

Apart from those portions of Mr. Moisseiff's paper which relate to abstract physical characteristics of the steel, the discussion concerning the necessity for considering deflections and buckling of plates develops a topic which it is impossible to emphasize too strongly.

With the growing importance of welding in connection with steel construction, the distribution of stress at joints acquires new importance. Questions of this character have arisen on numerous occasions in connection with riveted work, and their recurrence in connection with welded joints does not constitute a new problem; it merely represents a novel phase of one which has existed since the beginning of steel construction.

In conclusion, it may be remarked, as a matter of actual experience, that it is possible without increased cost to manufacture steel in conformity with Specifications A-7-34 of the American Society for Testing Materials, which form the basis of the Specifications for Steel Railway Bridges of the American Railway Engineering Association. At best, laboratory tests are only approximate indications of quality and behavior. As a final thought, it seems fitting to state that, all other factors being equal, the best structure

is that having the greatest mass. This final observation applies to the usual structure, and does not, of course, apply to movable bridges or to bridges of extremely long spans.

W. L. WARNER,[124] ESQ. (by letter).[124a]—Because of the fundamental metallurgical welding principles which are stated therein, the paper by Messrs. Bain and Llewellyn is of interest not only to the structural engineer and designer, but also to the welding engineer and metallurgist. From the welding standpoint it is believed that no criticism can be made. However, there are several statements in the paper to which attention should be directed.

In discussing the effect of carbon it is stated (see heading, "Characteristics of Individual Alloying Elements: Carbon") that "long experience indicates that at about 0.20% to 0.25% (with comparatively low content of other elements), a broadly applicable optimum is reached for structural carbon steels." This value of 0.25% carbon is the limit which has been set for alloy steel plate material used at the Watertown Arsenal, at Watertown, Mass., in building welded structures for gun carriages as determined experimentally from welding tests.

In the same discussion, the following statements should be emphasized as of particular interest to those concerned with the fabrication of welded structures:

"Furthermore, even with a minor content of other elements, carbon exerts a definite influence toward hardenability; only in its presence, and in proportion to its concentration, are other elements able to exert substantial influences toward hardenability. * * * However, where welding is to be utilized, the usual structural steel derives about all the strength that carbon alone can provide without excessively restricting other valuable properties."

The latter statement means simply that higher strength should be obtained by adding alloy elements rather than carbon when a weldable higher tensile steel is desired.

Another point in connection with the composition of weldable structural alloy steels is brought out by the following statement in the paper by Messrs. Bain and Llewellyn (see heading, "Complex Steels: Simultaneous Addition of Several Elements"): "A fortunate circumstance is that the concurrent use of moderate quantities of several elements appears to produce a more favorable combination of properties than would result from a single element in an amount sufficient to produce an equivalent strength increase." At Watertown Arsenal, it has been found, for example, that in welding structural nickel steel plate containing approximately $3\frac{1}{2}$% of nickel, better physical properties are obtained by using a combination of nickel and molybdenum in the electrode than by using even larger percentages of either of these two elements alone.

In the discussion of carbon content of the steels shown in Table 3, the following statement is made: "At the same time wherever welding is not used, or in cases amenable to stress-relief annealing following welding, the

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[124] Welding Engr., Watertown Arsenal, Watertown, Mass.

[124a] Received by the Secretary December 4, 1936.

higher carbon steels are applicable, and they may possess still higher strength." In this connection, it is desired to inject a word of caution that, even when stress-relief annealing is used, this treatment cannot prevent trouble from cracks due to hardness adjacent to the weld during the period between the time when the welding is done and the moment when the heat is turned on in the annealing furnace. Sometimes, this interval may be as much as ten days or two weeks. Therefore, even if the heat treatment is practiced, a carbon content of more than 0.25% in an alloy steel makes the freedom from cracks an uncertain proposition unless some preheating is done before welding.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

ANALYSIS OF VIERENDEEL TRUSSES

Discussion

BY JOHN E. GOLDBERG, JUN. AM. SOC. C. E.

JOHN E. GOLDBERG,[33] JUN. AM. SOC. C. E. (by letter).[33a]—Slightly complex though it may at first appear, it is unlikely that a formular method for the exact analysis of the general case of open-panel or Vierendeel truss can be simplified to any greater extent than the fine method presented by Professor Young in his creditable paper Some simplification is possible, however, when cautious use is made of assumptions and approximations. Simplification is also possible in the analysis of special cases, although it may be necessary or desirable to abandon the general method of consistent deflections in favor of some special theory

Using a totally different method of attack, a special slope deflection method developed by the writer in 1933 for the analysis of symmetrical, parallel chord bents and trusses, a perfect check of Professor Young's Example 1, the frame shown in Fig. 3, was obtained. The basis of the writer's method is an exact and easily set-up working equation which gives the rotation, θ_n, of the nth panel points in terms of the rotations of the two adjacent panel points:

$$\theta_n \left(6 \frac{I_n}{D} + \frac{I'_{mn}}{L_{mn}} + \frac{I'_{no}}{L_{no}} \right) = \frac{V_{mn}\, L_{mn} + V_{no}\, L_{no}}{2} + \frac{I'_{mn}}{L_{mn}}\, \theta_m + \frac{I'_{no}}{L_{no}}\, \theta_o \ldots (148)$$

In actual use, an equation of this type is set up for each panel point along one chord, the resulting simultaneous equations then being solved by algebraic methods or by successive approximations. When successive approximations are used, the equations are reduced to the form:

$$\theta_n = A + B\,\theta_m + C\,\theta_o \ldots\ldots\ldots\ldots\ldots\ldots\ldots (149)$$

For Points 1 and 2 (Fig. 3) Equation (148) yields:

$$\theta_1\,(6 + 1) = \frac{1.5\,(10)}{2} + 1\,(\theta_2) \ldots\ldots\ldots\ldots\ldots (150)$$

NOTE.—The paper by Dana Young, Assoc. M. Am. Soc. C. E., was published in August, 1936, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1936, by Messrs. L. J. Mensch, A. A. Eremin, Leon Blog, A. W. Fischer, and L. C. Maugh.

[33] With Dept. of Buildings, Chicago, Ill.

[33a] Received by the Secretary November 23, 1936.

and,

$$\theta_2 (6 + 1 + 1) = \frac{1.5 (10) + 0.5 (10)}{2} + 1 (\theta_1) \quad \text{.........(151)}$$

in which the shears are in kips. The simultaneous solution of Equations (150) and (151) (although the method of successive approximations using the form of Equation (149) would have been practically as easy) gives: $\theta_1 = 1.272$; and $\theta_2 = 1.409$. Final chord and cross-member moments are obtained by substituting the θ-values in the appropriate equations. For the moment at the end, n, of the chord member, n-m:

$$M_{nm} = \frac{V_{nm} L_{nm}}{4} + \frac{I'_{nm}}{L_{nm}} \frac{(\theta_m - \theta_n)}{2} \quad \text{..............(152)}$$

and, for the moment at either end of the nth cross-member:

$$M_n = -3 \frac{I_n}{D} (\theta_n) \quad \text{......................(153)}$$

Thus, by Equation (153), the moment at each end of the first cross-member is $-3(1.272) = -3.816$ kip-ft, and at each end of the second cross-member, $-3(1.409) = -4.227$ kip-ft. Respective shears are, therefore, $\frac{2\ (3.816)}{10} = 0.763$ kips, and $\frac{2\ (4.227)}{10} = 0.845$ kips, which check exactly Professor Young's calculations.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FLOOD PROTECTION DATA

PROGRESS REPORT OF THE COMMITTEE

Discussion

BY MESSRS. CHARLES D. CURRAN, AND EDWARD N. WHITNEY

CHARLES D. CURRAN,[18] ASSOC. M. AM. SOC. C. E. (by letter).[18a]—The five recommendations of the Committee should receive endorsement from all members of the profession active in flood-damage prevention. Preferably, the program outlined should be expanded rather than even slightly curtailed.

Although the basic data already gathered are in need of tabulation, evaluation, co-ordination, and especially interpretation, there should be no slacking in the accumulation of data. Discontinuance of established stream-gaging and rainfall stations should be discouraged. Eight or ten stations in Northern Vermont were discontinued during the decade preceding the disastrous flood of November, 1927. These stations have since been re-established. However, the value of the intermittent records, with their particular gap, is much less than it might have been. Indeed, more stations should be established. On October 6, 1932, what is said to have been the most disastrous flood known occurred at Margaretville and Arkville, N. Y., on the upper East Branch of the Delaware River. No records of discharge in the immediate vicinity exist. Other records in the Delaware Valley show a high discharge at that time, but do not indicate that the flood was unprecedented in any community. It is true that the area damaged was slight and no national catastrophe occurred. The important point is that present-day gaging records do not yet give a complete picture even on the major waterways.

The United States Geological Survey cannot expand to gather more data without a great increase in personnel and expenditures. Additional precipitation and stream-gages could be established by local municipal governments.

NOTE.—The Progress Report of the Committee on Flood Protection Data was presented at the Annual Meeting, New York, N. Y., January 15, 1936, and published in February, 1936, *Proceedings*. Discussion on the report has appeared in *Proceedings*, as follows: April, 1936, by Messrs. Robert Follansbee, and LeRoy K. Sherman; August, 1936, by C. R. Pettis, M. Am. Soc. C. E.; September, 1936, by Messrs. John C. Hoyt, and C. S. Jarvis; and November, 1936, by Messrs. Gordon W. Williams, Merrill Bernard, and Glenn W. Holmes.

[18] Asst. Engr., 1st Dist., U. S. Engr. Office, New York, N. Y.

[18a] Received by the Secretary November 5, 1936.

These gages of value both to the locality in the study of its problems and to engineers in general in the study of water resources could be inspected and checked occasionally by a Federal advisory organization. Records could be transmitted to the U. S. Geological Survey for incorporation with all records in the annual *Water Supply Papers.*

Amateur observers have done considerable for astronomy. It seems probable that, were amateur observations of flood flows encouraged, additional information on the propagation of the flood wave, its rate of travel, change of surface slope during floods, and many other hydrologic phenomena would be gathered at times when the hydraulic engineer is otherwise occupied either in directing protective work or inspecting control works.

A search for records of historic floods and an inventory of them is certainly needed, since the only available information on many streams is a short-time record of a U. S. Geological Survey station. This is especially true of regions where the population is transient and industry is recreational.

Probably one of the most important problems and the one about which the least is known is that of the relationship of damages to benefits and of the economic worth of flood protection. The relationship of damages to benefits is disputable. There may be no measurable relationship. On the other hand, it may be that, with little error, direct damages can be used as a measure of indirect damage and then as a measure of benefits from protection. The factors involved are many, and possibly no widely accepted solution will be found. However, as an approach to a solution, direct damages can be measured and classified. Present tendencies are to exaggerate direct damages by the erroneous inclusion of all storm and miscellaneous damage with flood damage. When the rain and wind of a storm ruin a cotton crop, the value of the lost cotton should not be included in the figures for damages caused by an accompanying flood. Highway damage caused by frost boils and discovered at the time of a thaw and flood should not be added to the flood damage. Furthermore, is damage to unprotected highway fills that encroach on a river channel properly chargeable to flood damage or to highway maintenance? What is the flood damage when a condemned bridge is washed away?

With direct damages resulting from the flood waters accurately listed, there should again be a division to classify them as having been caused either by the intense run-off approaching a stream, or by the inundation due to the overflow of water from the stream. From this classification it may be determined whether drainage, run-off retardation, or stream regulation is the most urgent corrective measure needed from an economic viewpoint.

Indirect damages to the immediate locality should be carefully considered. Loss of business to a food store is a damage; but how serious is the loss of a week's business to a clothier, or to a hardware merchant, when the business is merely delayed? Possibly the latter gains by a flood. While considering the "indirect damages" to be omitted, one should remember to include the immeasurable damage to foundations by the operation of several hundred pumps trying to empty cellars for a week.

General indirect damages are not readily measurable. They are almost as vague as benefits when an effort is made to interpret them in terms of dollars and cents. However, an accurate measure of direct damages will be an approach to the problem's solution.

Having determined the damages of a given flood, or the benefits of its prevention, the next problem is to determine a value of annual average damages or annual average benefits. Two methods at present widely used are both faulty. The one involves adding the damages of all known floods and dividing this total by the number of years during which they occurred. The result is the average annual damage. A value for benefits could be used instead of damages, or a multiplier of two or three could be applied to damages to obtain benefits. The multiplier is certainly open to criticism. The other method is to develop value for the probable annual damage based on flood frequencies derived from probability treatment of observed discharges and gage heights. This places great weight on the determination of the 1% (100-yr) and less frequent floods. A little consideration will show that neither method is a proper one from which to obtain a value for capitilization and determination of allowable expenditure; but what is a better method? The sooner an economic research is begun, the sooner the engineer may expect a better indication as to how much flood control is worth.

Edward N. Whitney,[19] Assoc. M. Am. Soc. C. E. (by letter).[19a]—The recommendations in this report appear sound and complete as of the date published. It is believed that the following discussion is appropriate since it touches on Recommendation (3): "Make an inventory of cloudburst floods, their causes and frequency of occurrence, designating the areas in the United States chiefly affected and those rarely affected, or apparently immune."

It is suggested that, due to later developments, another recommendation be made, namely, that one agency of the Federal Government either be directed to co-operate with the Miami Conservancy District, Dayton, Ohio, in its storm studies, or be given the responsibility of continuing the plotting and study of great storms that has been done so well in the past by the Engineering Staff of that District.

It is understood (October, 1936) that the Miami Conservancy District does not intend to continue its storm studies in the near future, since the publication of the revision of Part V, of its Technical Report, in 1936. It is hoped that the one office designated to carry on this important work (possibly one office of the United States Army Engineers, since this organization is now charged with the duty of preparing flood-control reports) will adopt the same methods used by the District, so that comparisons can be made with past storms.

The District states in Part V (1936) of its Technical Report that it will supply, at cost of reproduction, copies of its detailed storm maps and time-area-depth computations. The District studied storms only of the 103d Meridian, about the western boundary of Nebraska. It is hoped that the

[19] Cons. Engr., Baltimore, Md.

[19a] Received by the Secretary November 25, 1936.

Government office taking over the work for the entire United States will also supply the results of its work to any one needing them.

At this time (November, 1936) the great storm of September 2 to 6, 1935, which centered on the eastern shore of Maryland, is being drawn up according to the Miami method in the U. S. Engineer Office of the War Department at Baltimore, Md. From preliminary data it is believed that this storm gave the greatest depth of rainfall for a three-day storm ever recorded in Maryland, or in any State farther north in this region, for areas as great as 10 000 sq miles. Therefore, it is a very important storm, to be studied in the preparation of flood-control reports for Maryland and neighboring States. This storm does not appear in Vol. V (1936) of the Technical Reports of the Miami Conservancy District. This advance information is given so that other offices or persons interested will not start duplicating this tedious work with attendant waste of public or private funds.

The following data are offered as an example of the practical use of publications of the U. S. Weather Bureau, the *Water Supply Papers* of the U. S. Geological Survey, and the Technical Reports of the Miami Conservancy District, in making estimates of the possible and probable peak flows of the Susquehanna River at Towanda and Wilkes-Barre, Pa. (Data showing the results from four superimposed storms, with accompanying explanation, have been placed on file with the Chairman of the Committee.) The study relates to the work of the Research Committees of the Society, on Flood Protection Data and on Meteorological Data. The town of Towanda is one of the eight stations in the United States for which distribution hydrographs have been computed and published by the U. S. Geological Survey.[20]

Shifting certain large storms or placing synthetic storms on to the drainage area above Towanda gives estimates of possible maximum flood flows. It is believed that these possible discharges should be made known to engineers and the general public as soon as they are estimated reliably.

It is not known at present what the gage height would be at Towanda or at Wilkes-Barre corresponding to these discharges. The gage heights should be, and doubtless will be, determined during the preparation of complete flood-control reports.

The frequency of these maximum floods likewise is not known. However, there are many people who prefer to live, and many business administrators who prefer to be located, above any possible maximum flood. They would be glad to know, and are entitled to know, what this maximum flood elevation is. It is recommended that one of the obligations placed on any organization charged with flood-control studies be that these maximum possible flood quantities and elevations be published in the local press and that the elevations be plainly marked in the field as soon as they are known definitely, for the benefit of all who pay the cost of the flood-control studies.

This is a type of flood warning designed to save not only lives and property, but the expense of flood-protection works desired by owners to protect their property located in an area later found to be a danger zone.

[20] "Rainfall and Run-Off in the United States", *Water Supply Paper No. 772*, U. S. Geological Survey.